Folded Supersymmetry

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EW symmetry breaking in the Standard Model is caused by the potential

$$V = -m^2 H^+ H + \lambda (H^+ H)^2$$

Radiative correction is quadratically divergent

The Higgs mass is UV sensitive

$$m^2 \quad m_B^2 + O(\frac{\Lambda^2}{16\pi^2})$$

Fine tune

Assuming λ=1, m needs to be about 100 GeV

Gauge hierarchy

$$\Lambda = 10^{16} GeV = M_G$$

Fine tuning : 10^{-26}

Little hierarchy

$$\Lambda = 10 \text{ TeV}$$

Fine tuning : 10^{-2}

Little Hierarchy or LEP Paradox

There can be no fine tuning if

- Λ (cutoff) is lower or/and

- λ (quartic coupling) is bigger

However

- LEP (and other) measurements have strongly constrained

  - The effective scale of non-renormalizable operators that contribute to precision electroweak measurements to be higher than about 5 TeV. $\Rightarrow$ Λ_{min}

  - The Higgs mass < 200 GeV. $\Rightarrow$ λ_{max}

- These two bounds together lead to a conclusion :If there is no new degree of freedom up to this cutoff scale, SM is fine tuned to few percent.
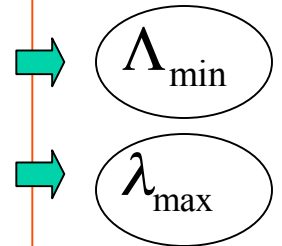
If you believe naturalness should work at this level, LEP paradox suggests that there should exist new degrees of freedom not much heavier than a TeV that, by symmetries,

- cancel the quadratic divergences,

- only have very small contributions to the non-renormalizable operators that contribute to precision electroweak measurements .
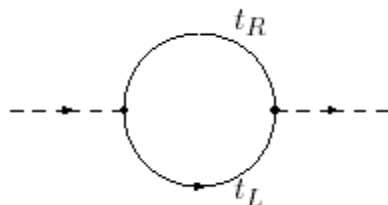
Here are some examples :

- SUSY with R-parity

- Litte Higgs with T-parity

- *Continuous symmetries cancel the quadratic dovergences*

- *Parities suppress the non-renormalizable operators*
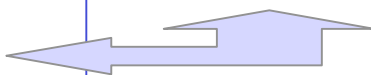
How do these cancellations work ?

Standard Model :

- Dominant contribution : Top loop,

- Since top Yukawa coupling is ~1 compared to EW gauge couplings which are g^2 ~0.4

$$\text{top} \quad \frac{3}{8\pi^2}\lambda_t^2\Lambda^2$$

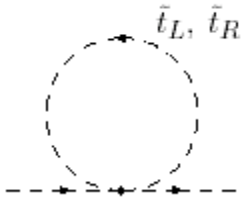$$\text{gauge} \quad \frac{9}{64\pi^2}g^2\Lambda^2$$

More degrees of freedom too

- Top is charged under color SU(3)

The first step to the solution of little hierarchy problem is canceling this top loop
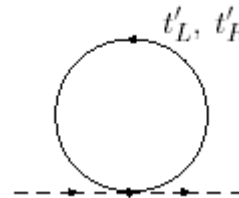
SUSY :

Little Higgs :

$$\tilde{t}_L,\ \tilde{t}_R$$

Stop loop cancels top loop.

SUSY commutes with SU(3) ⟹ *stop has color*

$$t^a_\alpha \xleftarrow{\quad Q^\alpha \quad} \tilde{t}^{\,a}$$

$$t'_L,\ t'_R$$

SU(2) is embedded in a larger global group which commute with color SU(3). Extra quarks that complete the representation with top, the top-partners, are then colored

$$t^a_i \xleftarrow{\quad g_i^{\,j} \quad} t^a_j$$
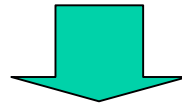
In these examples, the new particles that cancel the top-loop are charged under the Standard Model color.

Is this true in general ??

