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COMMENTS ON TECHNICOLOUR MODEL BUILDING

Thomas Appelquist and John Terning*
Department of Physics, Yale University
New Haven, CT 06511, U.S.A.

ABSTRACT
We discuss constraints on extended technicolour model building, mechanisms for producing small neutrino masses, and develop a potentially realistic model.

1. Introduction

Recent work[1,2] shows that technicolour (TC) theories can be made compatible with the observed particle mass spectrum (with ρ close to 1 and without excessive flavor changing neutral currents (FCNC's)). Though such an exercise is interesting as a sort of existence proof, one worries that since there are more parameters (gauge groups, gauge couplings, four-fermion couplings, etc.) than observables it is impossible to tell whether this success is the result of having the qualitatively correct physics or merely the power of parameter fitting.

To do better we have to construct models with fewer parameters than the standard model. Such models should make testable predictions for physics below 100 GeV, and have the potential to be ruled out by experiment. In principle an extended technicolour (ETC) model would fit this bill. In reality it is unlikely that we would be able to accurately analyze the non-perturbative dynamics involved, so we might be forced to parameterize some of our ignorance. Even so, we could still hope to have fewer parameters than observables, and hence a testable model.

2. Constraints on Model Building

There are several constraints that one might like to impose on a realistic ETC model. First of all, we expect that there should be more than one ETC scale. The absence of FCNC's requires that the mass of the gauge boson that connects the s quark to technifermions be at least 200 TeV, and that the mass of the gauge boson that connects the s quark to the d quark be at least 1000 TeV. On the other hand, to obtain a t quark mass above 100 GeV we expect an ETC scale of order 5 TeV. Such arguments suggest three different ETC scales, one for each family.

Another constraint arises from trying to obtain a large t-b mass splitting, while keeping ρ within 0.5% of 1. To do this without fine tuning the simplest possibility is to require that the right-handed t and right-handed b be in different reps

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of the ETC group ($SU(2)_L$ gauge invariance requires the left-handed t and b to be in the same representation).

The S parameter can also be worrisome, since experiments seem to be finding S to be around -1 ± 1, whereas QCD-like TC theories have large positive contributions to S (which grow with the number of technicolours (N_{TC})). Of course, S could be quite different in non-QCD-like theories.[3] However, the constraint on the S parameter seems to suggest that N_{TC} should be kept as small as possible, i.e. $N_{TC} = 2$. Recently Sundrum[4] has shown that vacuum alignment need not be a problem for $SU(2)_{TC}$.

Eichten and Lane,[5] have shown that the absence of a visible axion implies a limit on the number of spontaneously broken global U(1) symmetries. This led them to require quark-lepton unification in ETC models. In general, a realistic ETC theory should not have any exact, spontaneously broken, global symmetries, since this will lead to massless Goldstone bosons. Thus we cannot have repeated reps of the ETC gauge group.

Finally an ETC theory must explain why neutrinos are so light.

3. A Word About MAC

The established folklore states that fermion condensates occur in the most attractive channel (MAC),[6] which is determined by one gauge boson exchange. Peskin[6] has shown that when the gauge symmetry is unbroken, the MAC analysis is equivalent to minimizing an effective potential truncated at two loops. However, when the gauge bosons do acquire masses from the fermion condensate there are pure gauge boson contributions to the effective potential which have not yet been considered.

Since we have no reliable theoretical guide for which fermion condensates actually form, we will take a more phenomenological approach, i.e. we will allow condensates that are consistent with producing the standard model as a low energy effective theory.

4. Why Are Neutrinos Light?

A simple explanation for the fact that neutrinos (ν's) are extremely light was originally pointed out by Sikivie et. al.[7] Consider the following left-handed and charge-conjugated right-handed fermions labeled with their $SU(3)_{ETC} \otimes SU(2)_L \otimes U(1)_Y$ charges:

$$\begin{aligned} E_L, \tau_L; N_L, \nu_{\tau L} &\sim (\mathbf{3}, \mathbf{2})_{-1} \\ E^c_R, \tau^c_R &\sim (\overline{\mathbf{3}}, \mathbf{1})_2 \\ N^c_R, \nu^c_R &\sim (\overline{\mathbf{3}}, \mathbf{1})_0 \\ N^c_{1R}, \nu^c_{1R} &\sim (\overline{\mathbf{3}}, \mathbf{1})_0 \\ N^c_{2R}, \nu^c_{2R} &\sim (\mathbf{3}, \mathbf{1})_0 \ , \end{aligned}$$

