M Anber, LS, JCAP

N-flationary magnetic fields

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Plan of the talk

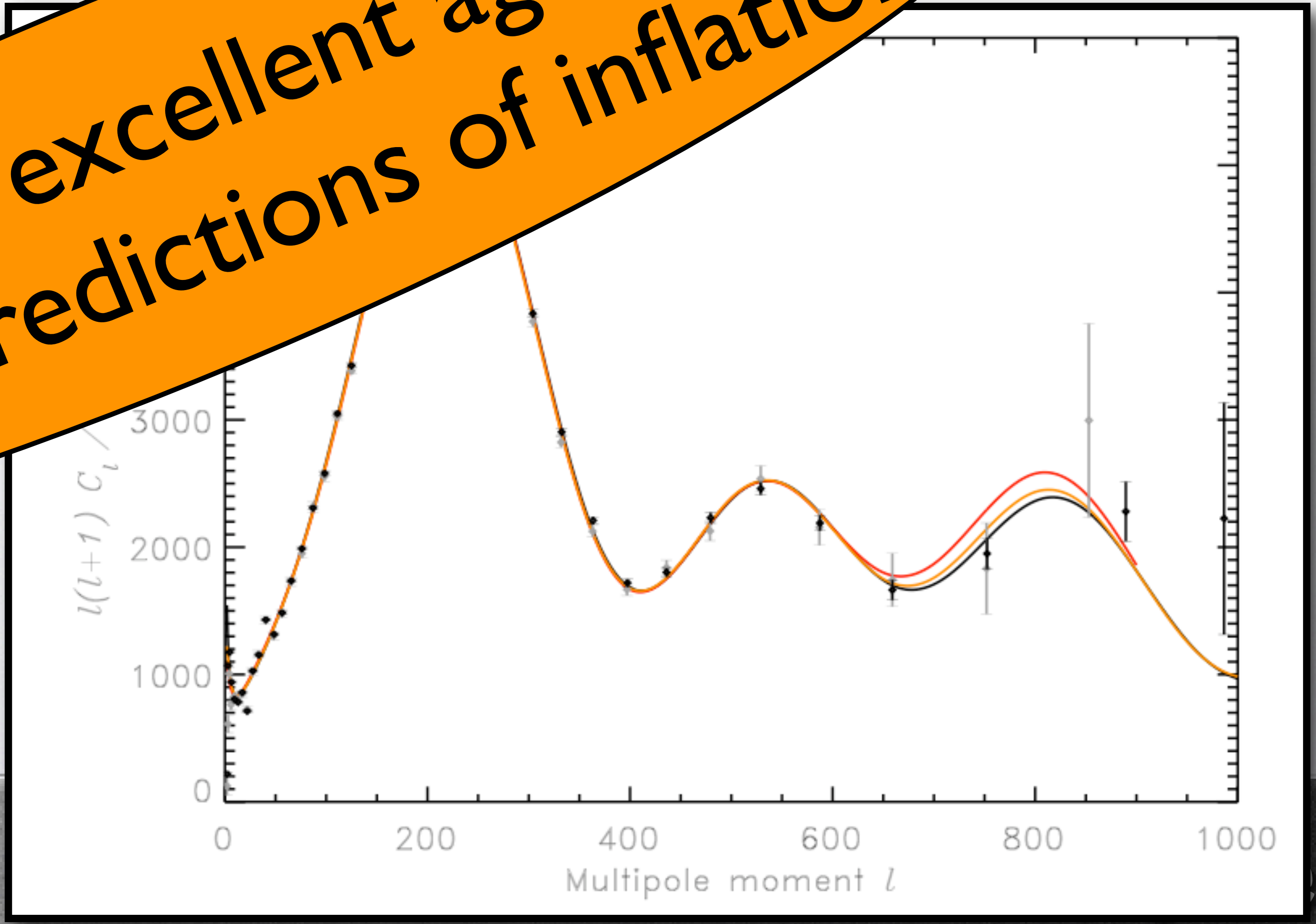
- Inflation in QFT and in String Theory
- pNGBs and inflation
- cosmological magnetic fields
- pNGB inflation produces cosmological magnetic fields

The success of inflation

Cosmological observations *strongly* support (at the % level)

- A spatially flat Universe
- A (quasi) scale inva
  of primordi

All of this in excellent agreement with the predictions of inflation



But what are the properties of inflation?

...and in particular, when did it take place?

...or, equivalently,
at what energy scales did it take place?

What is the energy scale of inflation?

An upper bound from CMB:

$$10^{-5} \sim \left(\frac{\delta T}{T}\right)_{\mathrm{CMB}} \sim \frac{\sqrt{\rho_{\mathrm{infl}}}}{M_P^2}\frac{1}{\sqrt{\varepsilon}}$$

Where $\varepsilon \simeq \frac{\rho_{\mathrm{infl}} + p_{\mathrm{infl}}}{\rho_{\mathrm{infl}}}$ has to be <1 during inflation

$\rho_{infl}^{1/4} < 10^{16}$ GeV

Unless the inflationary dynamics is very finely tuned,
ϵ is not "too small"

The inflationary scale might be just a factor
of ~ 1000 smaller than the gravity scale

Inflation can be a probe of the physics
of the fundamental theory of gravity

Strings

“Inflation in String Theory”

...a challenge!

At “low” energies String Theory must reduce to Quantum Field Theory...

...and finding good models of inflation in QFT is very difficult

...it is even more difficult to find which of those models can come from string theory

so let us start by looking at inflation in QFT...

