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β Functions of Orbifold Theories and the Hierarchy Problem

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Abstract

We examine a class of gauge theories obtained by projecting out certain fields from an $\mathcal{N} = 4$ supersymmetric $SU(N)$ gauge theory. These theories are non-supersymmetric and in the large N limit are known to be conformal. Recently it was proposed that the hierarchy problem could be solved by embedding the standard model in a theory of this kind with finite N . In order to check this claim one must find the conformal points of the theory. To do this we calculate the one-loop β functions for the Yukawa and quartic scalar couplings. We find that with the β functions set to zero the one-loop quadratic divergences are not canceled at sub-leading order in N ; thus the hierarchy between the weak scale and the Planck scale is not stabilized unless N is of the order 10^{28} or larger. We also find that at sub-leading orders in N renormalization induces new interactions, which were not present in the original Lagrangian.

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1 Introduction

The study of conformal symmetry has a long history in particle physics. Recently it has attracted renewed interest due to the work of Maldacena [1] on the correspondence between string theory on anti-de Sitter backgrounds and four dimensional conformal field theories, and further work on the orbifold projections of these theories [2-12]. An interesting result of this work [2-6] is that non-supersymmetric gauge theories obtained by orbifolding an $\mathcal{N} = 4$ SUSY $SU(N)$ gauge theory are conformal in the large N limit. Additional non-supersymmetric conformal theories can be obtained from a similar construction in type 0 string theories [13, 14]. Although conformal theories are seemingly quite esoteric, the idea of using static or slowly running couplings to generate a large hierarchy of scales has cropped up many times in particle phenomenology. Attempts to use approximate conformal symmetry in phenomenology have included such diverse topics as: electroweak symmetry breaking (walking technicolor) [15, 16, 17], the hunt for light composite scalars [16, 18, 19] (including the search for a Goldstone boson of spontaneously broken scale invariance* [18]), dynamical supersymmetry breaking [21], and the cosmological constant problem [22]. Most recently Frampton and Vafa [11, 12] have conjectured that orbifold theories are conformal at finite N , and further proposed that embedding the standard model in an orbifold theory can solve the naturalness problem of the electroweak scale (stabilizing the large hierarchy of scales without fine-tuning). This sudden appearance of such a simple solution to a long standing problem is quite surprising, so it seems worthwhile to discuss the underlying ideas of this scenario in some detail.

It has been previously noted [23] that conformal symmetry can remove the quadratic divergences that are responsible for destabilizing the hierarchy between the weak scale and a more fundamental scale like the Planck scale. In a conformal theory we must insist on regulators (like dimensional regularization) that respect conformal invariance or include counterterms that maintain the symmetry. With such a regularization quadratic divergences are impossible (since there is no cutoff scale on which they could depend). Such a resolution of the naturalness problem is of course only valid if the theory is exactly conformal (i.e. physics is the same at any length scale). In the

*The relation between scale invariance and conformal invariance is discussed in Ref. [20].

real world we know that physics is not conformal below the weak scale, and we expect that the fundamental theory of gravity will not be scale invariant since gravity has an intrinsic scale associated with it. Thus the best we can hope for phenomenologically is a theory that is approximately scale invariant in some energy range. That is we can only have an effective conformal theory that is valid above some infrared cutoff (which must be above the weak scale) and below some ultraviolet (UV) cutoff M (which must be at or below the Planck scale). From the perspective of the fundamental theory there is some non-conformal physics above (or near) the scale M (e.g. heavy particles or massive string modes) which we can integrate out of the theory. Studying the sensitivity of the effective theory to the cutoff M is equivalent to studying sensitivity of the low-energy physics to the details of the very high-energy physics. If we believe that there is a new fundamental scale of physics beyond the weak scale then in a “natural theory” we would like to see that the weak scale is not quadratically sensitive to changes in the high scale. The two known solutions to the naturalness problem are to either lower the UV cutoff of the effective theory to the weak scale (e.g. technicolor, large extra dimensions) or to arrange cancelations of the quadratic divergences order-by-order in perturbation theory (e.g. supersymmetry). One might expect that an effective conformal theory would fall into the latter category, however the vanishing of β functions does not imply the cancellation of quadratic divergences, they are independent [23]. To see that they are independent one need only consider supersymmetric theories where quadratic divergences cancel independently of the values of β functions.

In this paper we consider a class of $\mathcal{N} = 4$ orbifold theories [11, 12] at one loop. We explicitly calculate the β functions, solve for the couplings by imposing that the β functions vanish, and calculate the quadratic divergences. We find that the quadratic divergences do not cancel for finite N . We also discuss new interactions that are induced by renormalization group (RG) running, and remark on some open questions.

2 The Orbifold Theories

In this section we review the construction of $\mathcal{N} = 4$ orbifold theories, and present the matter content and Lagrangian for the particular models that we will be considering in this paper.

