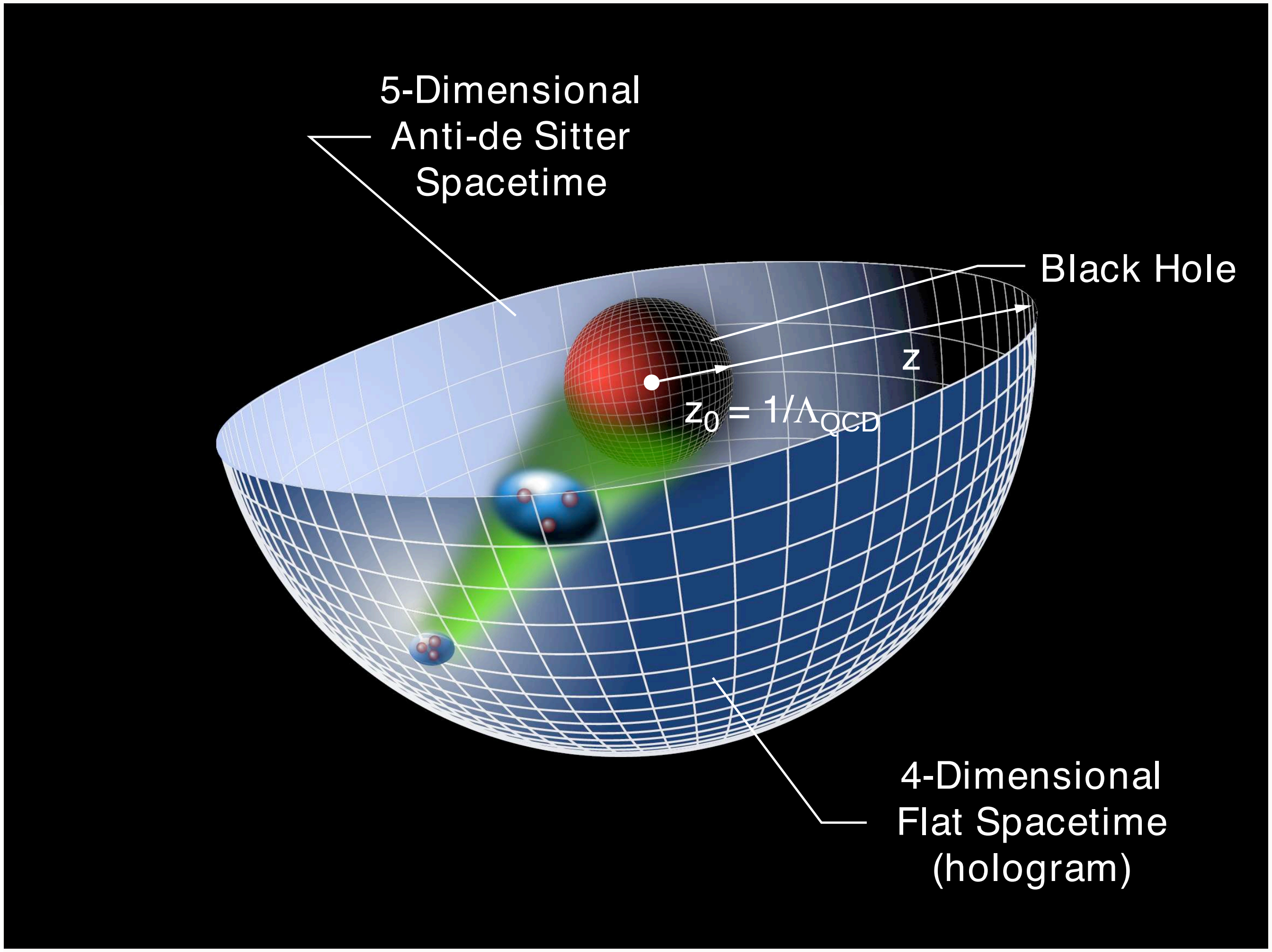
The Impact of AdS/CFT on QCD Phenomenology