Requirements for Inflation

In simplest models,
inflation is driven by a scalar field φ
with potential $V(\varphi)$.

Requirements on $V(\varphi)$:

$$\varepsilon = \frac{M_P^2}{2}\frac{V'(\phi)^2}{V(\phi)^2} \simeq 10^{-2} \qquad \eta = M_P^2 \frac{V''(\phi)}{V(\phi)} \simeq 10^{-2}$$

$\Rightarrow$ $V(\varphi)$ has to be ***flat***

The enemy: radiative corrections

Quantum effects bring couplings to be O(1)
in units of the cutoff of the theory (⇒M_P)

Spoil flatness of $V(\varphi)$

Our ally: symmetries

Supersymmetry is an option...

...but supergravity corrections generate
mass^2=O(V/M_P^2)

η=O(1)

(η problem)

A simple possibility...

A field φ has a *shift symmetry*
if the theory that describes it
is invariant under the transformation

$$\boxed{\varphi \longrightarrow \varphi + c}$$

If this symmetry is exact, the only possible potential for φ is $V(\varphi)$=constant

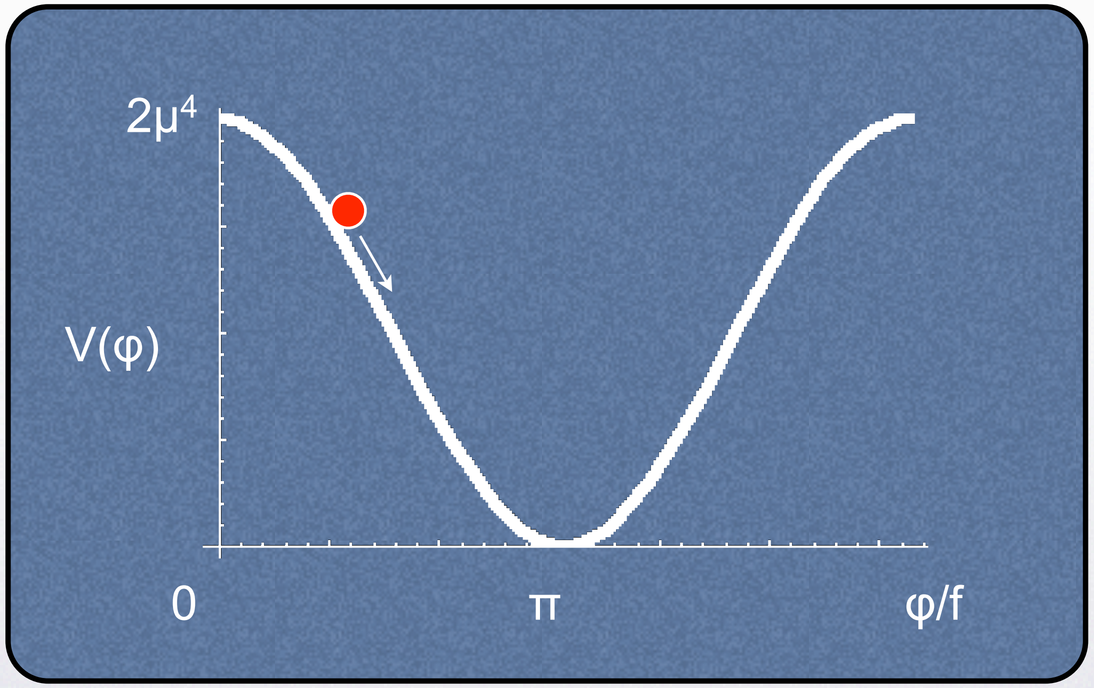
(i.e. a cosmological constant...)

now let us break the shift symmetry a little bit...
the potential for φ changes to

$$V(\varphi)=\mu^4\,[\,\cos(\varphi/f)+1\,]$$

Freese et al 1990

f measures the breaking of the symmetry ➡ in the limit $f \to \infty$ the symmetry is restored



The cosine potential: where does it come from?

- Theory with a spontaneously broken global U(1)

$$\mathcal{L} = \partial_\mu H^* \, \partial^\mu H - \lambda \left(|H|^2 - v^2 \right)^2$$

- Decompose $H = (v + \delta H) \, e^{i\phi/v}$

  where δH is massive and φ is a massless Goldstone boson

- The global U(1) is broken e.g. by gravitational interactions

$$\delta\mathcal{L} = \frac{1}{M_P} \left(H + H^* \right)^5$$

- A potential is generated: $\delta V(\varphi) \simeq \frac{v^5}{M_P} \cos\left(\frac{5\,\varphi}{v} \right)$

PSEUDO-NAMBU-GOLDSTONE BOSON
PNGB

Because of its radiative stability,

A pNGB is an excellent candidate for inflation in Quantum Field Theory

(Natural Inflation)

What about String Theory?

