Improving event generators with effective theories

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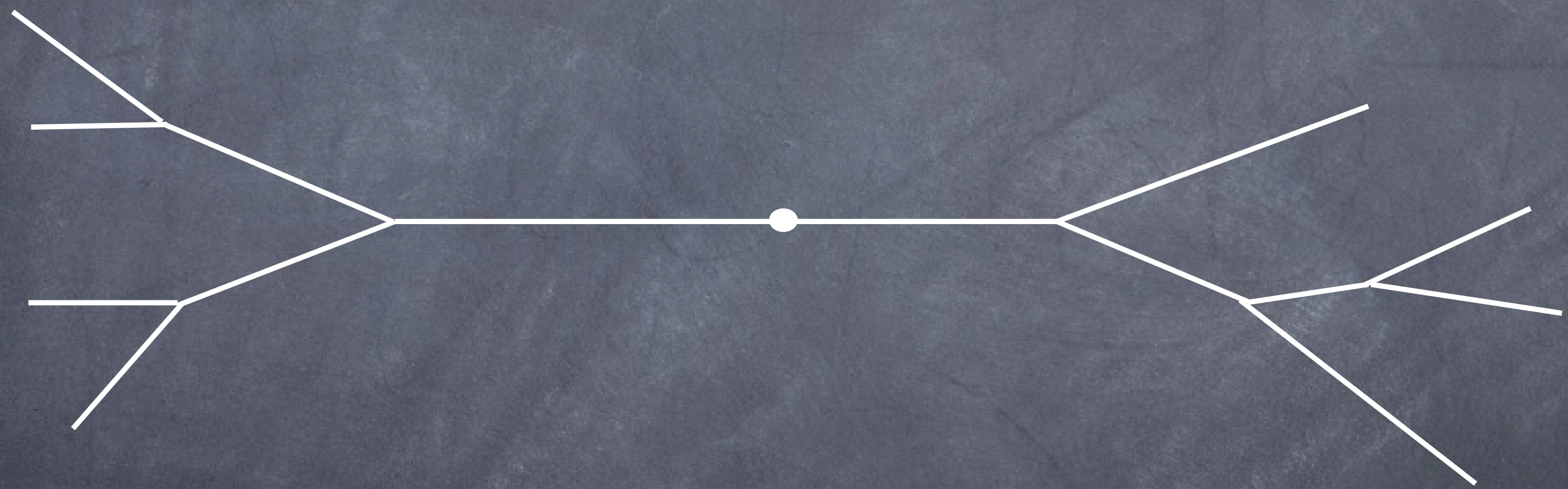
Outline

- The basics behind parton showers
- Jet distributions from SCET
- Systematically improving parton showers
- Implementation
- Conclusions

The basics behind parton showers

A simple example: $e^+e^- \to$ jets



Three perturbative steps

1. Calculate underlying process with limited number of partons in final state

   Use full QCD matrix elements including all interference

2. Add additional partons using splitting functions

   Splitting functions give naive probability for one particle to branch into two

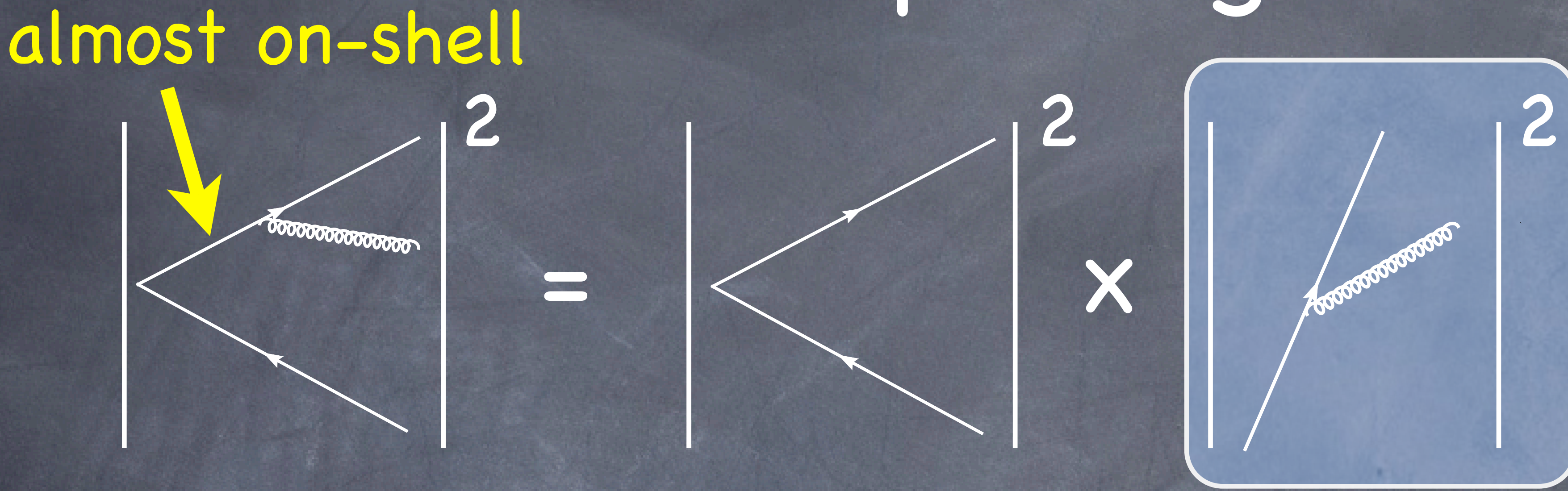
3. No-branching probability gives rise to Sudakov factors