where a **3** of $SU(3)_{ETC}$ corresponds to two technicolours and one family. Note that there are two extra sets of N_R^c's and ν_R^c's in conjugate reps. If (N_R^c,ν_R^c) condenses with (N_{1R}^c,ν_{1R}^c) then $SU(3)_{ETC}$ breaks down to $SU(2)_{TC}$, and it is (N_{2R}^c,ν_{2R}^c) which survive to be the partners of (N_L, $\nu_{\tau L}$). Now the usual one-ETC gauge boson exchange graph that feeds down masses to the τ will not give a mass to the ν_τ, since this graph is identically zero. To feed down a mass to the ν_τ there must be some mixing between different ETC gauge bosons. If the required mixing can only be produced by a loop containing ν's and N's, and we now consider the low-energy effective theory where the heavy ETC gauge bosons and N's have been integrated out, then we see that the diagram for the neutrino mass consists of a four-ν interaction vertex with two lines closed off by a mass insertion, a form familiar from the Nambu–Jona-Lasinio gap equation. Thus if the effective four-ν coupling constant is sub-critical, the ν_τ stays massless. However, if there are other fermions that can produce the mixing, then the mass of the ν_τ will be suppressed relative to the mass of the τ by a factor of the mixing mass squared over the ETC gauge boson mass squared.

5. A Recipe for an ETC Model

We will now proceed to construct an ETC model. We will assume that the TC group is $SU(2)_{TC}$, and that there is one family of technifermions. We will require that: 1) there are no exact non-Abelian global symmetries, 2) quarks and leptons are unified, 3) fermions appear only in antisymmetric irreps, (then the model only contains **3**'s and $\overline{\mathbf{3}}$'s of colour.[8]) 4) all gauge anomalies vanish. With these requirements we can proceed rather straightforwardly. We will gauge as many symmetries as possible without inducing proton decay; this can be done by putting all 20 $SU(2)_L$ doublets in a single representation. A search for the smallest irrep of the smallest gauge group produces the **36** of $SU(9)$. The simplest way to include the charge-conjugated right-handed fermions is to have two $\overline{\mathbf{36}}$'s, but this fermion content by itself would lead to no isospin breaking, and no mixing angles in the CKM matrix. This disaster can be avoided if we also add in extra fermions that can mix with some components of the $\overline{\mathbf{36}}$'s. The remaining antisymmetric irreps of $SU(9)$ are: the **9**, the **84**, and the **126**. The **9** is too small to be interesting, so we can add an **84** and an $\overline{\mathbf{84}}$ or a **126** and a $\overline{\mathbf{126}}$.

We can now explicitly write down an ETC model. The gauge group is $SU(9) \otimes SU(2)_L \otimes U(1)_R$, and we will take the fermion content to be:

$$\begin{array}{cc} (\mathbf{36},\mathbf{2})_0 & (\mathbf{126},\mathbf{1})_0 \\ (\overline{\mathbf{36}},\mathbf{1})_{-1}(\overline{\mathbf{36}},\mathbf{1})_1 & (\overline{\mathbf{126}},\mathbf{1})_0 \ . \end{array}$$

The remaining content of the model lies in specifying the pattern of symmetry breaking. As we have mentioned before we are forced to proceed phenomenologicaly, so without further adue we assume that a condensate in the channel $(\overline{\mathbf{36}},1)_1 \times (\overline{\mathbf{36}},1)_{-1} \to (\overline{\mathbf{126}},1,)_0$ forms around 10^4 TeV. Below 10^4TeV, the gauge symmetry is then broken down to $SU(5)_{ETC} \otimes SU(4)_{PS} \otimes SU(2)_L \otimes U(1)_R$, where PS denotes the usual Pati-Salam group.

The fermion content of the theory below 10^4 TeV is:

$$\begin{array}{cccc} & (\mathbf{5},\mathbf{4},\mathbf{2})_0 & (\mathbf{10},\mathbf{1},\mathbf{2})_0 & (\mathbf{1},\mathbf{6},\mathbf{2})_0 \\ (\overline{\mathbf{5}},\overline{\mathbf{4}},\mathbf{1})_{-1} & (\overline{\mathbf{5}},\overline{\mathbf{4}},\mathbf{1})_1 & (\overline{\mathbf{10}},\mathbf{1},\mathbf{1})_{-1}(\overline{\mathbf{10}},\mathbf{1},\mathbf{1})_1 & \\ & (\mathbf{5},\overline{\mathbf{4}},\mathbf{1})_0 & (\mathbf{10},\mathbf{6},\mathbf{1})_0(\overline{\mathbf{10}},\mathbf{4},\mathbf{1})_0 & (\overline{\mathbf{5}},\mathbf{1},\mathbf{1})_0(\mathbf{1},\mathbf{1},\mathbf{1})_0 \\ & (\overline{\mathbf{5}},\mathbf{4},\mathbf{1})_0 & (\overline{\mathbf{10}},\mathbf{6},\mathbf{1})_0(\mathbf{10},\overline{\mathbf{4}},\mathbf{1})_0 & (\mathbf{5},\mathbf{1},\mathbf{1})_0(\mathbf{1},\mathbf{1},\mathbf{1})_0 \ , \end{array}$$