☺ String Theory contains a lot of pNGBs (many inflaton candidates)

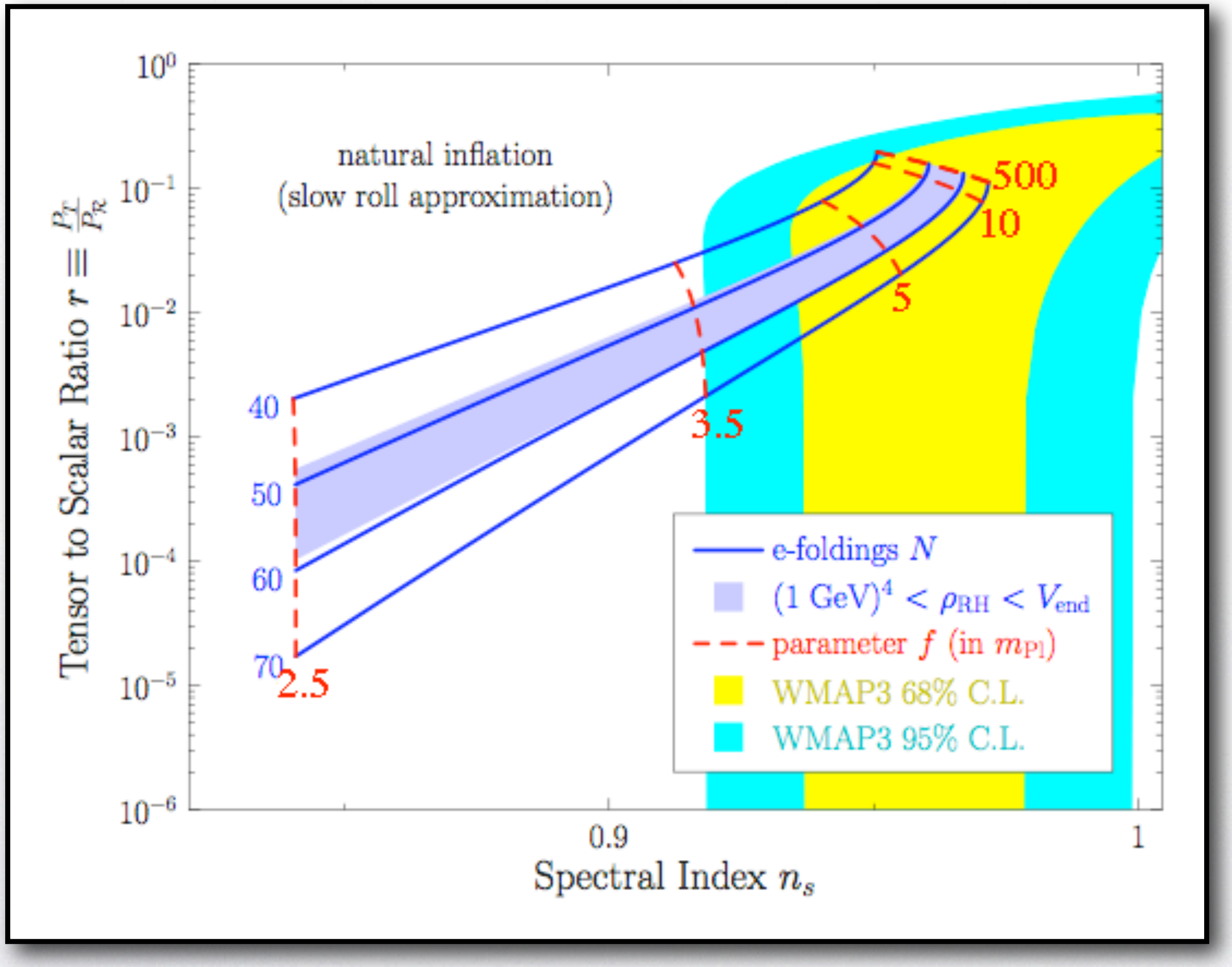
☹ Not every pNGB can come from String Theory *("swampland")*:

String theory appears to require

$$0<f\lesssim M_P$$

Banks, Dine,
Fox and Gorbatov 2003

The data:



from Savage et al, 2006

$f > 3.5\ M_P$

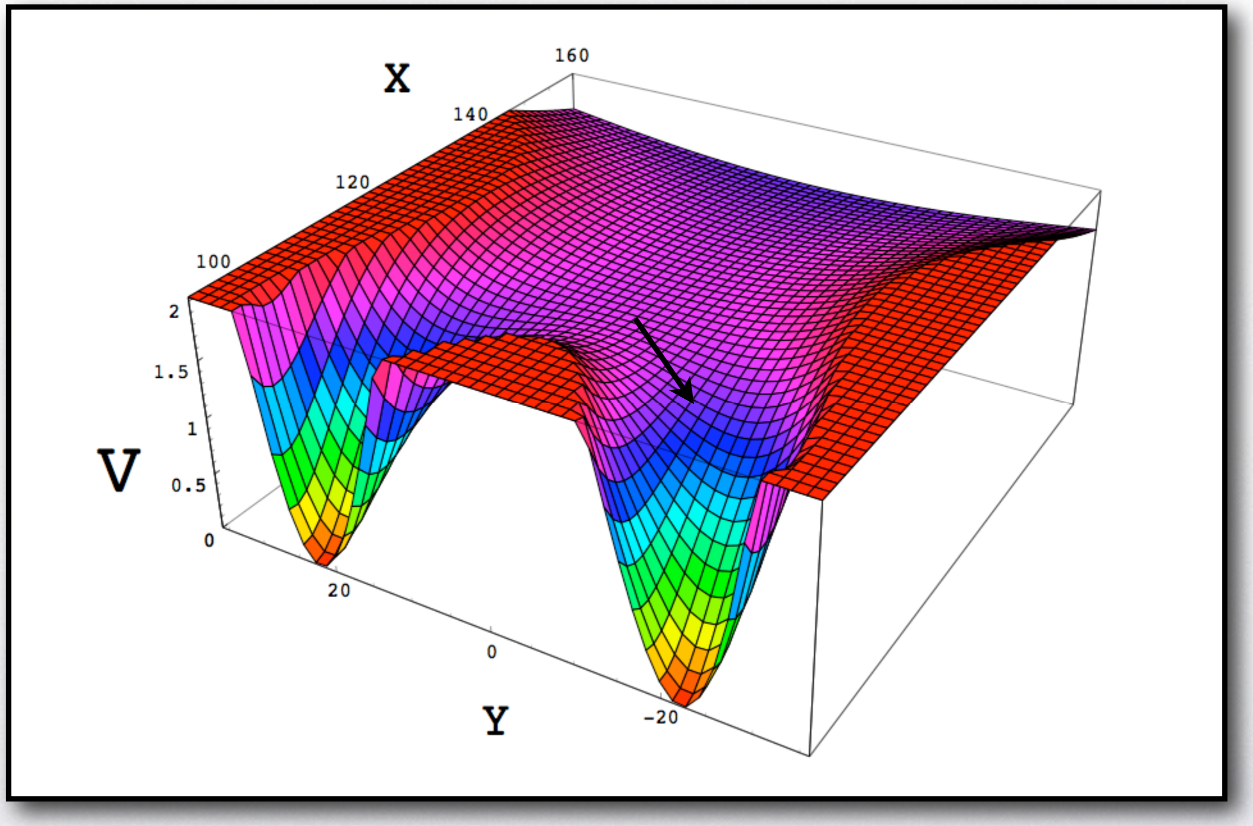
In contradiction with
the requirement
from String Theory!