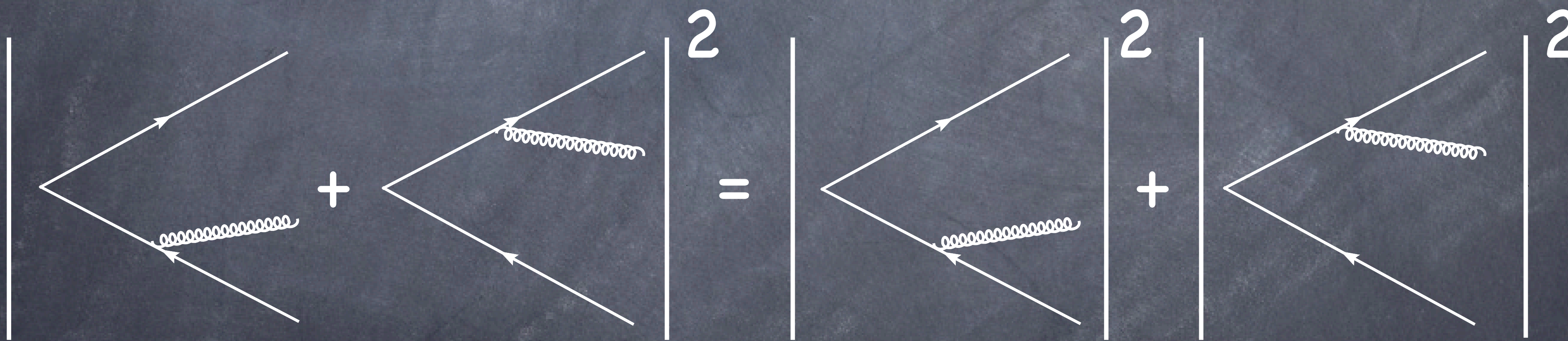
   Sudakov factors sum the leading logarithmic terms in perturbative expressions

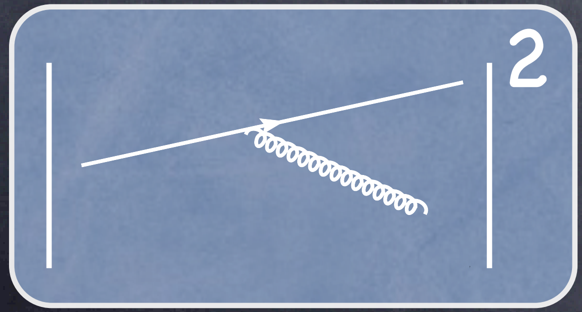
What needs to work...

- Factorization of cross section in collinear limit
  - $d\sigma_{n+1} = d\sigma_n \, d\varphi ds dz \, P(s,z)$
- Functions P(s,z) written as sum over contributions from individual particles
  - $P(s,z) = \Sigma_i \, P_i(s,z)$
- Expression derived in collinear limit also reproduces soft physics
  - Not trivial, since soft contribution entirely from interference

Factorization & splitting functions







$$= P(s,z) = \frac{1}{s}\,\frac{1+z^2}{1-z}$$

ME's vs PS's

Matrix elements	Parton Showers
Only limited number of final state partons possible	Arbitrary number of final state partons possible
Uses fixed order perturbation theory	Sudakov factors sum leading collinear logarithms
All regions of phase space are properly described	Only describes phase space with $Q \gg p_T^{(1)} \gg p_T^{(2)} \gg \ldots$
Straightforward to include higher loops	Only tree level can be incorporated
Better for large p_T emission	Better for small p_T emission