One starts with an $\mathcal{N} = 4$ supersymmetric $SU(N)$ gauge theory. The field content of this theory is (all fields are in the adjoint representation): gauge bosons A_μ , which are singlets of the $SU(4)_R$ global symmetry, four copies of (two-component) Weyl fermions $\Psi^i, i = 1, 2, 3, 4$, which transform as the fundamental $\mathbf{4}$ under the $SU(4)_R$, and six copies of (real) scalars Φ^{ij} which transform as the antisymmetric tensor $\mathbf{6}$ of $SU(4)_R$. In the procedure of orbifolding (discussed in detail in Refs. [2-10]) one chooses a discrete subgroup Γ of the $SU(4)_R$ symmetry of order $|\Gamma|$, and also embeds this subgroup into the gauge group (chosen here to be $SU(N|\Gamma|)$) as N copies of its regular representation (for a very clear explanation of this embedding see Ref. [9]). Orbifolding then means projecting out all fields from the theory which are not invariant under the action of the discrete group Γ . If Γ is a generic subgroup of $SU(4)_R$, then one obtains a non-supersymmetric theory. If Γ is embedded in an $SU(3)$ subgroup of $SU(4)_R$ then one obtains an $\mathcal{N} = 1$ supersymmetric theory, while if Γ is embedded in an $SU(2)$ subgroup of $SU(4)_R$ one obtains an $\mathcal{N} = 2$ supersymmetric theory. For a compilation of results on discrete subgroups of $SU(3)$ and $SU(4)$ see Refs. [24] and [25]. We are interested only in the non-supersymmetric theories, in which case Γ must be a subgroup of $SU(4)$. In order to simplify the analysis of the β functions, we restrict our attention in this paper to the case when Γ is Abelian, $\Gamma = Z_k$. In this case we start with an $SU(Nk)$ gauge group, and after orbifolding we obtain an $SU(N)^k$ theory.

Let us denote the k -th root of unity $e^{\frac{2\pi i}{k}}$ by ω . An embedding of Z_k into $SU(4)_R$ is specified by the transformation properties of the fundamental representation: $\mathbf{4} \rightarrow \text{diag}(\omega^{k_1}, \omega^{k_2}, \omega^{k_3}, \omega^{k_4}) \mathbf{4}$. This embedding is an $SU(4)$ subgroup if $k_1 + k_2 + k_3 + k_4 = 0 \pmod k$ (in order to insure that the determinant is one), moreover $k_1, k_2, k_3, k_4 \neq 0 \pmod k$ so that we obtain a non-supersymmetric theory. In order to simplify our calculations, we will assume in this paper that no two k_i 's are equal, and also that $k_i + k_j \neq 0 \pmod k$. With the assumption that $k_i + k_j \neq 0 \pmod k$ one can avoid the presence of adjoint scalars, and thus all fermions and scalars will be in bifundamental representations. The assumption that no two k_i 's are equal implies that there is only a single field with given gauge quantum numbers. This is probably the simplest and most symmetric orbifold theory that one can consider. However, we believe that the conclusions we draw from these particular orbifolds could be generalized to more complicated embeddings.

With this choice of embedding of the discrete group we get the following field content for our orbifold theory:

- gauge bosons A_μ for every gauge group $SU(N)$,
- (two-component) fermions $\Psi(m, \overline{m+k_i}) \equiv \Psi_{\beta_{m+i}}^{\alpha_m}$ which transform as fundamentals under the m -th $SU(N)$ factor in the $SU(N)^k$ product and as antifundamental under $m+k_i$ (m is arbitrary, $i = 1, 2, 3, 4$, and $m+i$ is a short hand for $m+k_i$),
- complex scalars $\Phi(m, \overline{m+l_i}) \equiv \Phi_{\beta_{m+i}}^{\alpha_m}$ which transform as fundamentals under the m -th group and as antifundamentals under $m+l_i$ (m is arbitrary, $l_i = k_i + k_4$ and $i = 1, 2, 3$). Note that for the scalars a different shorthand is employed, $m+i$ represents $m+k_i+k_4$.

The Lagrangian of orbifold theories is obtained from the original Lagrangian by retaining only terms containing fields invariant under the discrete symmetry. We give the $\mathcal{N} = 4$ Lagrangian in the Appendix. The Yukawa couplings in the orbifold theory are given by

$$\mathcal{L}_{Yukawa} = -Y \sum_{m,i < j} \left(\Psi_{\beta_{m+i}}^{\alpha_m} \Psi_{\gamma_p}^{\beta_{m+i}} \Phi_{\alpha_m}^{\dagger \gamma_p} + h.c. \right), \quad (2.1)$$

where in the above sum $m+i$ is again a shorthand for $m+k_i$, and $p = m+k_i+k_j$. Note that, unlike in the supersymmetric theory, there is no factor of $\sqrt{2}$ appearing in this coupling. The quartic scalar couplings are given by

$$\begin{aligned} \mathcal{L}_{\text{quart}} = & -\frac{1}{2} \sum_{m,j < i} \left[\lambda_1 \phi_{\beta_{m+i}}^{\alpha_m} \phi_{\alpha_m}^{\gamma_{m+i}} \phi_{\gamma_{m+i}}^{\delta_m} \phi_{\delta_m}^{\beta_{m+i}} - \lambda_3 \phi_{\beta_{m-i}}^{\alpha_m} \phi_{\alpha_m}^{\gamma_{m+i}} \phi_{\gamma_{m+i}}^{\delta_m} \phi_{\delta_m}^{\beta_{m-i}} \right. \\ & + \lambda_4 \left(\phi_{\beta_{m-i}}^{\alpha_m} \phi_{\alpha_m}^{\gamma_{m+j}} \phi_{\gamma_{m+j}}^{\delta_m} \phi_{\delta_m}^{\beta_{m-i}} + \phi_{\beta_{m+i}}^{\alpha_m} \phi_{\alpha_m}^{\gamma_{m-j}} \phi_{\gamma_{m-j}}^{\delta_m} \phi_{\delta_m}^{\beta_{m+i}} \right) \\ & + \lambda_5 \left(\phi_{\beta_{m+i}}^{\alpha_m} \phi_{\alpha_m}^{\gamma_{m+j}} \phi_{\gamma_{m+j}}^{\delta_m} \phi_{\delta_m}^{\beta_{m+i}} + \phi_{\beta_{m-i}}^{\alpha_m} \phi_{\alpha_m}^{\gamma_{m-j}} \phi_{\gamma_{m-j}}^{\delta_m} \phi_{\delta_m}^{\beta_{m-i}} \right) \\ & \left. - 2\lambda_2 \left(\phi_{\beta_{m-j}}^{\alpha_m} \phi_{\alpha_m}^{\gamma_{m+i}} \phi_{\gamma_{m+i}}^{\delta_{m+i-j}} \phi_{\delta_{m+i-j}}^{\beta_{m-j}} + \phi_{\beta_{m-i}}^{\alpha_m} \phi_{\alpha_m}^{\gamma_{m+j}} \phi_{\gamma_{m+j}}^{\delta_{m-i+j}} \phi_{\delta_{m-i+j}}^{\beta_{m-i}} \right) \right], \end{aligned} \quad (2.2)$$