Way out: use extra fields (i)

Racetrack inflation:

Blanco-Pillado et al 2004

inflaton is mixture of a pNGB and a modulus

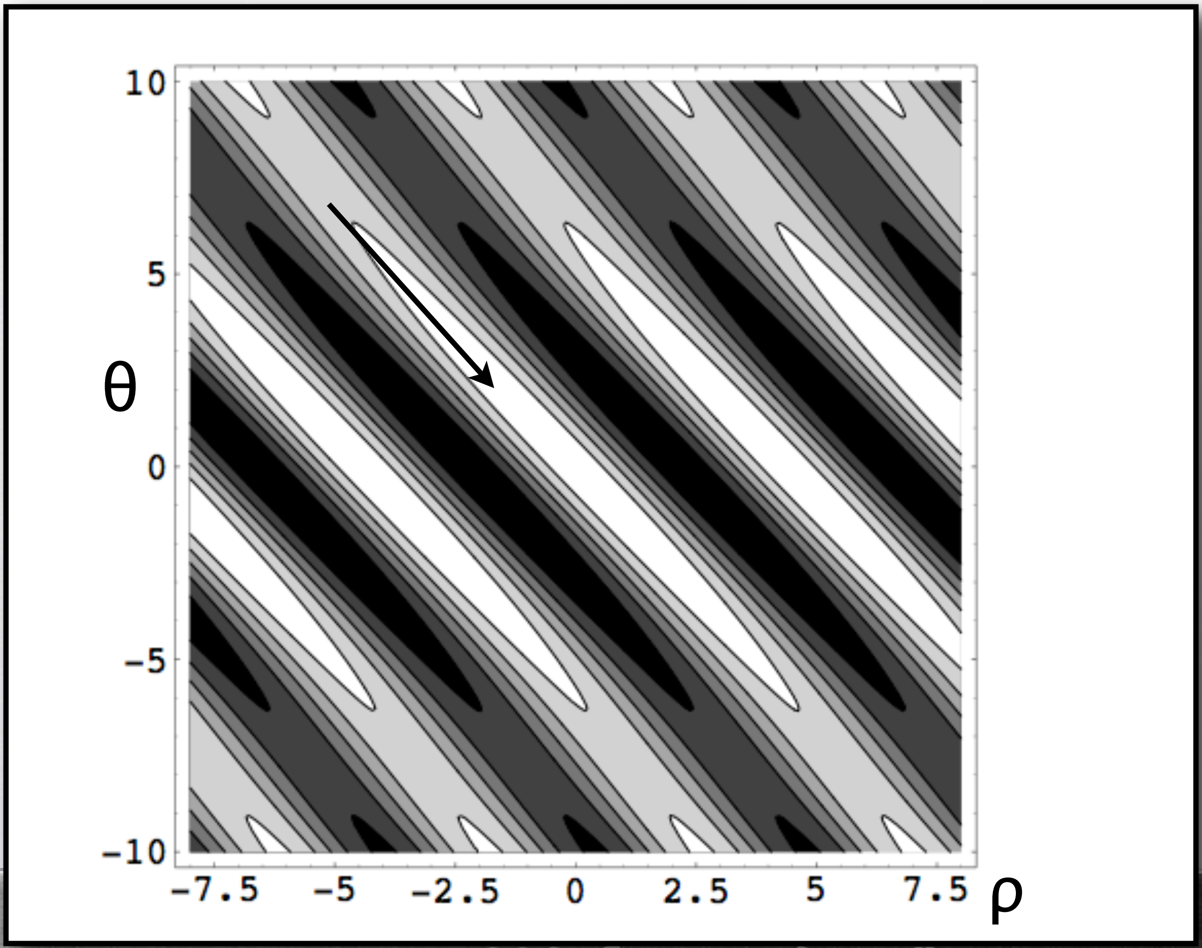


Way out: use extra fields (ii)

With two pNGBs:

Kim, Nilles and Peloso 2004

$$V = \Lambda_1^4 \left[1 - \cos\left(\frac{\theta}{f_1} + \frac{\rho}{g_1}\right)\right] + \Lambda_2^4 \left[1 - \cos\left(\frac{\theta}{f_2} + \frac{\rho}{g_2}\right)\right]$$



Way out: use extra fields (iii)

Use ***several*** pNGBs

Dimopoulos et al 2005

N-flation

(assisted inflation with pNGBs)

How does it work?