Jet distribtutions from SCET

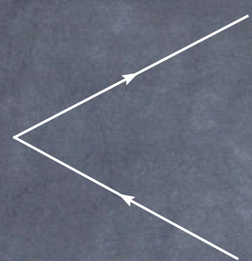
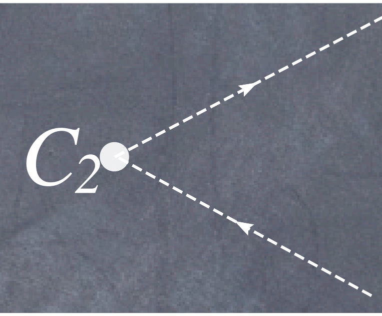
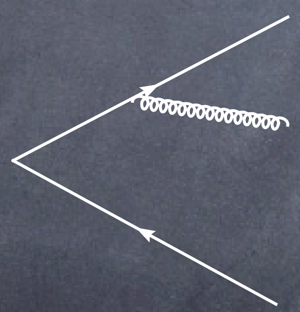
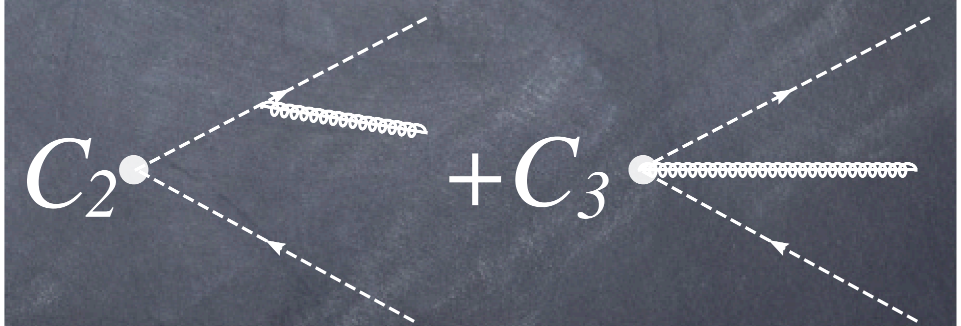
Basic idea of SCET

- Jet distributions contain many scales, such as Q, E_i, $p_T^{(i)}$...
- SCET is effective theory describing interactions between collinear and soft particles
  - Collinear particles have $p_T \ll E \sim Q$
  - Soft particles have $p_T \sim E \ll Q$
- SCET separates E and p_T by keeping only soft and collinear fields
- SCET describes physics in the limit $p_T \ll Q$
- Matching calculations allow to incorporate physics with $p_T \sim Q$ into matching coefficients

Parton showers from SCET

- Start with SCET at high scale, where fermions have large virtuality
- Evolve the operator to lower scales, lowering virtuality
- If additional partons can be resolved, perform threshold matching
- Keep evolving to lower scales

Obtain SCET at the high scale

QCD	SCET
j_{QCD}	$C_2 O_2 + C_3 O_3 + \ldots$
	
	

Operators O_n and coefficients C_n chosen such that we reproduce full QCD with up to n partons

Evolve SCET to lower scales

- Operators O_n depend on renormalization scale μ
- Sets the scale for p_T in emission
- Product of operator and coefficient independent of μ
- Allows to obtain coefficient at arbitrary scale μ

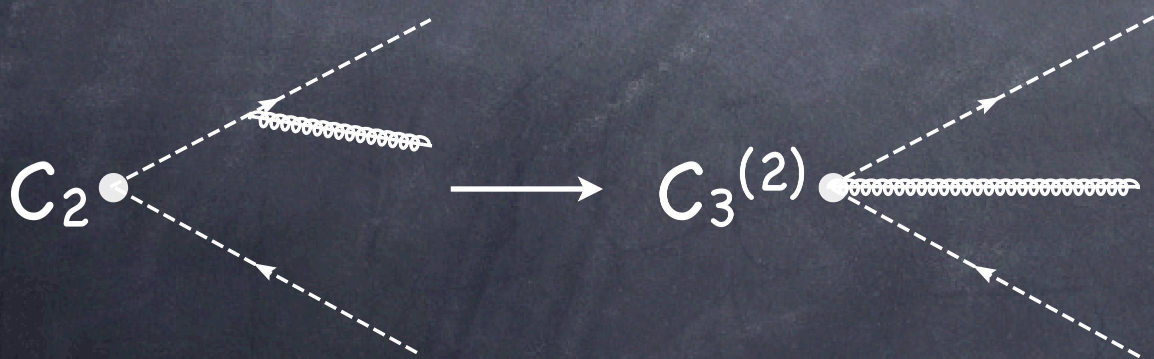
$$\mu \frac{d}{d\mu} [C_n(\mu) \ \langle O_n \rangle_\mu] = 0$$