Is it true that any solution to the little hierarchy problem must necessarily involve new colored particles ?

Why is this question important ?

Because the LHC is a hadron collider, and thus colored particles can be abundantly produced.

i.e.
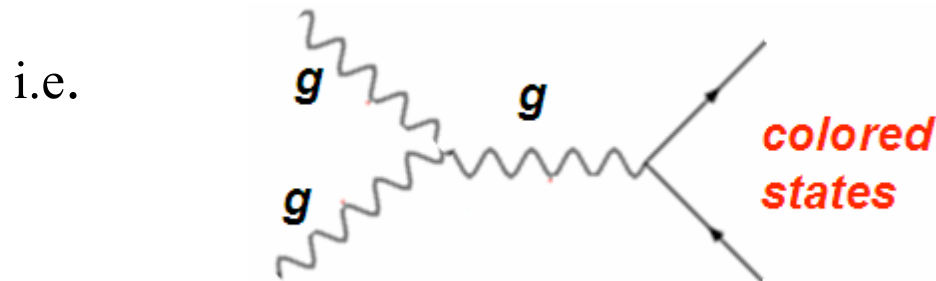
Since in these examples the "top-partners", the particles that cancel the top loop, are colored, the chance of their discovery at the LHC is good.

So, the LHC will tell us if the electroweak scale is simply a result of fine tuning or can be understood naturally by new physics at energy around TeV.

This is a good news.
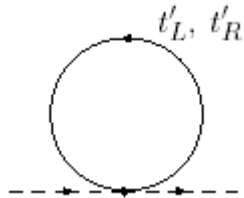
It is not always true !

It turns out in general the new particles need not be colored.

Crucial point : The top-partners need be related to the top only by a discrete
symmetry instead of a continuous symmetry as in the previous
examples

For example,   In the mirror twin Higgs   [Chacko, HSG and Harnik]

Extra quarks relate to top by a discrete symmetry.

It doesn't have to have the same color as the top.

This diagram is exactly the same as that of the little Higgs model but the
top-partner is charged under the mirror SU(3). The cancellation works
independently of the color of the particles running in the loop.

Color is just a dummy index.

It doesn't affect how the cancellation works.

This is the first example showing that the top-partners can be singlets under the SM SU(3).
This finding is significant since we now have to be more careful when interpreting the
LHC results. The LHC may not be able to reveal the true dynamics of electroweak
symmetry breaking.

This situation leads us to a more general consideration :

	fermion	boson	
color	Little Higgs	SUSY	Global symmetry
Non-color	Twin Higgs - mirror	?????????	Discrete symmetry

Question :
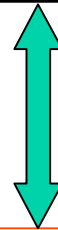
Can Non-colored bosons cancel the top loop contribution to Higgs mass ??

It is certainly possible, again, the diagram in
supersymmetry theories that cancel the top loop is

$$\tilde{t}_L,\ \tilde{t}_R$$
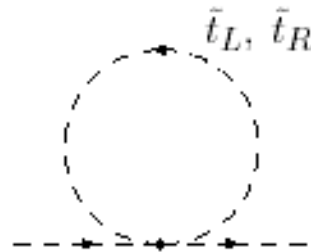
The stop running in the loop has "color" index which is, in this diagram, dummy

Can we find a theory that realize this situation ??

Correspondence in Large-N supersymmetric theories

In the large N limit a relation exists between the correlation functions of supersymmetric theories and those of their orbifold daughters, which can have lesser supersymmetry, that holds to all orders in perturbation theory. The masses of scalars in the daughter theory are protected against quadratic divergences by the supersymmetry of the mother theory.

•Kachru & Silverstein,
•Lawrence, Nekrasov & Vafa,
•Bershadsky, Kakushadze & Vafa,
•Kakaushadze,
•Bershadsky & Johansen,
•Schmaltz.

Mother theory

N-Supersymmetry

SU(ΓN)

A special case

N=1 SUSY
SU(2N)

Project out
certain states

$N \rightarrow \infty$

Daughter theory

$\leq N$-Supersymmetry

SU(N) ×SU(N) ×..