where we have used the shorthand notation $m+i = m+l_i = m+k_i+k_4$ in the above sums. In $\mathcal{N} = 1$ language, the λ_1 , λ_3 , and λ_4 couplings are descendants of the D-terms, while the λ_2 coupling is a descendant of the superpotential term, and λ_5 receives contributions from both terms. In our

normalization λ_5 is twice the superpotential coupling minus the D-term coupling. The Lagrangian obtained by orbifolding the $\mathcal{N} = 4$ theory corresponds to “degenerate” values of couplings: $Y^2 = \lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = \lambda_5 = g^2$, where g is the gauge coupling. However, as we will see below, for these values of the couplings the β functions do not vanish. Therefore, if the theory is indeed conformal for finite N , one has to assume that there will be a different set of couplings for which all the β functions vanish. However, for generic values of the quartic scalar couplings the potential is unbounded from below, while when all couplings are identical the potential is positive definite (as guaranteed by the supersymmetry of the theory it was projected from). We will assume that the ratios of the couplings are sufficiently close to one at the zeros of the β functions so that the potential is bounded. We will see later that this is true in the large N limit.

3 The Renormalization Group Equations

To calculate the one-loop β functions we rely heavily on the work of Machacek and Vaughn [26] who summarized one-loop results and derived two-loop β functions for a general field theory. We first calculated the $\mathcal{N} = 4$ SUSY β functions for the gauge, Yukawa and quartic couplings despite the fact that they are related by supersymmetry. In order for this calculation to be useful for the non-supersymmetric orbifold theories one has to refrain from using the superfield formalism and instead deal separately with component scalar, fermion, and gauge boson fields. There is a term by term correspondence between the $\mathcal{N} = 4$ theory and the orbifolded theory in the large N limit [6]. The fact that all the β functions vanish when SUSY relations are imposed between the various couplings provides strong cross checks on the calculation.

At one-loop the gauge β function vanishes identically [11], so at one-loop the gauge coupling is a free parameter. The general one-loop β function for the Yukawa couplings is [26]:

$$(4\pi)^2 \beta_Y^a = \frac{1}{2} [Y_2^\dagger(F)Y^a + Y^a Y_2(F)] + 2Y^b Y^{\dagger a} Y^b + Y^b \text{Tr} Y^{\dagger b} Y^a - 3g_m^2 \{C_2^m(F), Y^a\} \quad (3.1)$$

where Y_{ij}^a is the Yukawa coupling of scalar a to fermions i and j ,

$$Y_2(F) = Y^{\dagger a} Y^a, \quad (3.2)$$

and $C_2^m(F)$ is the quadratic Casimir of the fermion fields transforming under the m -th gauge group. Thus the first term in Eq. (3.1) represents scalar loop corrections to the fermion legs, the second term 1PI corrections from the Yukawa interactions themselves, the third term fermion loop corrections to the scalar leg, and the last term represents gauge loop corrections to the fermion legs.

The Yukawa β function can be derived by projecting the $\mathcal{N} = 4$ result graph by graph (see the Appendix). The only changes are that $|\Gamma|N$ is replaced by N and the fermions are in bifundamental representations rather than the adjoint. Thus we find:

$$(4\pi)^2\beta_Y = 6NY^3 - 6\frac{N^2 - 1}{N}g^2Y, \quad (3.3)$$

so β_Y vanishes when

$$Y = Y_* \equiv g\sqrt{1 - \frac{1}{N^2}}. \quad (3.4)$$

Note that this result is independent of the values of the quartic scalar couplings.

In the notation of Machacek and Vaughn [26] the β function for a quartic scalar coupling at one-loop is given by

$$(4\pi)^2\beta_\lambda = \Lambda^2 - 4H + 3A + \Lambda^Y - 3\Lambda^S, \quad (3.5)$$

where Λ^2 corresponds to the 1PI contribution from the quartic interactions themselves and should not be confused with a mass scale, H corresponds to the fermion box graphs, A to the two gauge boson exchange graphs, Λ^Y to the Yukawa leg corrections, and finally Λ^S corresponds to the gauge leg corrections. The contributions to Λ^2 , H , and Λ^Y can be found by simply projecting the $\mathcal{N} = 4$ results (see the Appendix). The contributions to Λ^S can be found by noting that the scalars are bifundamentals rather than adjoints. The gauge boson exchange term, A , can be calculated by a simple manipulation of the gauge generators, which is explained in the Appendix. We find:

$$\begin{aligned} (4\pi)^2\beta_{\lambda_1} &= N(4\lambda_1^2 + \lambda_3^2 + 2\lambda_4^2 + 2\lambda_5^2 - 16Y^4 + 16\lambda_1Y^2) \\ &\quad + 3\frac{N^2 - 4}{N}g^4 - 12\frac{N^2 - 1}{N}g^2\lambda_1, \end{aligned} \quad (3.6)$$