$$\mu \frac{d}{d\mu} \ C_n(\mu) = C_n(\mu) \ \gamma_n(\mu)$$

$$C_n(\mu) = C_n(Q) \ \Pi_n(Q,\mu)$$

Perform threshold matching

- Operator O_n contributes to matrix elements with more final states by emitting particles in SCET
- Allowed p_T set by scale μ
- For $\mu<p_T$ particle not described by SCET any more
- Need to perform threshold matching



Keep evolving to lower scales

Sequence of matching and running

Match at $\mu=Q$	$C_2^{(2)}\ O_2^{(2)} + C_3^{(3)}\ O_3^{(3)} + \ldots$
Evolve $p_T<\mu<Q$	$C_2^{(2)}\ \Pi_2\ O_2^{(2)} + C_3^{(3)}\ \Pi_3\ O_3^{(3)} + \ldots$
Match at $\mu=p_T$	$C_3^{(2)}\ \Pi_2\ O_3^{(2)} + C_3^{(3)}\ \Pi_3\ O_3^{(3)} + \ldots$
Evolve $\mu<p_T$	$C_3^{(2)}\ \Pi_2\ \Pi_3\ O_3^{(2)} + C_3^{(3)}\ \Pi_3\ O_3^{(3)} + \ldots$

SCET vs PS/ME

Possible to show from first principles

1. Matrix elements in SCET satisfy

Standard splitting function

$$\sum_{\substack{\text{spins}\\ \text{pol}}} |O_3^{(2)}|^2 = \sum_{\substack{\text{spins}\\ \text{pol}}} |O_2^{(2)}|^2 \, P(s,z)$$

2. Evolution kernels equivalent to Sudakov factors

$$\Pi_n(\mu_1,\mu_2) = \Delta_q^{N_q/2}(\mu_1,\mu_2)\, \Delta_g^{N_g/2}(\mu_1,\mu_2)$$