Non- SUSY
SU(N) ×SU(N)

By using this special relation, we can build a class of non-SUSY models that cancel the quadratic divergences of the Higgs mass due to the SUSY of the mother theory
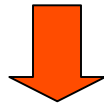
Folded Supersymmetry

Outline of the rest of the talk

- More on the correspondence
  - Yukawa sector cancellation
  - Gauge sector cancellation
- Folded SUSY mechanism
- UV completion—5D model
- Phenomenology
- Conclusion

More on the light bulb

- The general statement :
  - Begin with SU(N) theory : Mother theory
  - Choose a discrete group Z which is a symmetry of the mother theory
  - Keep only states that are invariant under Z : daughter theory
  - In the large *N* limit, the correlation functions of the mother and daughter theories are the same up to a rescaling of the couplings.
- The proof is based on following
  - The contributions are dominated by planar diagrams
  - And the dominant planar diagrams are those with maximal number of inner loop
  - Other diagrams are suppressed by 1/N
  - The correspondence is proved up to these 1/N corections
- For us, we need this correspondence only at one-loop
  - They are always planar.
  - If all the diagrams have a inner loop unbroken,
  - This theory will have the correspondence and there will be no 1/N correction
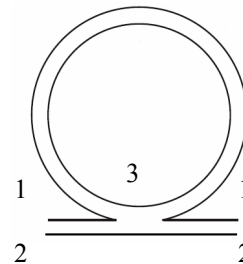 - Otherwise, there are power of 1/N corrections and the cancellations are not complete

Let's see how to apply the correspondence to
Yukawa coupling:

$$\lambda\, Q_{12}\, Q_{23}\, Q_{31}$$

Charges of Q's under the symmetry groups SU(2N)$_1$ X SU(2N)$_2$ X SU(2N)$_3$ are

$$Q_{23} = (1, 2N, \overline{2N}) \qquad Q_{12} = (2N, \overline{2N}, 1) \qquad Q_{31} = (\overline{2N}, 1, 2N)$$

The diagrams contribute to the mass of scalar field, Q$_{12}$ for example, are
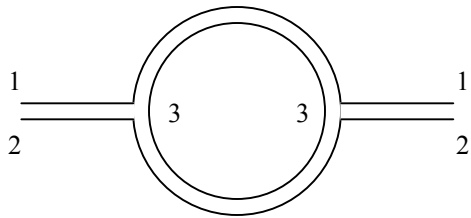
Double line notation just to keep tract of the double indexes

There is no diagram without inner loop.

Quadratic divergences in the daughter theory can be cancelled with no 1/N correction.

The projection breaks the gauge group
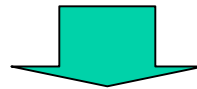
$$SU(2N)_i \longrightarrow SU(N)_{iA} \times SU(N)_{iB}$$

and the parity of various fields are

Under

$Z = Z_{2\Gamma} \times Z_{2R}$

$$\tilde{q}_{i,j} = \begin{pmatrix} \tilde{q}_{iA,jA}(+) & \tilde{q}_{iA,jB}(-) \\ \tilde{q}_{iB,jA}(-) & \tilde{q}_{iB,jB}(+) \end{pmatrix}$$

$$q_{i,j} = \begin{pmatrix} q_{iA,jA}(-) & q_{iA,jB}(+) \\ q_{iB,jA}(+) & q_{iB,jB}(-) \end{pmatrix}$$