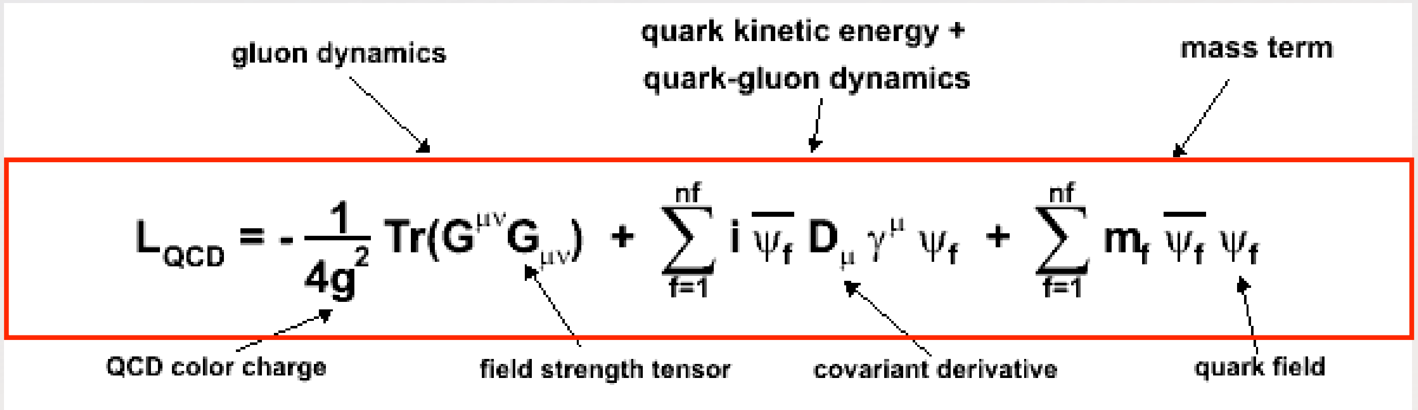
Changes in physical length scale mapped to evolution in the 5th dimension z

U.C. Davis
March 13, 2007

in collaboration with Guy de Teramond

Stan Brodsky
SLAC

QCD Lagrangian



QCD: $N_C = 3$ Quarks: 3_C Gluons: 8_C.

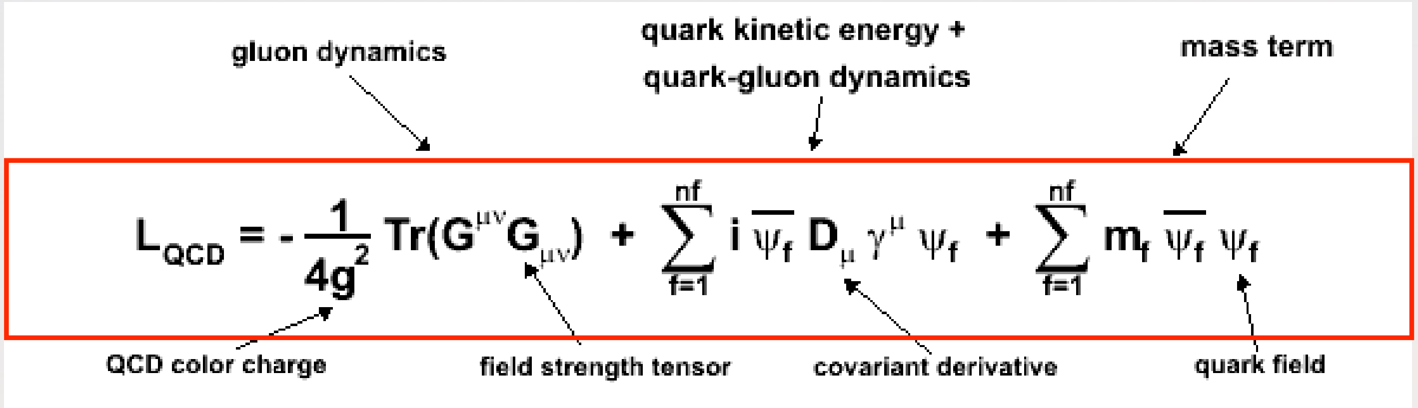
$\alpha_s = \frac{g^2}{4\pi}$ is dimensionless