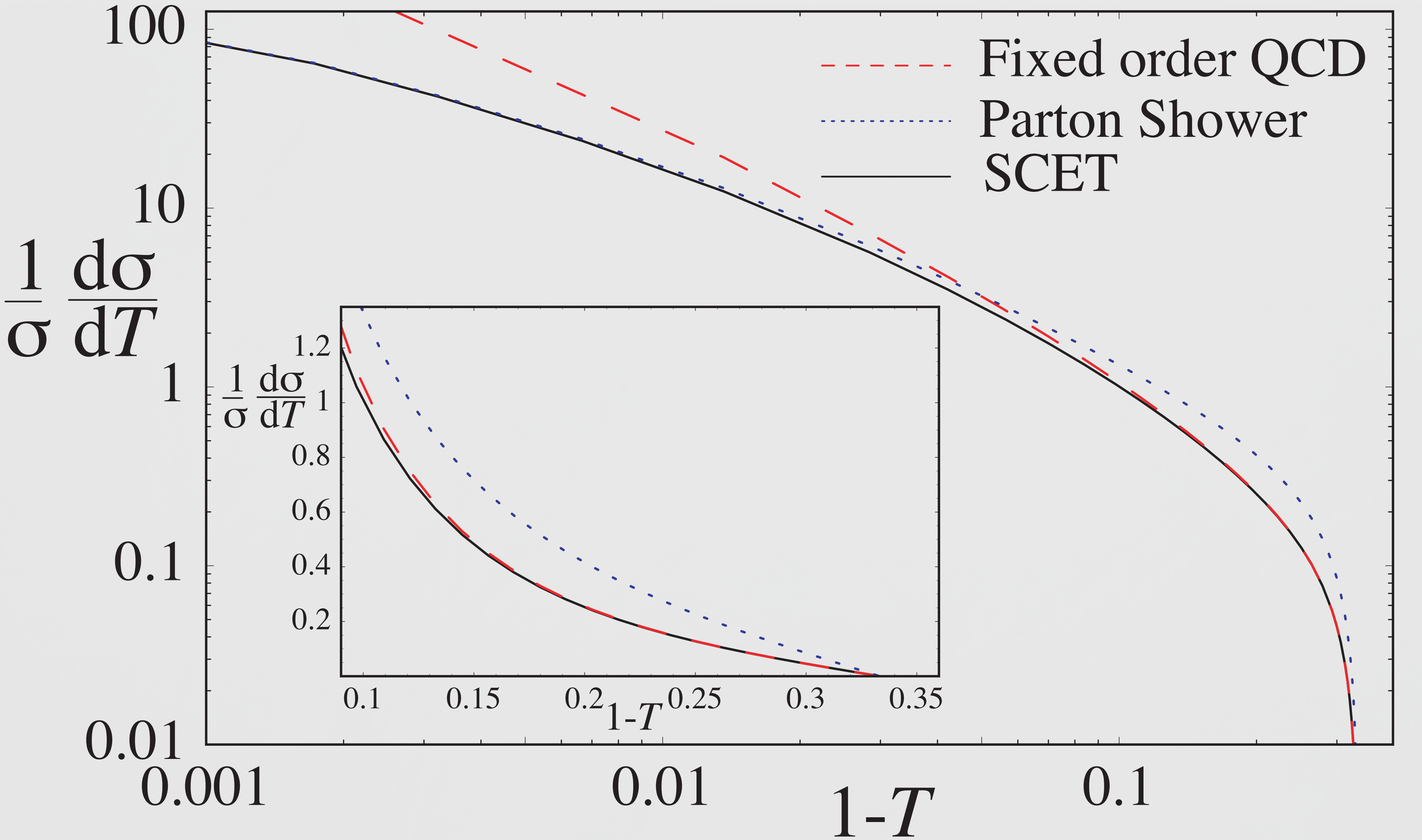
SCET vs PS/ME

$SCET = C_2^{(2)} \Pi_2 O_3^{(2)} + C_3^{(3)} \Pi_3 O_3^{(3)} + \dots$

large p_T	small p_T
$\Pi_n = 1$	$O_3^{(3)} = 0$
$C_2^{(2)} O_3^{(2)} + C_3^{(3)} O_3^{(3)}$	$C_2^{(2)} \Pi_2 O_3^{(2)}$
QCD	Parton Shower

SCET has the right limits to interpolate between QCD and the parton shower

SCET vs PS/ME



Fixed order QCD
Parton Shower
SCET
$\frac{1}{\sigma}\frac{d\sigma}{dT}$
100
10
1
0.1
0.01
0.001
0.01
0.1
1-T
1.2
1
0.8
0.6
0.4
0.2
0.1
0.15
0.2
0.25
0.3
0.35

Systematically improving parton showers

Naive parton showers...

Naive parton shower algorithms take into account

- Matrix elements for 2→2 interactions at tree level
- Additional partons generated by LO splitting functions
- LL resummation from Sudakov factors
- Correct at LO and LL in limit $Q \gg p_T^{(1)} \gg p_T^{(2)} \gg p_T^{(3)} \gg \ldots$

Algorithms exist to add the following:

- Add matrix elements with more final states (Sherpa)
- Add matrix elements at 1-loop order (MC@NLO)

Possible improvements with SCET

- Add matrix elements with more partons
  - Include more operators O_n in QCD→SCET matching
- Add loop corrections to matrix elements
  - Calculate matching at higher order in PT
- Add subleading logarithms to "Sudakov" factors
  - Calculate anomalous dimensions at higher order in PT

Requires straightforward, well defined calculations

Add more partons

To reproduce matrix elements with up to four large p_T partons need more operators in matching

$$QCD = C_2\, O_2 + C_3\, O_3 + C_4\, O_4$$

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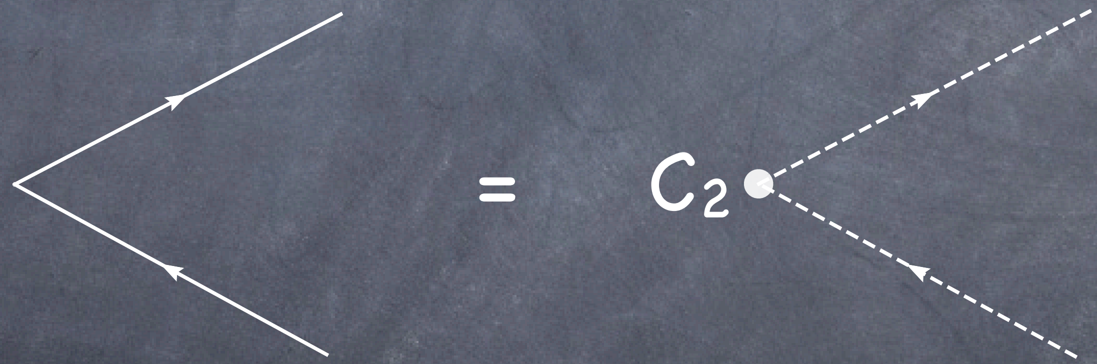