Keeping only the even fields and terms that involve $\tilde{q}_{1A,2A}$, the Yukawa term becomes

$$\left[\sqrt{2}\lambda\, \tilde{q}_{1A,2A}\, q_{2A,3B}\, q_{3B,1A} \; + \; \text{h.c.} \right] \; +$$

$$2\,\lambda^2\, |\tilde{q}_{1A,2A}|^2\, |\tilde{q}_{3A,1A}|^2 + 2\,\lambda^2\, |\tilde{q}_{1A,2A}|^2\, |\tilde{q}_{2A,3A}|^2$$
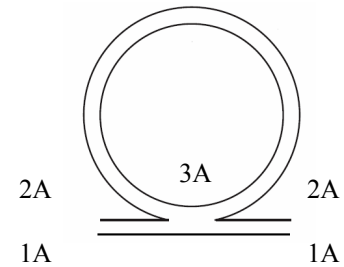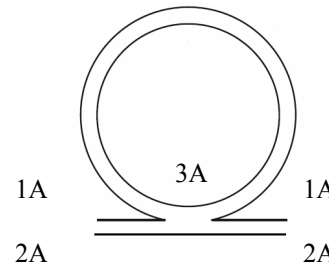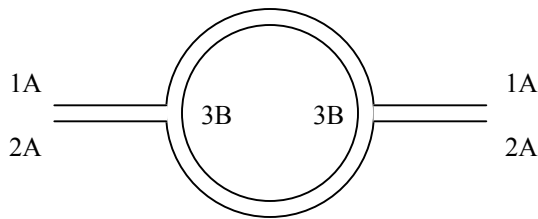
Notice that :

• 3B in the first diagram and 3A in the last two diagrams are dummies.

• So we can replace 3A by 3B without affecting the result.

• By doing this, the interactions above have exactly the form of supersymmetric Yukawa interactions. This implies that the diagrams cancel exactly as in a supersymmetric theory.

Now let's see how to apply the correspondence to Gauge interactions :

the usual scalar-gauge boson interactions give the following diagrams

• first diagram has a inner loop

• the second diagram has no inner loop and so is 1/N suppressed

The gauginos contribute to diagrams as the following
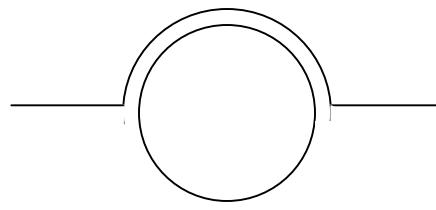
Also a possible diagram with 1/N suppression

The 1/N corrections exist and there is no guarantee that the quadratic divergences will be cancelled completely.

Again, we can check the argument explicitly

- Begin with SU(2N) gauge theory with one fundamental and one anti-fundamental

- The mother theory is invariant under the same discrete symmetry : $Z_{2\Gamma} \times Z_{2R}$ with

$$Q \to \Gamma\, Q \qquad\qquad \bar{Q} \to \Gamma^* \bar{Q} \qquad\qquad V \to \Gamma V \Gamma^\dagger$$

So, the parity of various fields

$$\begin{pmatrix} (-)^{\sigma\sigma\chi} & (+)^{V\sigma\chi} \\ (+)^{\sigma V\chi} & (-)^{VV\chi} \end{pmatrix} = \chi$$

$$\begin{pmatrix} (+)^{\sigma\sigma'\eta} & (-)^{V\sigma'\eta} \\ (-)^{\sigma V'\eta} & (+)^{VV'\eta} \end{pmatrix} = V'^{\eta}$$

$$\tilde{q} = \begin{pmatrix} \tilde{q}_A(+) \\ \tilde{q}_B(-) \end{pmatrix} \quad ; \quad q = \begin{pmatrix} q_A(-) \\ q_B(+) \end{pmatrix}$$

From gauge bosons we get
$$\frac{3}{32\pi^2} \frac{2N^2 - 1}{2N} g^2 \Lambda^2$$

From `gauginos' we get
$$-\frac{1}{8\pi^2} N g^2 \Lambda^2 \qquad \Longrightarrow \qquad -\frac{1}{16\pi^2} \frac{1}{N} g^2 \Lambda^2$$

From scalar quartic interactions we get
$$\frac{1}{32\pi^2} \frac{2N^2 - 1}{2N} g^2 \Lambda^2$$

So, the gauge contributions are not completely cancelled.

This can be understand by counting the number of degree of freedoms.