Classical Lagrangian is scale invariant for massless quarks

If $\beta = \frac{d\alpha_s(Q^2)}{d \log Q^2} = 0$ then QCD is invariant under conformal transformations:

Parisi

QCD Lagrangian and Conformal Symmetry



Conformal Symmetry – Property of classical renormalizable Lagrangian

(massless quarks)

Poincare transformations plus

$$\text{dilatation} : x^\mu \to \lambda x^\mu$$

plus

$$\text{conformal transformations} : \text{inversion}[x^\mu \to -\frac{x^\mu}{x^2}] \times \text{translation} \times \text{inversion}$$

Strongly Coupled Conformal QCD and Holography

- Conformal Theories are invariant under the Poincaré and conformal transformations with $M^{\mu\nu}$, P^{μ}, D, K^{μ}, the generators of $SO(4,2)$.

- QCD appears as a nearly-conformal theory in the energy regimes accessible to experiment. Invariance of conformal QCD is broken by quark masses and quantum loops.

- Growing theoretical and empirical evidence that $\alpha_s(Q^2)$ has an IR fixed point: von Smekal, Alkofer and Hauck, arXiv:hep-ph/9705242; Alkofer, Fischer and Llanes-Estrada, hep-th/0412330; Deur, Burkert, Chen and Korsch, hep-ph/0509113 . . .

- Phenomenological success of dimensional scaling laws for exclusive processes

$$d\sigma/dt \sim 1/s^{n-2}, \quad n = n_A + n_B + n_C + n_D,$$

implies QCD is a strongly coupled conformal theory at moderate but not asymptotic energies

Brodsky and Farrar, Phys. Rev. Lett. **31**, 1153 (1973); Matveev *et al.*, Lett. Nuovo Cim. **7**, 719 (1973).

Maldacena:

AdS/CFT: mapping of AdS_5 X S_5 to conformal N=4 SUSY

- QCD not conformal; however, it has some manifestations of a scale-invariant theory: Bjorken scaling, dimensional counting for hard exclusive processes
- IR fixed point? $\alpha_s(Q^2) \simeq \text{const at small } Q^2$
- "Semi-classical" approximation to QCD
- Use mapping of conformal group SO(4,2) to AdS_5

- Polchinski & Strassler: AdS/CFT builds in conformal symmetry at short distances, counting, rules for form factors and hard exclusive processes; non-perturbative derivation

- Goal: Use AdS/CFT to provide models of hadron structure: confinement at large distances, near conformal behavior at short distances

- de Teramond, sjb: AdS/QCD Holographic Model: Initial "semi-classical" approximation to QCD: Remarkable agreement with light hadron spectroscopy

- Mapping to 3+ 1 Light-Front equations, wavefunctions

- Use AdS/CFT wavefunctions as expansion basis for diagonalizing $H^{LF}{}_{QCD}$; variational methods

Prediction from AdS/QCD

Only one parameter!