Start from N pNGBs:

$$\mathcal{L} = -\sqrt{-g}\sum_{i=1}^{N}\left\{\frac{1}{2}\left(\partial\phi_i\right)^2 + \Lambda_i^4\left[1+\cos(\phi_i/f_i)\right]\right\}$$

Assume that all the φ_i, all the f_i and all the Λ_i are equal:

$$\mathcal{L} = -\sqrt{-g}\left\{\frac{N}{2}\left(\partial\phi\right)^2 + N\,\Lambda^4\left[1+\cos(\phi/f)\right]\right\}$$

Canonically normalized field $\Phi = \sqrt{N}\varphi$

$$\mathcal{L} = -\sqrt{-g}\left\{\frac{1}{2}\left(\partial\Phi\right)^2 + N\Lambda^4\left[1+\cos\left(\frac{\Phi}{\sqrt{N}f}\right)\right]\right\}$$

Can be $> M_P$ even if $f < M_P$!

How many pNGBs can String Theory have?

Dimopoulos et al 2005

In principle, up to 10^5

...but if we want to keep radiative corrections to M_P under control,

$$\boxed{N \lesssim 200}$$

is needed

How many pNGBs do we need?

Dimopoulos et al 2005

Assuming $\varphi_1 = \varphi_2 = \ldots = \varphi_N << f$:

$$N \sim 200$$

$\left(\begin{array}{c}\text{marginally compatible with}\\ \text{radiative stability requirements}\end{array}\right)$ (!)

Liddle & Kim 2006

Assuming $\varphi_1, \varphi_2, \ldots, \varphi_N << f$,
$\varphi_1, \varphi_2, \ldots, \varphi_N$ homogeneously distributed

$$N \sim 600$$

(things get worse)

However things are not so bad if we drop the requirement $\varphi_i << f$...

This requirement corresponds to approximating

$1-\cos(x) \sim x^2/2$...

...that corresponds to requiring a large effective f

...that corresponds to requiring a large N

If we drop this requirement

N as small as **50** is enough to have enough N-flation

VERY PRELIMINARY!

In any case, pNBGs seem to play a role in many models of inflation in String Theory...

pNGBs are coupled to the electromagnetic field

M. Anber, LS 2006

Stick to N-flation to fix ideas, but these arguments are good for any model of pNGB inflation

$$\mathcal{L} \supset \sum_{i=1}^{N} \alpha \frac{\phi_i}{4\, M_P} F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$[\alpha = O(1)]$

Magnetic fields can be produced by the rolling pNGBs at inflation

Cosmological magnetic fields

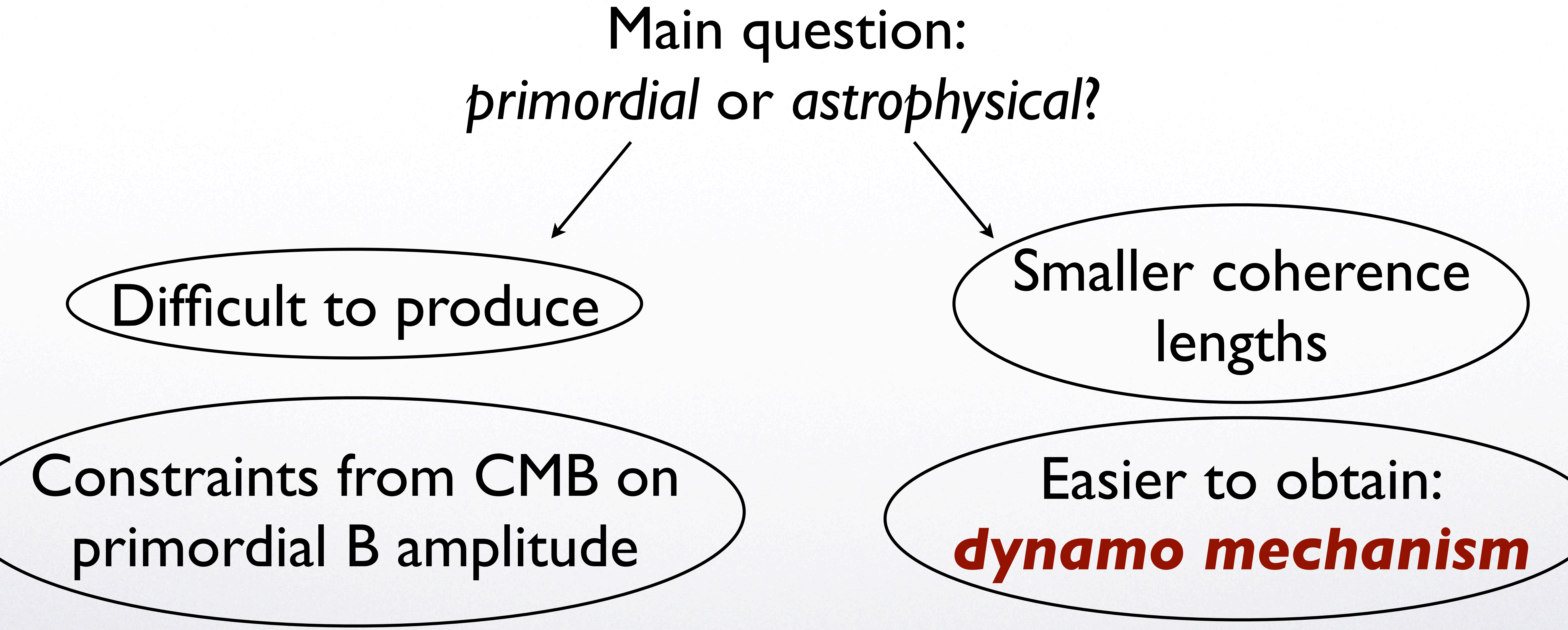
Observed with a number of techniques

(Zeeman splitting
Faraday rotation
Synchrotron emission)

- In the Galaxy (~kpc), solid evidence of $B \cong 2\text{-}4\ \mu G$
- In clusters (~10-100 kpc), some evidence of $B \cong 1\ \mu G$

  (but people consider also B~nG)
- At larger scales, situation more confused

...and their origin is unknown!