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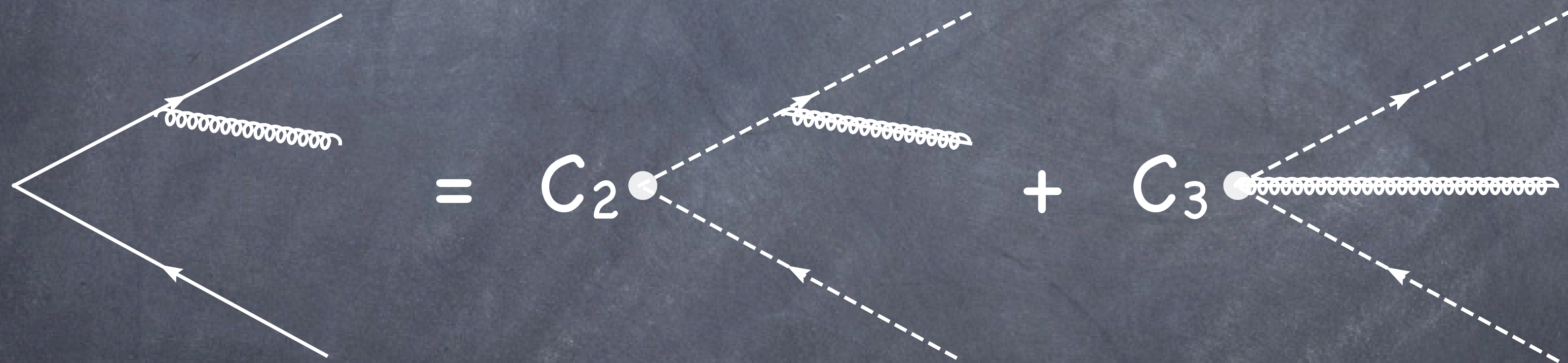

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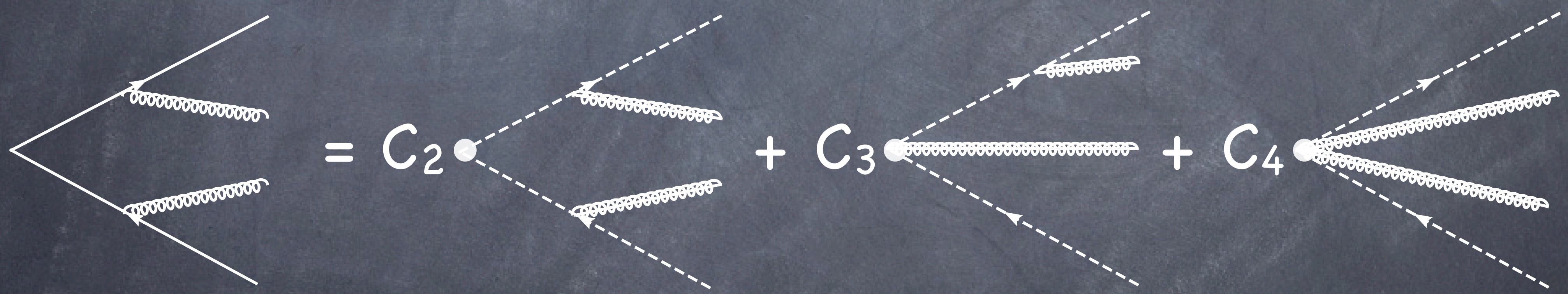
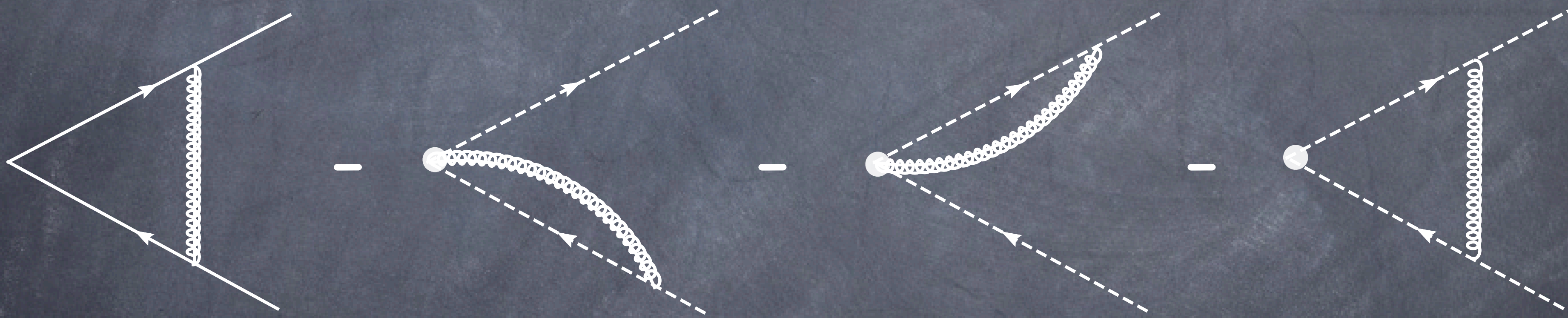