where the first two lines correspond to the $\mathbf{36}$ and $\overline{\mathbf{36}}$'s respectively. Next we imagine that a condensate forms in the channel $(\mathbf{10},\mathbf{6},\mathbf{1})_0 \times (\overline{\mathbf{5}},\overline{\mathbf{4}},\mathbf{1})_1 \to (\mathbf{5},\mathbf{4},\mathbf{1})_1$ around 1000 TeV. This breaks the gauge symmetry down to $SU(4)_{ETC} \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. We can now see that the $(\mathbf{5},\mathbf{4},\mathbf{2})_0$, the $(\overline{\mathbf{5}},\overline{\mathbf{4}},\mathbf{1})_{-1}$, and the $(\overline{\mathbf{5}},\overline{\mathbf{4}},\mathbf{1})_{-1}$ contain particles with quantum numbers corresponding to three families of ordinary fermions (plus ν_R's) and one family of technifermions. Note that the first family splits off at this scale. We can also see why ν^c_R's and right-handed down-type quarks (d^c_R's) are special in this model. Singlets under $SU(4)_{PS} \otimes SU(2)_L \otimes U(1)_R$ will have the standard model quantum numbers of ν_R's, while particles that transform as $(\mathbf{6},\mathbf{1})_0$ under $SU(4)_{PS} \otimes SU(2)_L \otimes U(1)_R$ will split into particles with standard model quantum numbers $(\overline{\mathbf{3}},\mathbf{1})_{2/3}$ and $(\mathbf{3},\mathbf{1})_{-2/3}$ which correspond respectively to d^c_R's and an exotic quark which can obtain a mass with a d^c_R that is gauge invariant under $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. In fact the condensate which forms at 1000 TeV generates just such a mass connecting a $(\overline{\mathbf{4}},\overline{\mathbf{3}},\mathbf{1})_{2/3}$ with a $a(\mathbf{4},\mathbf{3},\mathbf{1})_{-2/3}$. Thus there will be extra mixing available for ν^c_R's and d^c_R's.

The next stage of breaking is taken to occur around 100 TeV, where a condensate forms in the channel $(\mathbf{4},\mathbf{1},\mathbf{2})_0 \times (\mathbf{6},\mathbf{1},\mathbf{2})_0 \to (\overline{\mathbf{4}},\mathbf{1},\mathbf{1})_0$, which breaks the gauge symmetry down to $SU(3)_{ETC} \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. The second family splits off at this scale. Once all the group decompositions are performed we see that we have the correct fermion content to perform the trick discussed in section 4, not only for ν^c_R's, but also for the d^c_R's. Thus at the lowest ETC scale, around 10 TeV, we have condensates forming in the following channels: $(\overline{\mathbf{3}},\overline{\mathbf{3}},\mathbf{1})_{2/3} \times (\overline{\mathbf{3}},\mathbf{3},1)_{-2/3} \to (\mathbf{3},\mathbf{1},\mathbf{1})_0$ (i.e. "d^c_R"'s), $(\overline{\mathbf{3}},\mathbf{1},\mathbf{1})_0 \times (\overline{\mathbf{3}},\mathbf{1},\mathbf{1})_0 \to (\mathbf{3},\mathbf{1},\mathbf{1})_0$ (i.e. "ν^c_R"'s), and $(\mathbf{3},\mathbf{1},\mathbf{2})_0 \times (\mathbf{3},\mathbf{1},\mathbf{2})_0 \to (\overline{\mathbf{3}},\mathbf{1},\mathbf{1})_0$. This breaks the gauge symmetry down to $SU(2)_{TC} \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$, and we have a one family TC theory.

It should be noted that the mechanisms for generating the masses of the t and the b are totally different. The t gets its mass through the standard one ETC gauge boson exchange, while the b mass is suppressed by the trick of Sikivie et. al., also the b^c_R mixes with exotic quarks. Thus this model has no problem accomodating a large t-b mass splitting, but it still remains to explain the t-τ mass splitting. As with the first two families, we can expect about a factor of 10 enhancement from QCD and walking effects.[9] The remaining factor of 10 ehancement could arise from a 10% "fine" tuning of four-fermion couplings in the effective theory below 10 TeV.[2]

We also note that our choice of fermion content allows us to implement the Nelson-Barr solution to the strong-CP problem.[10] The fundamental theory is assumed to be CP conserving. If CP is spontaneously broken by complex phases in the masses of particles coming from the $\mathbf{126}$, then CP violating phases will appear in the CKM matrix, but the determinant of the mass matrix can be real, so the effective strong CP violating parameter $\overline{\theta}$ is identically zero.

6. Conclusions

We have constructed a potentially realistic (i.e. not obviously wrong) ETC model, which incorporates all the right ingredients: $m_t \gg m_b$, $m_\nu \approx 0$, a family hierarchy, no bad FCNC's, no massless techniaxion, and no strong CP problem. It remains to be seen whether this model can survive a more detailed scrutiny, in particular whether it can produce the observed masses and mixing angles.

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8. References

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