The projection breaks SU(2N) to SU(N)$_A$ × SU(N)$_B$ × U(1)$_{A-B}$.

Now focus on the contributions to the mass of $\tilde{q}_A$

Bosons (gauge bosons and D terms)

- SU(N)$_A$: N^2-1 (each boson contributes 1)

- U(1)$_{A-B}$: ½ (each boson contributes ½)

Fermions (off diagonal) : have 2N^2 degree of freedoms and each fermion contributes 1/2

$$-N^2$$

If we would have begun with U(2N), an extra U(1) gauge boson and D contribute another ½. The cancellation will be complete. But we cannot do that since the equality of the gauge couplings of SU(N) and the U(1) cannot be justified.

Summary

- The gauge contributions are suppressed by ~1/N

- The Yukawa contributions can be cancelled completely but we will need all superfields to be bi-fundamental.

- This is what we know so far but not really useful practically since the Higgs in the Standard Model is not bi-fundamental.

The mechanism works only on very limited specific class of models. How can we generalize it to include a larger class of models ?

(by paying a certain price, of course)

Folded Supersymmetry

Based on this observation, we outline a set of procedures which suitably
extend the particle content and vertices of a theory so as to cancel the one loop
quadratically divergent contribution to the mass of a scalar arising from
a specific interaction, to leading order in N. These `rules' apply to Yukawa
interactions and also to SU(N) gauge interactions.

- Supersymmetrize.
- In the relevant graphs identify an index as being summed from 1 to N. By
  suitably expanding the particle content and gauge, global and discrete
  symmetries of the theory, extend this sum from 1 to $2N$. For Yukawa
  interactions and gauge interactions this can always be done in such a way
  that the resulting theory is invariant under $Z_{2\Gamma}$ and Z_{2R} symmetries.
- Project out states odd under the combined $Z_{2\Gamma}$ and Z_{2R} symmetries. The
  resulting daughter theory will be free of quadratic divergences to leading
  order in N.

We now apply these rules to construct a model where the top loop is cancelled by new scalars not charged under SM color.

- First, the supersymmetrized top Yukawa term is

$$\lambda_t \, (3,2)_{Q3} \, (1,2)_{HU} \, (\bar{3},1)_{U3}$$

- For U_3, the dummy index is 2.

- For H_u, the dummy index is 3.

- For Q_3, there is no dummy index.

    - So Q_3 is not protected, but what we need is the protection on H_u and that can be achieved by doubling the sum of the color, i.e. treat 3 as large N

    - So we can extend the SU(3) to SU(6),

    - And break it down to $SU(3)_A \times SU(3)_B$ by the projection we have discussed in the begining

We can also double the sum of color by extending the SU(3) to SU(3)$_\mathbf{A}$ × SU(3)$_\mathbf{B}$ × Z$_\mathbf{2}$

The top Yukawa in both cases are

$$\lambda_t \left[Q_{3A} H_U U_{3A} + Q_{3B} H_U U_{3B} \right]$$

It is invariant under both SU(6) and SU(3)$_\mathbf{A}$ × SU(3)$_\mathbf{B}$ × Z$_\mathbf{2}$

- Since H$_\mathbf{u}$ is color singlet, both cases are equivalent for our purpose.

- So we can make the simplest model by choosing the gauge group to be SU(3)$_\mathbf{A}$ × SU(3)$_\mathbf{B}$ and add a Z$_\mathbf{2}$ that interchanges A and B.

- We can also consider extending the SU(2) to SU(4) that will help suppressing the one loop gauge contributions and protect U$_\mathbf{3}$

We will concentrate on the simplest model where SU(2) is not extended in the rest of the talk.