Main question:

primordial or *astrophysical*?

Difficult to produce

Constraints from CMB on primordial B amplitude

Smaller coherence lengths

Easier to obtain: ***dynamo mechanism***

The dynamo

Uses differential rotation of plasma in galaxies
to amplify an existing “seed” B field

$$\frac{\partial \mathbf{B}}{\partial t} = \boldsymbol{\nabla} \times (\mathbf{v} \times \mathbf{B}) + \frac{1}{4\pi\sigma}\nabla^2\mathbf{B} ,$$

velocity of plasma

conductivity

How large a seed field is needed?

|B| doubles at every galaxy rotation

↓

Exponential amplification

↓

Exponential uncertainties:

~~~~~~~~~~~~~~~~~~~~~~~~~~~

Giovannini → Need 10^{-23} G at 1 Mpc

Davis → Enough 10^{-30} G at 10 kpc ✓

very difficult to produce even such weak fields...
~~~~~~~~~~~~~~~~~~~~~~~~~~~

back to our model...

$$\mathcal{L} \supset \sum_{i=1}^{N} \alpha \frac{\phi_i}{4\,M_P} F_{\mu\nu}\tilde{F}^{\mu\nu}$$

As in Dimopoulos et al 2005, simplify analysis by assuming $|\varphi_1| = |\varphi_2| = ... = \Phi/\sqrt{N}$

Electromagnetic field coupled to the *sum* of the pNGBs

the *direction* of rolling of the pNGBs matters:

define $\gamma=(N_+-N_-)/N$ where

N_+= # of pNGBs with $\phi>0$

N_-= # of pNGBs with $\phi<0$

$[-1<\gamma<1]$

Main equation:

$$\boxed{\frac{\partial^2 F_\pm}{\partial \tau^2} + \left(k^2 \pm \frac{\alpha\,\gamma\sqrt{N}}{M_P}\,\frac{d\Phi}{d\tau}\,k\right) F_\pm = 0}$$

$F_\pm$= >ve and <ve helicity comoving modes of the magnetic field

(τ=conformal time)

One of the two modes has
a ***negative***, ***time dependent*** “mass term”

↓

Exponential amplification
of one helicity mode

The result depends only on
one combination of parameters

$$\boxed{\xi \equiv |\alpha\gamma| \sqrt{N\epsilon/2}}$$

where ε is the slow-roll parameter

$$\varepsilon \sim \dot{\phi}^2/V(\phi)$$

Our result

$$\boxed{F(\tau,\vec{k}) \;\simeq\; \sqrt{\frac{k}{2}}\left(\frac{k}{2\xi\, aH}\right)^{1/4} e^{-2\sqrt{2\xi\, k/aH}}\, e^{\pi\,\xi}}$$

Exponential amplification term!

(Comoving) Energy density in magnetic modes

$$\rho_{\rm magn} \sim k^3\, F^2$$

$$\rho_{\rm magn} \simeq H^4\,\left(k/H\right)^{4.5}\,e^{2\pi\xi}$$

for $k \lesssim H$

Power is concentrated in short wavelength modes

A Constraint

The energy in the magnetic field should not exceed the energy in the inflaton condensate!

If insist on COBE normalization (H~10^{13}GeV),

$\xi<7$

If require just H>10^{-3}eV,

$\xi<25$

...implication for model building:

In models of pNGB inflation in String Theory

If $\alpha\gamma\sqrt{N} \gtrsim 100$

the backreaction of the magnetic field
during inflation

cannot be neglected!

(difficult to tell what happens - nonlinearities)

Can we start the dynamo with these fields?

Garretson, Field and Carroll 1992

If we obey the constraints above

AND

~~If after inflation the magnetic field does not evolve (apart from effects related to expansion of the Universe)~~

THEN

The resulting magnetic field today is too weak to be the one we observe

Evolving the field in the cosmic plasma

The magnetic field produced has *maximal helicity* (generated by parity-violating background)

$$\mathcal{H} \equiv \int_V d^3x\, \mathbf{B} \cdot \mathbf{A}$$

and helicity is (almost) conserved for large conductivities

$$\frac{d\mathcal{H}}{dt} = -\frac{1}{4\pi\sigma} \int_V d^3x \mathbf{B} \cdot (\nabla \times \mathbf{B}) \cong 0$$

Dissipative processes suppress power at small scales

In order to conserve helicity,
power has to go to larger scales:

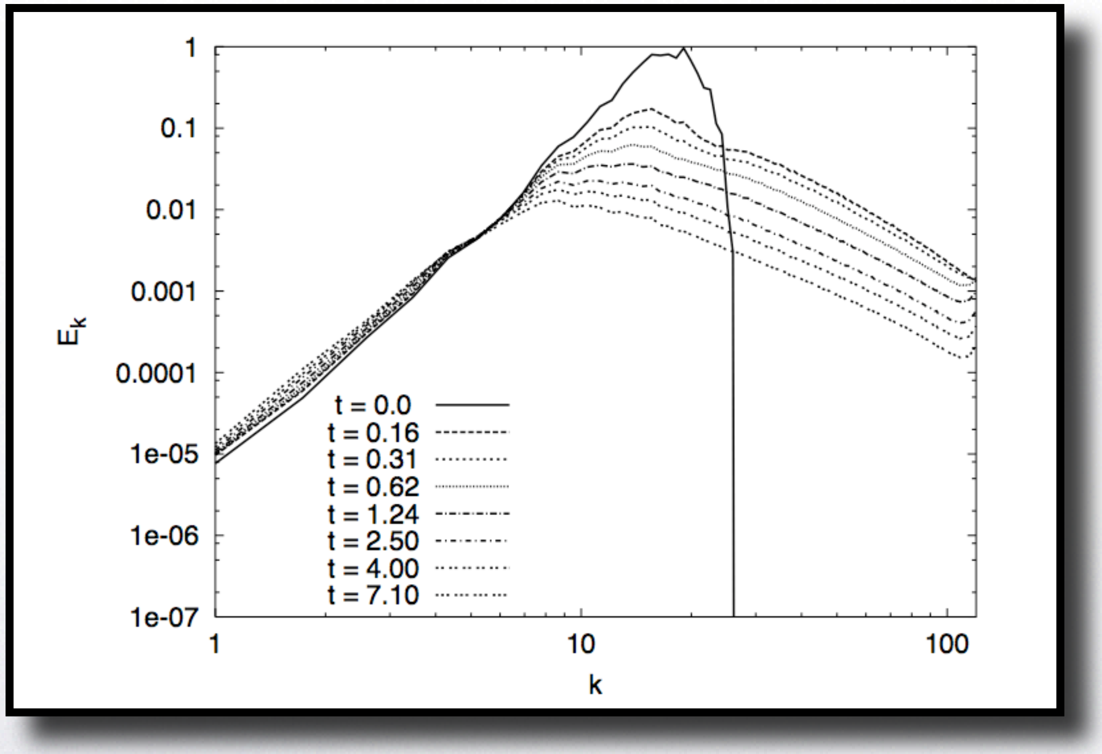
Inverse cascade

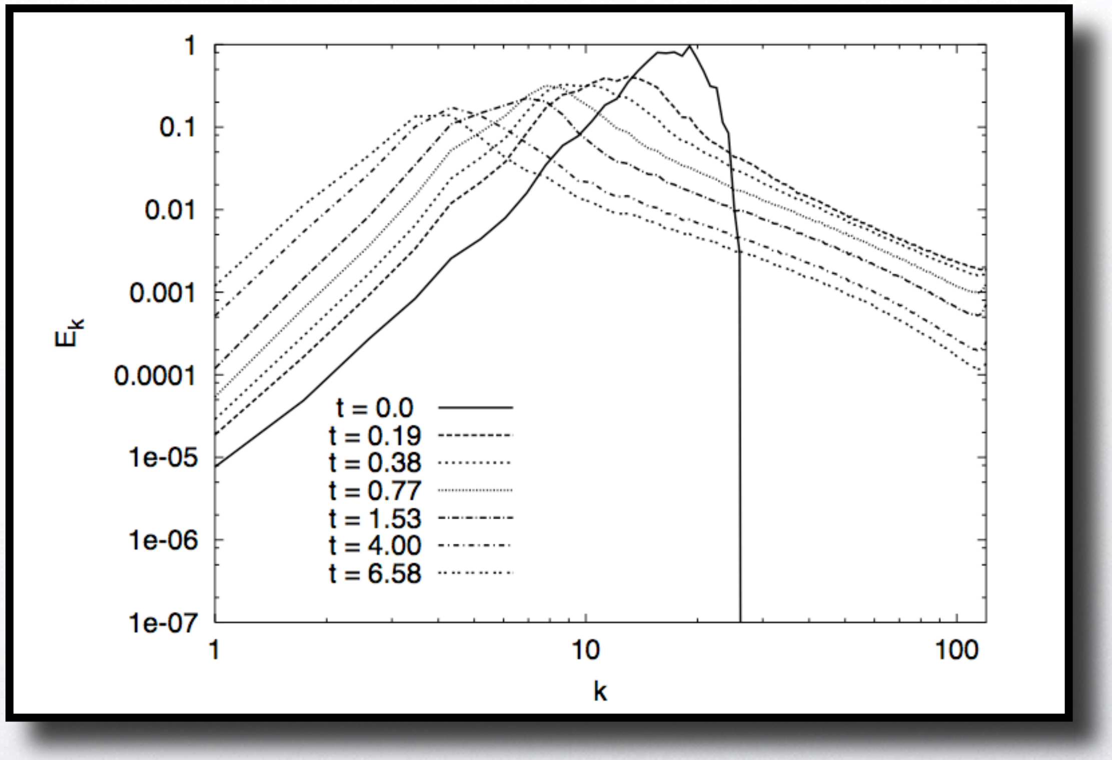
Numerical solutions

Evolution of the comoving magnetic field:

without helicity

with helicity





From Jedamzik and Banerjee 2004

Scalings:

- ☛ Coherence length $\propto \tau^{2/3}$
- ☛ Magnetic field strength $\propto \tau^{-2/3}$
- ☛ Spectral index for scales>coherence length:

  constant

(property of *self-similarity*)

Final value of the magnetic field

(before the dynamo)

$$B \simeq 10^{-33}\,\frac{e^{\pi\,\xi}}{\xi^{17/12}}\left(\frac{T_{\rm RH}}{10^{9}{\rm GeV}}\right)^{11/36}\left(\frac{l_{\rm phys}}{10\,{\rm kpc}}\right)^{-9/4}\,{\rm G}$$

$\xi \gtrsim 2$

is sufficient to initiate the dynamo

In terms of the original parameters

$$\alpha\,\gamma\,\sqrt{N} \gtrsim 10$$

Enough magnetic field for α and/or $\gamma\,\sqrt{\mathrm{N}}$ of *O*(few)!