$$(4\pi)^2\beta_{\lambda_2} = N(-2\lambda_2\lambda_4 - 2\lambda_2\lambda_5 + 8Y^4 - 16\lambda_2Y^2) + 12\frac{N^2 - 1}{N}g^2\lambda_2, \quad (3.7)$$

$$(4\pi)^2\beta_{\lambda_3} = N\left(\frac{1}{2}\lambda_3^2 - 2\lambda_1\lambda_3 + 2\lambda_4\lambda_5 - 8\lambda_3Y^2\right) + 3\frac{N^2 - 4}{2N}g^4 + 6\frac{N^2 - 1}{N}g^2\lambda_3, \quad (3.8)$$

$$(4\pi)^2\beta_{\lambda_4} = N\left(\frac{1}{2}\lambda_5^2 + 2\lambda_2^2 + \lambda_4^2 + 2\lambda_1\lambda_4 - \lambda_3\lambda_5 - 8Y^4 + 8\lambda_4Y^2\right) + 3\frac{N^2 - 4}{2N}g^4 - 6\frac{N^2 - 1}{N}g^2\lambda_4, \quad (3.9)$$

$$(4\pi)^2\beta_{\lambda_5} = N\left(\frac{1}{2}\lambda_5^2 + 2\lambda_2^2 - \lambda_3\lambda_4 + \lambda_4\lambda_5 + 2\lambda_1\lambda_5 - 8Y^4 + 8\lambda_5Y^2\right) + 3\frac{N^2 - 4}{2N}g^4 - 6\frac{N^2 - 1}{N}g^2\lambda_5. \quad (3.10)$$

Finding the general solution for the simultaneous zeroes of the β_λ functions is obviously a complicated problem, here we choose to focus on the solutions for the couplings that reduce in the large N limit to the $\mathcal{N} = 4$ SUSY fixed point, i.e. $\lambda_{i*} \rightarrow g^2$. At order $1/N^4$ there are two such solutions which are given by:

$$\begin{aligned} \lambda_{1*} &\approx g^2 \left(1 - \frac{5}{8N^2} + \frac{459}{1024N^4} + \dots\right) \\ \lambda_{2*} &\approx g^2 \left(1 - \frac{19}{16N^2} - \frac{387}{2048N^4} + \dots\right) \\ \lambda_{3*} &\approx g^2 \left(1 - \frac{7}{4N^2} - \frac{423}{512N^4} + \dots\right) \\ \lambda_{4*} &\approx g^2 \left(1 - \frac{5}{8N^2} + \frac{459}{1024N^4} + \dots\right) \\ \lambda_{5*} &\approx g^2 \left(1 - \frac{5}{8N^2} + \frac{459}{1024N^4} + \dots\right) \end{aligned} \quad (3.11)$$

and

$$\begin{aligned} \lambda_{1*} &\approx g^2 \left(1 - \frac{19}{16N^2} + \frac{225}{8192N^4} + \dots\right) \\ \lambda_{2*} &\approx g^2 \left(1 - \frac{47}{32N^2} - \frac{1467}{16384N^4} + \dots\right) \end{aligned}$$

$$\begin{aligned}
\lambda_{3*} &\approx g^2 \left(1 - \frac{5}{8N^2} - \frac{153}{4096N^4} + \dots \right) \\
\lambda_{4*} &\approx g^2 \left(1 - \frac{1}{16N^2} + \frac{5067}{8192N^4} + \dots \right) \\
\lambda_{5*} &\approx g^2 \left(1 - \frac{1}{16N^2} + \frac{5067}{8192N^4} + \dots \right)
\end{aligned} \tag{3.12}$$

We should note that the zeroes of the β functions are not true fixed points. This is because we have not included all possible quartic couplings allowed by gauge invariance, we have only included the quartic couplings that arise from the projection from the $\mathcal{N} = 4$ theory. Examples of operators that do not appear in the tree-level Lagrangian of these orbifold theories include

$$\phi_{\beta_{m+i}}^{\alpha_m} \phi_{\alpha_m}^{\beta_{m+i}} \phi_{\delta_{m+i}}^{\gamma_m} \phi_{\gamma_m}^{\delta_{m+i}} \quad \text{and} \quad \phi_{\beta_{m+i}}^{\alpha_m} \phi_{\alpha_m}^{\beta_{m+i}} \phi_{\delta_{m-i}}^{\gamma_m} \phi_{\gamma_m}^{\delta_{m-i}}. \tag{3.13}$$

Such gauge invariant operators are induced, for example, by two gauge boson exchange diagrams. In the non-supersymmetric theory there is no symmetry or non-renormalization theorem that prevents these operators from appearing via RG evolution. A full calculation would require considering all possible quartic interactions, and finding the simultaneous zeroes of all β functions. However, if the fixed point values of some of these new couplings are non-zero then, as we will see, we lose the special large N behavior of the pure projected theory.

We will proceed as follows: we assume that the effective ‘‘conformal’’ theory is embedded in a more fundamental theory at a scale M (e.g. some set of particles of mass M are integrated out of the theory at this scale), we assume that the theory has been arranged such that the β functions for Y and λ_i vanish, and that at this particular renormalization scale, M , all other quartic couplings vanish. We can then compute the proper 1PI contribution to the mass of any particular scalar. We will only keep the quadratically divergent piece.