UV Completion : 5D model

y=πR

y=0

$SU(3)_A \times SU(3)_B \times SU(2)_L \times U(1)_Y \times Z_2$

H_u, H_d

Q_A, U_A, D_A, L_A, E_A

Q_B, U_B, D_B, L_B, E_B

$S_1/Z_2 \times Z'_2$
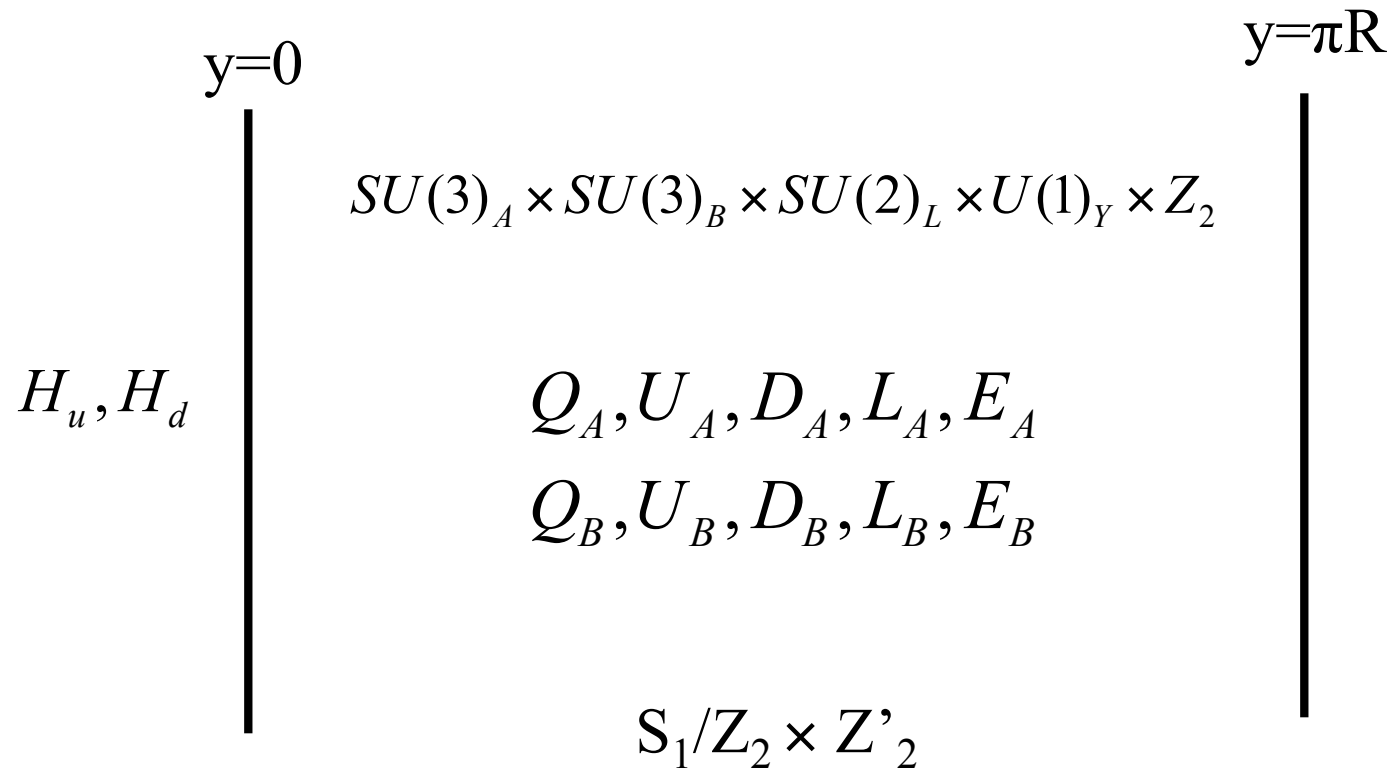
The projections we discussed before is now replaced by the orbifolding conditions.

$$\text{5D, } N=1 \text{ SUSY} \ = \ \text{4D, } N=2 \text{ SUSY}$$

The orbifolding conditions are chosen in such a way that

- SUSY is totally broken (Scherk-Schwarz breaking)
- Fermions in the SM sector (A) and bosons in the mirror sector (B)

  have zero modes
- Gauge bosons have zero modes

Low energy effective 4D theory has :

SM particles, F-gluon, F-sparticles and Higgsino

All other sparticles are much heavier.