Operators O_3 and O_4 contain difference between QCD matrix elements and SCET matrix elements

Add loop corrections

To reproduce matrix elements at higher loops need coefficients to higher order in perturbation theory

To obtain C_2 at one loop, calculate



$$C_2 = 1 - \frac{\alpha_s C_F}{4\pi}\left(8 - \frac{7\pi^2}{6} + 3i\pi\right)$$

Similar calculations possible for C_3, C_4, ...

Add subleading logarithms

- Logarithms summed by regular RG evolution
- Anomalous dimensions can be calculated at higher orders in perturbation theory
- For consistent resummation, need cofficient of log at (n+1)-loops and constant term at n-loops
- NLL resummation possible with existing calculations, but some subtleties due to operator mixing

Implementing the idea

(work in progress with Frank Tackmann)

Correcting event generators

- Event generators populate phase space according to a certain differential distribution
- SCET allows to calculate the differential distributions, with systematically improvable uncertainties
- Combine both approaches
  - Create N events with your favorite event generator
  - For each event, calculate the differential weight for event generator ($d\sigma_{PS}$) and SCET result ($d\sigma_{SCET}$)
  - Reweigh the event by $d\sigma_{SCET}/d\sigma_{PS}$

Correcting event generators

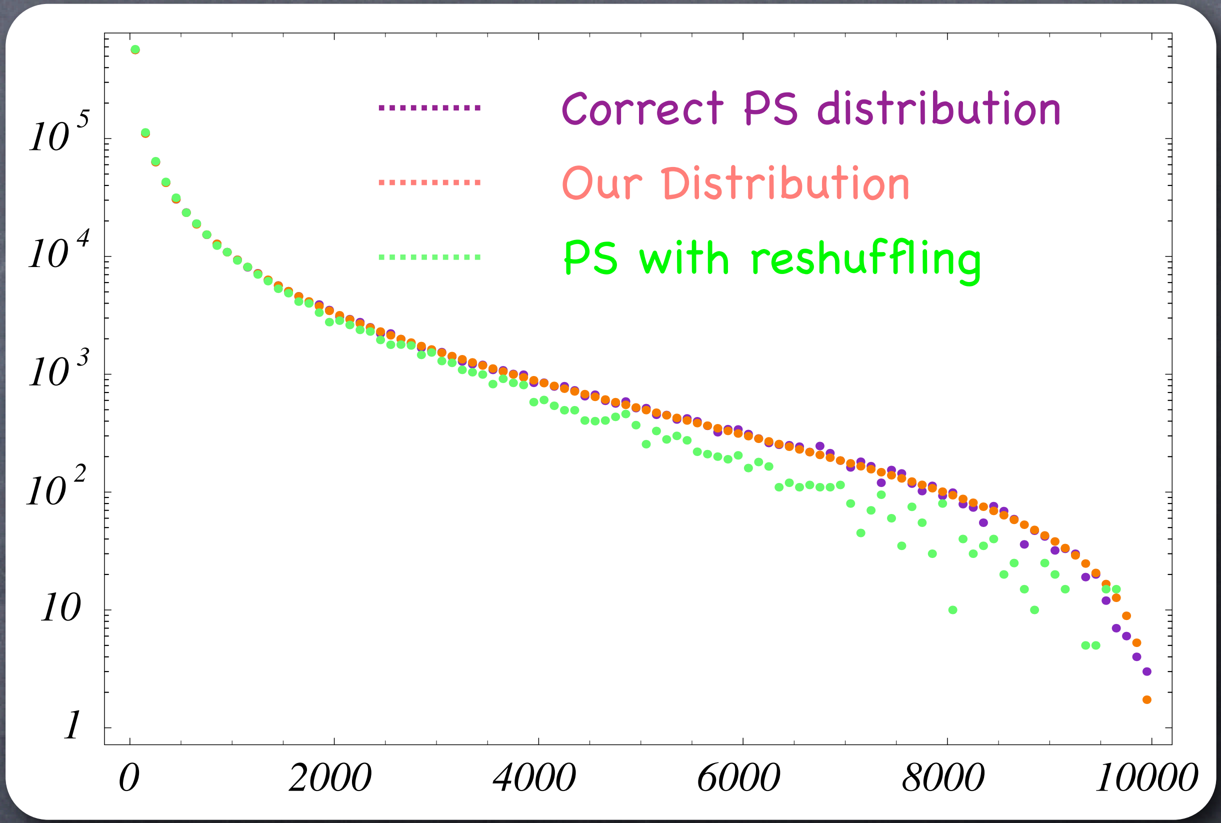
- Advantages:
  - Very general method, can incorporate arbitrary effects
  - Allows to rely on all the previous work
  - Allows to estimate uncertainties in the same way
- Challanges:
  - Need to know the precise event weight for the used event generator
  - To obtain small weights generator used should be close to correct