Entire light quark baryon spectrum

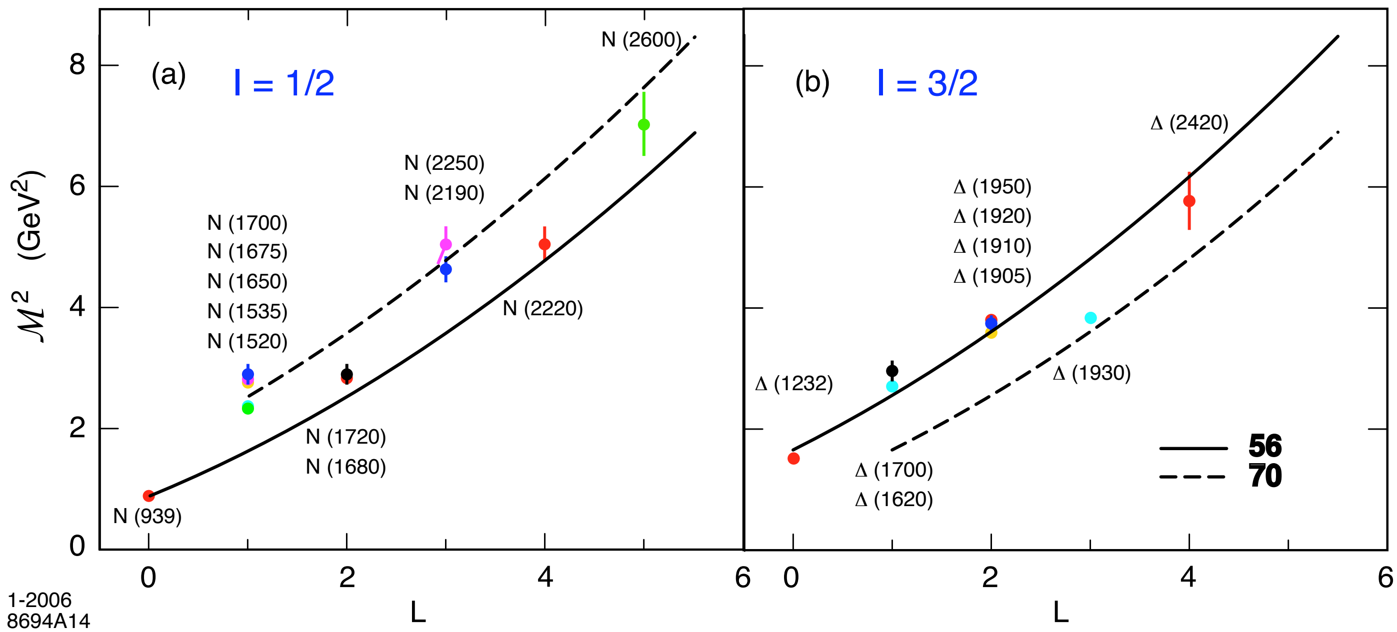


Fig: Predictions for the light baryon orbital spectrum for Λ_{QCD} = 0.25 GeV. The $\mathbf{56}$ trajectory corresponds to L even $P = +$ states, and the $\mathbf{70}$ to L odd $P = -$ states.

Guy de Teramond
SJB

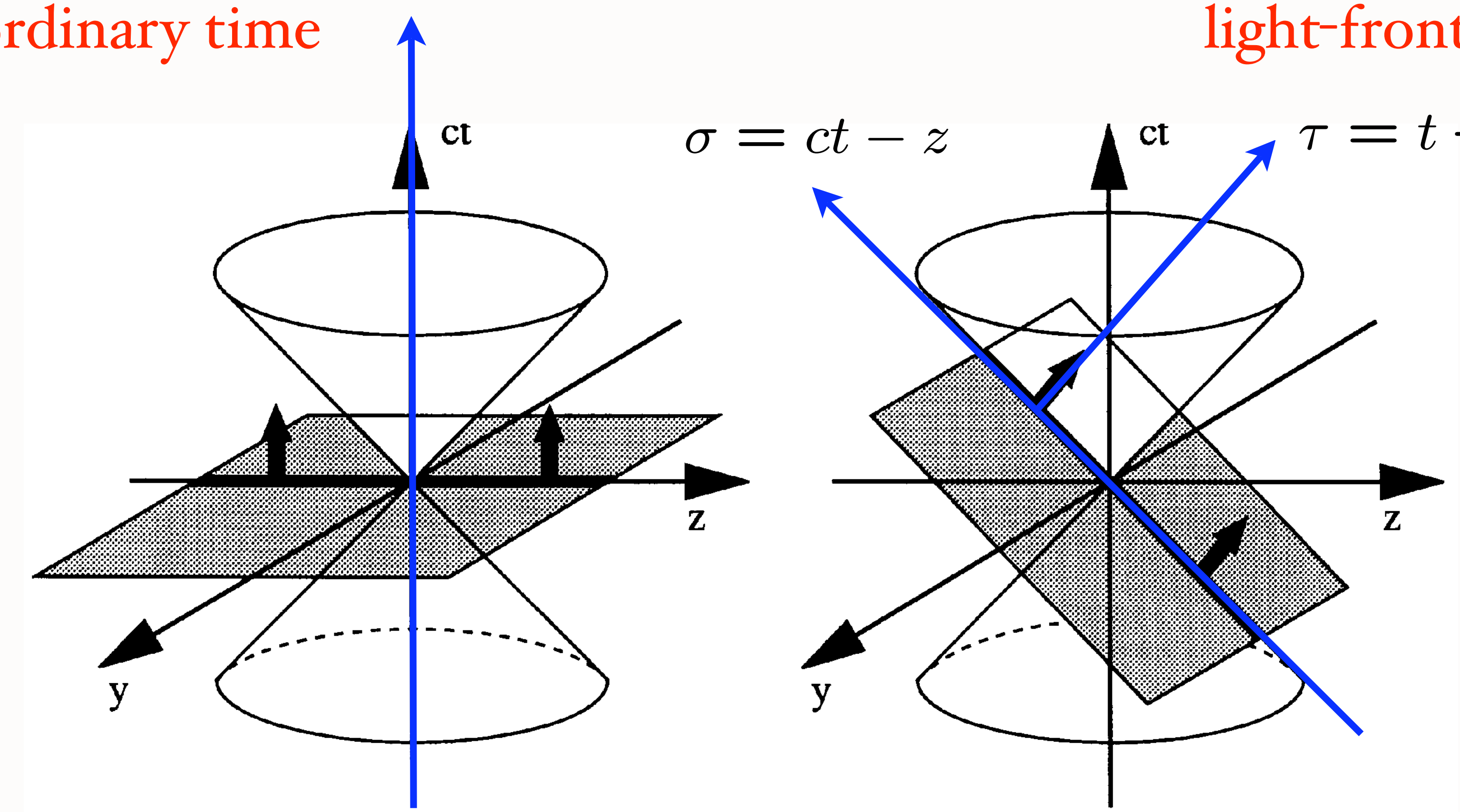
- $SU(6)$ multiplet structure for N and Δ orbital states, including internal spin S and L.