The quadratic divergence is given by

$$m_\phi^2 = \left[N(2\lambda_1 - \lambda_3 + 2\lambda_4 + 2\lambda_5) + 3\frac{N^2 - 1}{N}g^2 - 8Y^2N \right] \int^M \frac{d^4p}{(2\pi)^4} \frac{1}{p^2}. \tag{3.14}$$

Plugging in our solutions for the zeroes of the β functions we have (to lowest

non-vanishing order in N) for both cases:

$$m_\phi^2 = \frac{3g^2}{N} \frac{M^2}{16\pi^2}. \quad (3.15)$$

Note that, as expected, the terms linear in N canceled. Thus we see that there is a technically unnatural hierarchy in this set of theories. In order to keep the scalars light a mass counterterm must be tuned, order by order, to cancel quadratic divergences. Alternatively, N has to be taken extremely large. For $m = m_{weak} \approx 1$ TeV, $M = M_{Pl} \approx 10^{18}$ GeV, $\frac{g^2}{4\pi} \approx \frac{1}{30}$ we find that one would need $N \approx 10^{28}$.

We now briefly comment on the possible effects of including other quartic operators like those displayed in Eq. (3.13). There is a contribution to Λ^2 of (3.5) of order $N^2(\lambda^{new})^2$, the contribution to A is of order g^4 (see Appendix). Thus the form of the β function is:

$$\begin{aligned} (4\pi)^2 \beta_k^{new} &= N^2 a_k^{ij} \lambda_i^{new} \lambda_j^{new} + N b_k^{ij} \lambda_i^{new} \lambda_j + c_k^{ij} \lambda_i \lambda_j + 16N \lambda_k^{new} Y^2 \\ &\quad + d_k 3 \left(1 + \frac{2}{N^2}\right) g^4 - 12 \frac{N^2 - 1}{N} g^2 \lambda_k^{new}, \end{aligned} \quad (3.16)$$

where we have taken the coupling λ_k^{new} to have the same sign and normalization as λ_1 . In the above formula, d_k is an integer, depending on how many gauge groups the scalar fields share (see Appendix). Thus we expect λ_k^{new} to be of order g^2/N at a fixed point. The contribution of the graphs arising from these operators to the quadratic divergence is of order N^2 , so the contribution to m_ϕ^2 is of order $g^2 N$. Thus the inclusion of these additional operators seems to make the naturalness problem much worse. It may be possible to cancel the quadratic divergence order by order, but a priori there seems to be no reason for such a cancellation to occur at a fixed point of the theory.

Using the methods presented above one can also calculate the two-loop gauge β function. The two-loop piece of the gauge β function in a general gauge theory is given by [26]:

$$\begin{aligned} \beta_g^{(2)} &= -\frac{g^3}{(4\pi)^4} \left[\left\{ \frac{34}{3} (C_2(G))^2 - \frac{1}{2} \left(4C_2(F) + \frac{20}{3} C_2(G) \right) S_2(F) \right. \right. \\ &\quad \left. \left. - \left(4C_2(S) + \frac{2}{3} C_2(G) \right) S_2(S) \right\} g^2 + Y_4(F) \right], \end{aligned} \quad (3.17)$$

where $C_2(G)$ is the Casimir of the adjoint, $C_2(F)S_2(F)$ is the sum over (two-component) fermions of the Casimir times the Dynkin index in the given representation, $C_2(S)S_2(S)$ is the same for complex scalars, while $Y_4(F)$ is the contribution of the Yukawa couplings defined by

$$\text{Tr}Y^a Y^{\dagger a} t^A t^B = Y_4(F)\delta^{AB}, \quad (3.18)$$

where Y^a are the Yukawa coupling matrices for the scalar field a , and t^A are the gauge generators in the representation of the fermion fields.

For the orbifold theory considered above these expressions are given by

$$\begin{aligned} C_2(G) &= N, \\ C_2(F)S_2(F) &= 4(N^2 - 1), \\ C_2(G)S_2(F) &= 4N^2, \\ C_2(S)S_2(S) &= 3(N^2 - 1), \\ C_2(G)S_2(S) &= 3N^2, \\ Y_4(F) &= 24N^2Y^2. \end{aligned} \quad (3.19)$$

Note that Eq. (3.17) is independent of the quartic scalar couplings. At the one-loop fixed point of the Yukawa coupling, which is also independent of the values of the quartic scalar couplings, $Y^2 = \frac{N^2-1}{N^2}g^2$. Using this value we find that the leading order terms in N cancel, and the sub-leading pieces give

$$\beta_g^{(2)} = \frac{4g^5}{(4\pi)^4} > 0, \quad (3.20)$$

thus the theory is not asymptotically free. If the theory is indeed conformal, then the fixed point would necessarily be a UV fixed point. In order to check whether the theory is conformal or not, one would need to study the three-loop gauge β function. If the three-loop term turns out to be negative and of $\mathcal{O}(N^2)$, then there will be a perturbative UV fixed point, since the fixed point will be $g^2 = \mathcal{O}(1/N^2)$ and higher loop corrections to the gauge β function can be neglected. For any other case there cannot be a perturbative fixed point. For example if the three-loop term is $\mathcal{O}(N)$, then any putative fixed point can only be seen by summing all planar diagrams. Such a fixed point could exist independent of the sign of the three-loop term.

If this theory turns out to be conformal with a perturbative fixed point, then this could provide an interesting example of a theory with a non-trivial UV fixed point. Such a theory could then serve as a counter example to the conjecture presented in Ref. [28].

4 Conclusions

In this paper we have considered a particular class of non-supersymmetric orbifold theories obtained from finite $\mathcal{N} = 4$ theories. Our calculations are summarized by equations (3.3), (3.6)–(3.10) and (3.15). We calculated the one-loop β functions and found the simultaneous zeroes that approach the SUSY fixed point in the large N limit. At one-loop the theory possesses quadratic divergences in sub-leading orders in N and therefore cannot stabilize the weak scale without N being unreasonably large.