Current Work: Create standalone program that takes output from event generators and reweighs (with errors) according to precise SCET distribution.

Very preliminary results

Some issues with PS implementation

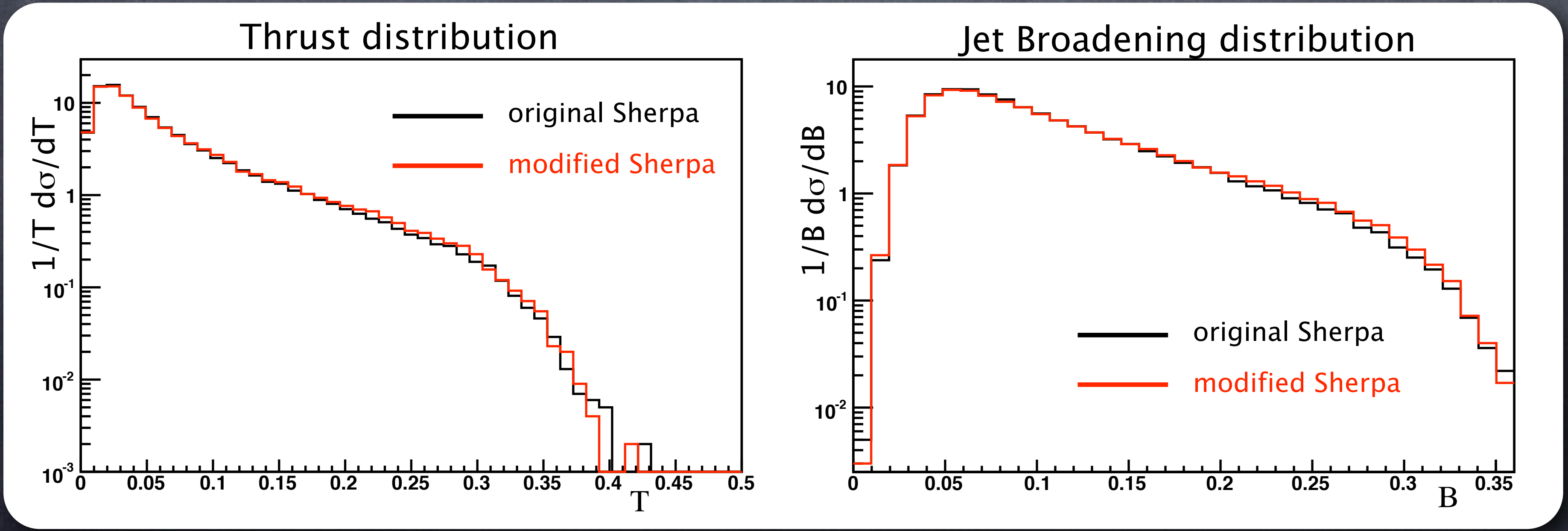
- Momentum is not conserved at leading order in parton shower
- Need to make somewhat ad-hoc corrections
- Can change the shape of distribution functions



Very preliminary results

- While they are power suppressed, need to understand them to get proper weight
- They destroy the factorization of probabilities for an event
- Need to use different reshuffling

Need to check this does not affect distributions much



Things to do

- Finish implementation
- Extend to hadronic collisions
- Finish calculation of subleading logarithms
- Do matching onto higher order operators
- Do detailed studies comparing to experimental and other theoretical predictions