Supersymmetry is not manifest at low energy

The cancellations

Roughly

- The gauge contributions are cancelled by gauginos like MSSM

- The top contribution is cancelled completely by using Folded-SUSY mechanism at one loop.

- Regain top contribution at two-loop trigger EW symmetry breaking

Radiative corrections

At one-loop

Top Yukawa contribution to the Higgs mass is cancelled, even for the contributions from all the heavy KK modes. Since at every KK level, there are equal number of fermions and bosons with the same coupling to the Higgs.

Gauge contributions : Just like MSSM, the contribution is proportional to the   gaugino mass squared

$$\delta m_H^2|_{\text{gauge}} = \text{K}\,\frac{3g_2^2 + g_1^2}{16\pi^4}\,\frac{1}{R^2}$$

F-stops are not protected. They get a mass at one loop. The dominant contributions come from the color gauge coupling and Yukawa coupling.

$$m_Q^2 = \text{K}\,\frac{1}{4\pi^4}\left(\frac{4}{3}g_3^2 + \frac{3}{4}g_2^2 + \frac{1}{36}g_1^2\right)\frac{1}{R^2} \quad + \quad \text{K}\,\frac{\lambda_t^2}{8\pi^4}\frac{1}{R^2}$$

$$m_U^2 = \text{K}\,\frac{1}{4\pi^4}\left(\frac{4}{3}g_3^2 + \frac{4}{9}g_1^2\right)\frac{1}{R^2} \quad + \quad \text{K}\,\frac{\lambda_t^2}{4\pi^4}\frac{1}{R^2}$$

At two loop

The F-stops that cancel the top loop get masses at one loop. This means that the Higgs will get a two-loop contribution

$$\delta m_H^2 |_{\text{top}} \approx -\frac{3\lambda_t^2}{4\pi^2} \tilde{m}_t^2 \log\left(\frac{1}{R\,\tilde{m}_t}\right)$$

The Fine tuning in this model is

$$\frac{m_{H,\text{phys}}^2}{2\delta m_H^2 |_{\text{top}}} \times 100\%$$

For 1/R of order 5 TeV, a cutoff Λ of 20 TeV and a Higgs mass of 115 GeV the fine-tuning is of order 12%. For a Higgs mass of 200 GeV this falls to 40%.

In the absence of further interactions between the A and B sectors the lightest F-spartner, the F-slepton is stable. To avoid the cosmological bound on stable charged particles we add to the theory the non-renormalizable interactions

$$\delta\left(y\right)\int d^2\theta\left(\frac{Q_A Q_A Q_A L_B}{\Lambda}+\frac{Q_B Q_B Q_B L_A}{\Lambda}\right)$$

and

$$\delta\left(y\right)\int d^2\theta\left(\frac{U_A U_A D_A E_B}{\Lambda}+\frac{U_B U_B D_B E_A}{\Lambda}\right)$$

This allows F-sleptons to decay to 3 quarks and the LSP, which in this case is mostly Higgsino. F-baryons are also no longer stable, and decay before nucleosynthesis. Standard Model baryons are still stable because decays to F-leptons are kinematically forbidden.