RG running also generates new operators (quartic scalar couplings) which are not present in the tree-level orbifold Lagrangian. These new couplings will shift the fixed point values of the original operators, and also contribute to the quadratic divergences themselves. It is possible, but unlikely, that with these new couplings all quadratic divergences vanish. The difficulty in canceling the quadratic divergences stems from the fact that the contributions of the new operators to the quadratic divergence is more important in the $1/N$ expansion than the divergences we have discussed here. We think that a cancellation is unlikely to occur, but the importance of the problem merits further investigation which would involve the renormalization of the full set of operators allowed by symmetries.

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Appendix A $\mathcal{N} = 4$ β Functions

$\mathcal{N} = 4$ supersymmetric theories are finite, therefore the β function vanishes to all orders in perturbation theory. In terms of component fields the $\mathcal{N} = 4$ Lagrangian has three different kinds of couplings: gauge, Yukawa, and quartic scalar. Even though these couplings are related by $\mathcal{N} = 4$ supersymmetry it is useful to calculate their β functions separately. In the orbifold theories different couplings are not related by supersymmetry, yet $\mathcal{N} = 4$ results are helpful in the calculation of the non-supersymmetric β functions.

The $\mathcal{N} = 4$ theory can be thought of as an $\mathcal{N} = 1$ theory with three adjoint chiral superfields and a superpotential for these fields. When the $\mathcal{N} = 4$ theory is expressed in terms of $\mathcal{N} = 1$ component fields the $SU(4)_R$ global symmetry is not explicit in the Lagrangian, only its $SU(3) \times U(1)$ subgroup is manifest. In terms of components the Lagrangian is given by

$$\begin{aligned} \mathcal{L}_{\mathcal{N}=4} = & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - i\bar{\lambda}^a\sigma^\mu D_\mu\lambda^a - i\bar{\Psi}_i^a\sigma^\mu D_\mu\Psi_i^a + D^\mu\phi_i^{\dagger a}D_\mu\phi_i^a + \\ & -\sqrt{2}gf^{abc}(\phi_i^{\dagger c}\lambda^a\Psi_i^b - \bar{\Psi}_i^c\bar{\lambda}^a\phi_i^b) - \frac{Y}{\sqrt{2}}\epsilon_{ijk}f^{abc}(\phi_i^c\Psi_j^a\Psi_k^b + \bar{\Psi}_i^c\bar{\Psi}_j^a\phi_k^{\dagger b}) \\ & + \frac{g^2}{2}(f^{abc}\phi_i^b\phi_i^{\dagger c})(f^{ade}\phi_j^d\phi_j^{\dagger e}) - \frac{Y^2}{2}\epsilon_{ijk}\epsilon_{ilm}(f^{abc}\phi_j^b\phi_k^c)(f^{ade}\phi_l^{\dagger d}\phi_m^{\dagger e}), \quad (\text{A.1}) \end{aligned}$$

where $a, \dots, e = 1, \dots, N^2 - 1$ are the adjoint gauge indices, while $i, \dots, m = 1, 2, 3$ are $SU(3)$ flavor indices. The $SU(N)$ structure constant is denoted by f^{abc} , λ is the (two-component) gaugino, Ψ_i are the (two-component) adjoint fermions, and ϕ_i are the three complex adjoint scalars. Meanwhile g is the gauge coupling and Y is the coupling of the superpotential term for the chiral superfields. The above Lagrangian is $\mathcal{N} = 4$ supersymmetric for $Y = g$. In order to easily identify the origin of different terms in the calculation it is instructive to keep Y explicit in the Lagrangian.

The one-loop (as well as two-loop) β functions are known for a general field theory [26]. In order to use the formulae given in Ref. [26] one needs to calculate certain group theoretic factors. This calculation can be conveniently carried out using the method of Cvitanovic [27], in which one draws a separate ‘‘group theory diagram’’ for every Feynman diagram. Evaluating these group theory diagrams will then amount to calculating the group theory coefficients needed for the general formulas of the β functions of [26]. Since all fields are in the adjoint representation every Yukawa coupling carries a factor f^{abc} while

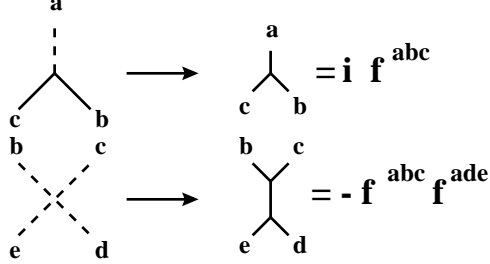


Figure 1: The group theory Feynman diagrams for the Yukawa couplings and the quartic scalar couplings of the $\mathcal{N} = 4$ theory.

every quartic scalar coupling carries a factor $f^{abc} f^{ade}$. In order to obtain the group theory diagrams one replaces every factor of $i f^{abc}$ with a cubic vertex (see Fig. 1). The diagram obtained this way does not have to coincide with the actual form of the Feynman diagram that one is evaluating.

Using the Lagrangian (A.1) and the above rules of calculating the group theory factors one can obtain the various β functions for the $\mathcal{N} = 4$ theory. The one-loop β function for the gauge coupling is given by

$$(4\pi)^2 \beta_g = -g^3 \left(\frac{11}{3} C_2(G) - \frac{2}{3} S_2(F) - \frac{1}{3} S_2(S) \right), \quad (\text{A.2})$$

where $C_2(G)$ is the Casimir of the adjoint, $S_2(F)$ is the Dynkin index of the (two-component) fermions, and $S_2(S)$ is the Dynkin index for the complex scalars. For the $\mathcal{N} = 4$ theory $C_2(G) = N$, $S_2(F) = 4N$, $S_2(S) = 3N$, and thus $\beta_g = 0$.