Discussion...

→ One obvious possibility: N=few, $\alpha \sim 10$

→ More difficult: insist on $\alpha = 1$

e.g. for N=600 (as required by Liddle and Kim 2006),
need $N_+ \sim 420$ and $N_- \sim 180$...

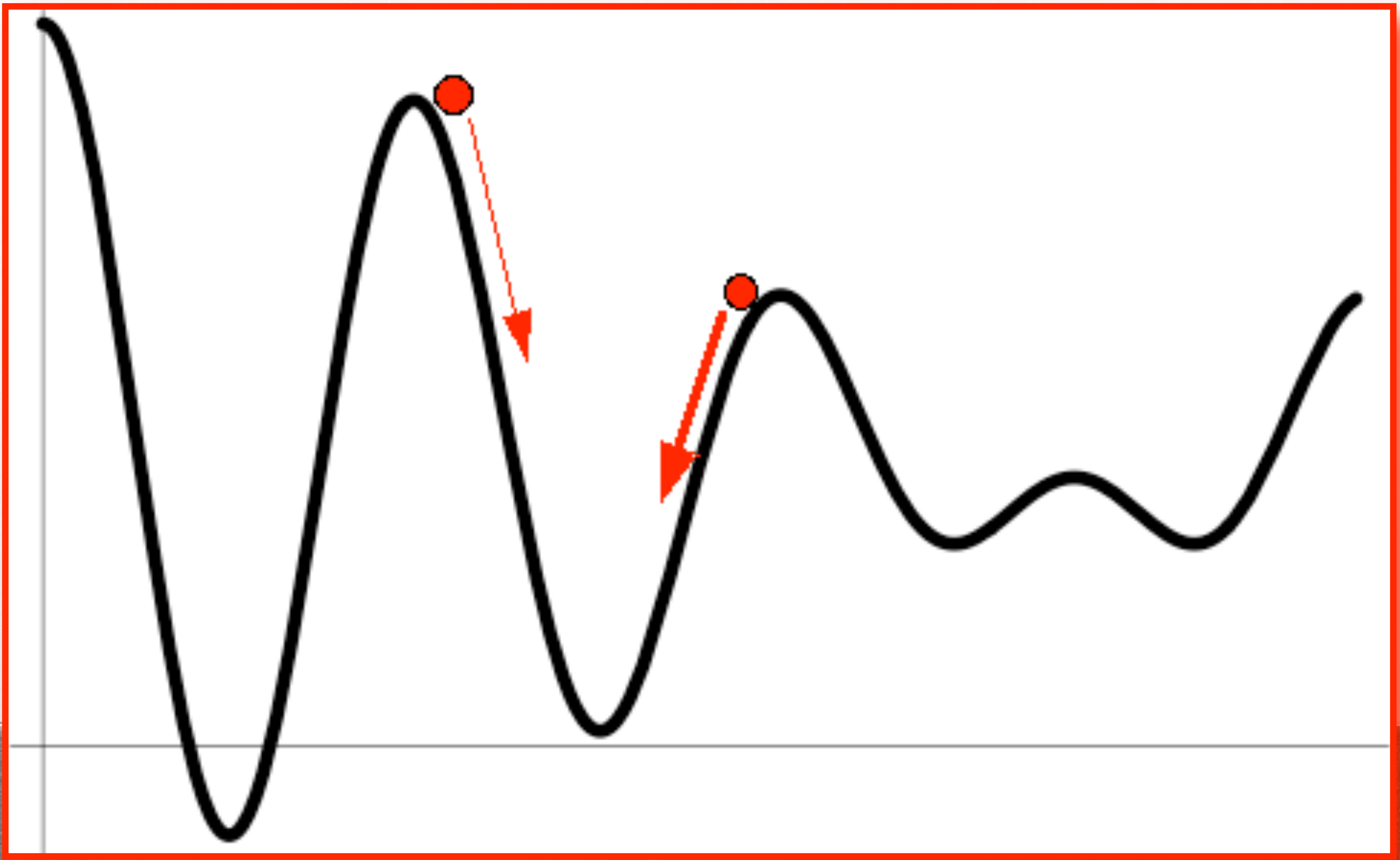
...rather improbable, if the theory is exactly symmetric wrt $\phi_i \rightarrow -\phi_i$

Probability $\propto \exp\{-(N_+ - N_-)^2/2N\} \approx 10^{-20}$!

...but an asymmetry can exist:

(Blanco-Pillado et al 2004)

$$V(\phi) = \Lambda_1^4 \cos a\phi + \Lambda_2^4 \cos b\phi + \Lambda_3^4 \cos(a-b)\phi$$



Conclusions

- pNGBs are very well motivated candidates for inflation
- By taking into account MHD effects, they can lead to the production of the observed cosmic magnetic fields
- To do: effects of pNGB perturbations on the magnetic fields (flat spectrum)