Collider Phenomenology

F-slepton

can be pair produced at the LHC through their couplings to the W,Z and photon. These cascade down to the lightest right-handed F-slepton (F-slepton of about 100 GeV). This eventually decays to three jets and the LSP after traveling anywhere from a few mm. to tens of meters, depending on the exact parameters of the model. The collider signatures therefore include either six jet events with missing energy, or highly ionizing tracks.
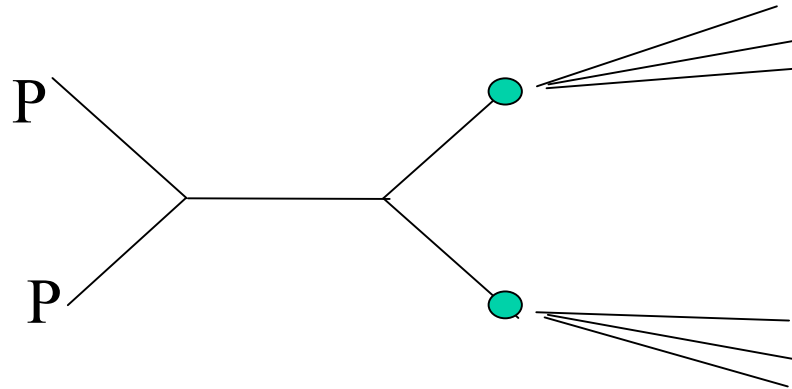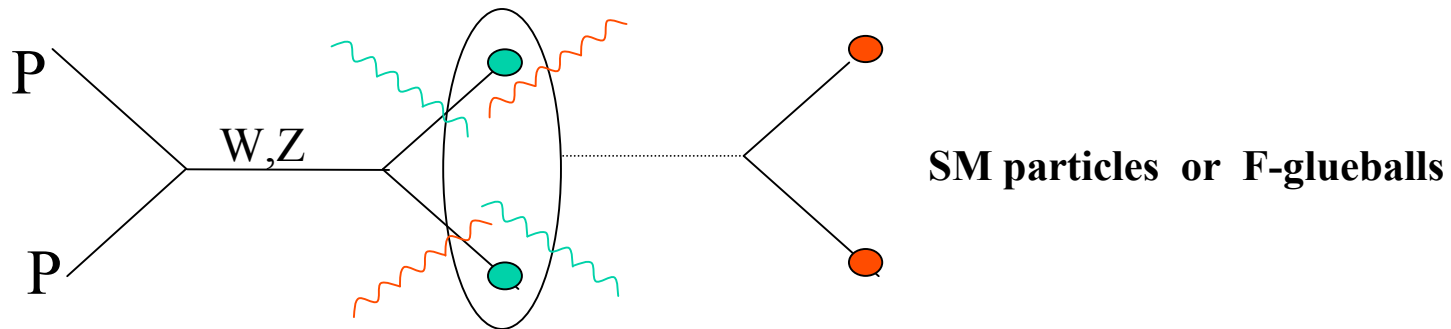
F-squarks

Are pair produced Drell-Yan. However, the absence of light states (compared with $\Lambda_{F\text{-}QCD}\sim$ 10 GeV) charged under F-color implies that they cannot hadronize individually. They therefore behave like scalar quirks or `squirks'.

[Strassler & Zurek; Luty, Kang & Nasri]

The two F-squirks are connected by an F-QCD string and together form a bound state. This bound state is initially in a very excited state but promptly decays down to its ground state by the emission of soft F-glueballs and soft photons. Eventually the two F-squarks pair annihilate into 2 F-glueballs, 2 hard W's, 2 hard Z's or 2 hard photons. They could also annihilate into SM fermions through an off-shell W,Z or photon.

P

P

W,Z

SM particles or F-glueballs

- Below the mass of the F-squarks this is essentially a hidden valley model and the F-glueballs decay back to Standard model states. Unfortunately, this decay is very slow and the F-glueballs decay well outside the detector.

  [Strassler and Zurek]

- However, the other decay modes are very characteristic, and given enough events it should be straightforward to determine the masses of the F-squarks from the energy distributions of outgoing leptons and photons.

Conclusion

- Folded-SUSY solves the LEP paradox without the usual stop.
- The collider signatures of this class of models are very different from the traditional supersymmetric models.
- This provides another counter-example to the conventional wisdom that new particles that cancel the one-loop top contribution to the Higgs mass must be colored.

Spectrum

- F-squarks are ~500-700 GeV

- lightest F-slepton ~ 100 GeV

- gaugino ~ 1TeV

- 1/R = 5TeV

- Λ of 5D theory ~ 20 TeV

Folded gauge contribution = SM *(4/3)/(2(N^2-1))

D-term   U(1)

SU(N)