The one-loop β function for the Yukawa coupling Y^a in a general gauge theory is given by the formula

$$(4\pi)^2 \beta_Y^a = \frac{1}{2} (Y_2^\dagger(F) Y^a + Y^a Y_2(F)) + Y^b Y^{\dagger a} Y^b + Y^b \text{Tr} Y^{\dagger b} Y^a - 3g^2 \{C_2(F), Y^a\}. \quad (\text{A.3})$$

In the case of the $\mathcal{N} = 4$ theory we evaluate the β function of the vertex $-\sqrt{2} g f^{abc} \phi_i^\dagger c \lambda^a \Psi_i^b$. In the projected orbifold theory all Yukawa couplings are equal due to the Z_k symmetry of the theory, thus we can use any of the $\mathcal{N} = 4$ vertices to obtain the projected result. For this coupling the different

$$\begin{aligned}
\Lambda^2 &= (8N \text{ (triangle)} + 2 \text{ (square)} + 2 \text{ (square with X)}) g^4 \\
&+ (-8N \text{ (triangle)} + 8 \text{ (square with X)} - 8 \text{ (square with X)} - 4N \text{ (triangle with X)}) g^2 Y^2 \\
&+ (16 \text{ (square)} + 8N \text{ (triangle with X)}) Y^4 \\
A &= 2 (\text{ (square)} + \text{ (square with X)}) g^4 \\
H &= 4(g^4 + Y^4) \text{ (square)} + 8g^2 Y^2 (\text{ (square with X)} - \text{ (square with X)}) \\
\Lambda^S &= 4N g^4 \text{ (triangle)} + 8N g^2 Y^2 \text{ (triangle with X)} \\
\Lambda^Y &= 16N g^2 Y^2 \text{ (triangle)} + 32N Y^4 \text{ (triangle with X)}
\end{aligned}$$

Figure 2: Contributions to the β function of the quartic scalar couplings of the fields $\phi_1^a \phi_1^{\dagger b} \phi_2^c \phi_2^{\dagger d}$ in the $\mathcal{N} = 4$ theory. The ordering of the fields in the above diagrams is clockwise, with ϕ_1^a in the upper left corner. The meaning of the above group theory diagrams is explained in Fig. 1.

terms in the above β function are:

$$\begin{aligned}
\frac{1}{2}(Y_2^\dagger(F)Y^a + Y^a Y_2(F)) &= (4Ng^2 + 2NY^2)\sqrt{2}g, \\
Y^b Y^{\dagger a} Y^b &= (-4NY^2)\sqrt{2}g, \\
Y^b \text{Tr} Y^{\dagger b} Y^a &= (2Ng^2 + 2NY^2)\sqrt{2}g, \\
-3g^2 \{C_2(F), Y^a\} &= (-6Ng^2)\sqrt{2}g.
\end{aligned} \tag{A.4}$$

The sum of these terms adds up to zero independently of the value of Y , which can be understood in the following way: for $Y \neq g$ we have an $\mathcal{N} = 1$ supersymmetric theory with three adjoint fermions and a non-vanishing superpotential. Since we have chosen the β function of the Yukawa coupling involving the gaugino, therefore the Yukawa β function has to be proportional to the gauge β function for any value of Y . The one-loop β function of the gauge coupling is independent of Y therefore the cancellation has to happen for a generic value of Y . This provides an independent check of our result.

Finally we calculate the one loop β functions for the quartic scalar couplings. The general formula for an arbitrary gauge theory is given by

$$(4\pi)^2 \beta_{\text{quartic}} = \Lambda^2 + 3A - 4H + \Lambda^Y - 3\Lambda^S. \tag{A.5}$$

$$\begin{aligned}
\Lambda^2 &= (8N \text{ (Y-shape)} + 8N \text{ (X-shape)} + 2 \text{ (square)} + \text{ (square with X)}) \\
&+ \text{ (square with X)} \text{) } g^4 - 8Ng^2 Y^2 (\text{ (Y-shape)} + \text{ (X-shape)}) \\
&+ 32 Y^4 \text{ (square)} \\
\mathbf{A} &= 2 (2 \text{ (square)} + \text{ (square with X)} + \text{ (square with X)}) g^4 \\
\mathbf{H} &= 8 (g^4 + Y^4) \text{ (square)} \\
\Lambda^S &= 4Ng^4 (\text{ (Y-shape)} + \text{ (X-shape)}) \\
\Lambda^Y &= 8Ng^2 (g^2 + Y^2) (\text{ (Y-shape)} + \text{ (X-shape)})
\end{aligned}$$

Figure 3: Contributions to the β function of the quartic scalar couplings of the fields $\phi_1^a \phi_1^{\dagger b} \phi_1^c \phi_1^{\dagger d}$ in the $\mathcal{N} = 4$ theory. The ordering of the fields in the above diagrams is clockwise, with ϕ_1^a in the upper left corner. The meaning of the above group theory diagrams is explained in Fig. 1.

We calculate two different combinations of quartic β functions in the $\mathcal{N} = 4$ theory: one for the coupling of the operator $\phi_1^a \phi_1^{\dagger b} \phi_2^c \phi_2^{\dagger d}$, for which the contributions are given in Fig. 2, and another for the operator $\phi_1^a \phi_1^{\dagger b} \phi_1^c \phi_1^{\dagger d}$ the contributions to which are given in Fig. 3. Combining these results according to Eq. (A.5) one finds that these β functions indeed vanish for the $\mathcal{N} = 4$ theory. Cancellation of various terms occurs after decomposing the “gluon box” diagrams [27] in a complete basis of group theory tensors using the results given in Fig. 4.

In order to project the $\mathcal{N} = 4$ theory down to the orbifolded theory it is convenient to make use of large N double-line notation, since all our fields are bifundamentals. To do this we need two $SU(N)$ identities:

$$if^{abc} = 2\text{Tr} (T^a T^b T^c - T^c T^b T^a) , \quad (\text{A.6})$$

and

$$(T^a)_j^i (T^b)_n^m = \frac{1}{2} (\delta_n^i \delta_j^m - \frac{1}{N} \delta_j^i \delta_n^m) . \quad (\text{A.7})$$

$$\begin{aligned}
\text{Box} &= \frac{1}{2}(\parallel + \equiv + 2\text{X}) + \frac{N}{4}(\text{Y} + \text{Z} + \text{D}) \\
\text{V} &= \text{Y} + \text{X} \\
\text{V} &= \frac{2}{N}(\equiv - \text{X}) + \text{D} - \text{D} \\
\text{V} &= \mathbf{i} \mathbf{d}^{abc}
\end{aligned}$$

Figure 4: The diagrammatic representation of the $SU(N)$ group theory identities needed to show that the $\mathcal{N} = 4$ β functions of the quartic couplings do indeed vanish. The first line gives the decomposition of the “gluon box diagram” in terms of a complete set of tensors, the second line is the Jacobi identity, while the third line is an identity relating different combinations of the d and f tensors. A single unconnected line corresponds to δ_b^a . These results are taken from [27].

To keep the fields canonically normalized after changing from the single index basis to the double index basis we need to rescale

$$\phi^a = \sqrt{2}\phi_i^j (T^a)_j^i . \tag{A.8}$$

Using these identities and representing δ_j^i by a line with an arrow we can obtain the large N results given in Fig. 5.

At tree-level the effect of orbifolding is similar to taking the above large N limit, the only difference is that different oriented lines can correspond to different gauge groups. The appropriate combination of gauge groups for each vertex can be read off from the projected Lagrangian (2.2). Once we have the tree-level vertices we can simply calculate all the diagrams relevant to the β functions. Additionally we can apply the projection rules to the $\mathcal{N} = 4$ diagrams involving quartic or Yukawa couplings, however sub-leading

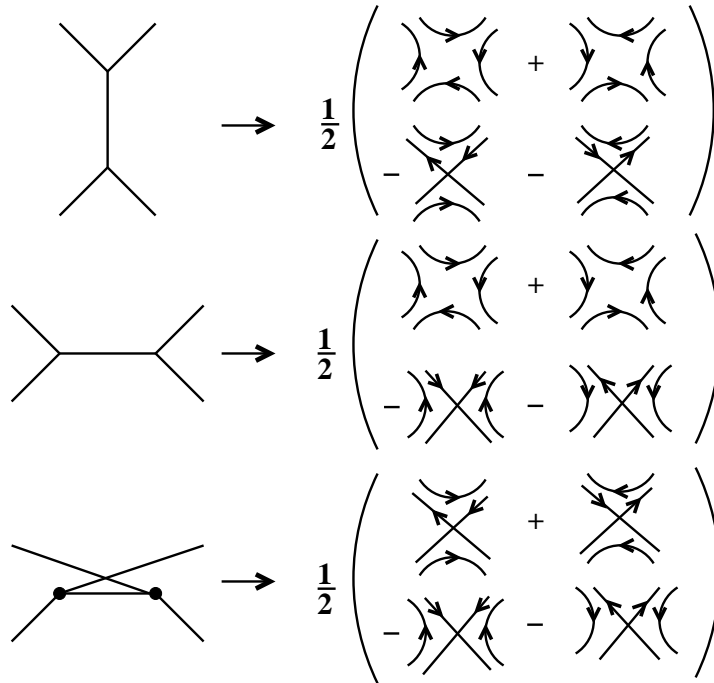


Figure 5: The large N rules for adjoints.

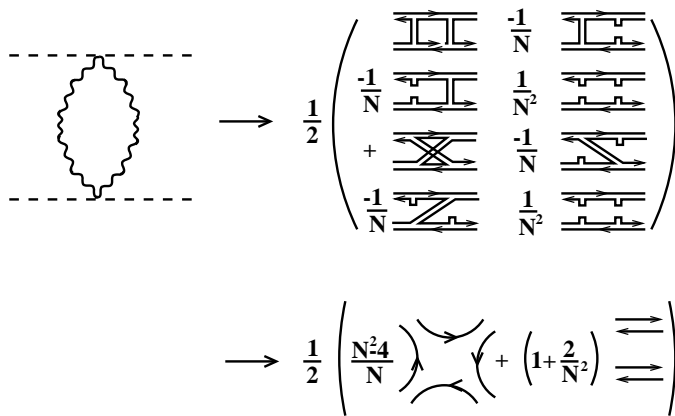


Figure 6: The proper correction to quartic couplings from gauge boson exchange.

terms in N can be generated in loops, and these terms must be kept. This procedure provides a check on the calculation.

The double line notation is also convenient for gauge diagrams, however $1/N$ terms are already present in the gauge boson propagator so a little more care must be taken. We illustrate the use of the double line notation in the calculation of the proper correction to the quartic coupling from two gauge boson exchange. For simplicity, we consider the case of two different scalar fields that share one gauge group. The calculation proceeds by using the identity (A.7) and is depicted in Fig. (6).

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