$SU(6)$	S	L	Baryon State
56	$\frac{1}{2}$	0	$N\frac{1}{2}^+(939)$
	$\frac{3}{2}$	0	$\Delta\frac{3}{2}^+(1232)$
70	$\frac{1}{2}$	1	$N\frac{1}{2}^-(1535)$ $N\frac{3}{2}^-(1520)$
	$\frac{3}{2}$	1	$N\frac{1}{2}^-(1650)$ $N\frac{3}{2}^-(1700)$ $N\frac{5}{2}^-(1675)$
	$\frac{1}{2}$	1	$\Delta\frac{1}{2}^-(1620)$ $\Delta\frac{3}{2}^-(1700)$
56	$\frac{1}{2}$	2	$N\frac{3}{2}^+(1720)$ $N\frac{5}{2}^+(1680)$
	$\frac{3}{2}$	2	$\Delta\frac{1}{2}^+(1910)$ $\Delta\frac{3}{2}^+(1920)$ $\Delta\frac{5}{2}^+(1905)$ $\Delta\frac{7}{2}^+(1950)$
70	$\frac{1}{2}$	3	$N\frac{5}{2}^-$ $N\frac{7}{2}^-$
	$\frac{3}{2}$	3	$N\frac{3}{2}^-$ $N\frac{5}{2}^-$ $N\frac{7}{2}^-(2190)$ $N\frac{9}{2}^-(2250)$
	$\frac{1}{2}$	3	$\Delta\frac{5}{2}^-(1930)$ $\Delta\frac{7}{2}^-$
56	$\frac{1}{2}$	4	$N\frac{7}{2}^+$ $N\frac{9}{2}^+(2220)$
	$\frac{3}{2}$	4	$\Delta\frac{5}{2}^+$ $\Delta\frac{7}{2}^+$ $\Delta\frac{9}{2}^+$ $\Delta\frac{11}{2}^+(2420)$
70	$\frac{1}{2}$	5	$N\frac{9}{2}^-$ $N\frac{11}{2}^-$
	$\frac{3}{2}$	5	$N\frac{7}{2}^-$ $N\frac{9}{2}^-$ $N\frac{11}{2}^-(2600)$ $N\frac{13}{2}^-$

Dirac's Amazing Idea:
The "Front Form"

Evolve in ordinary time

Evolve in light-front time!



Instant Form

Front Form

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$

$$\psi(x, k_\perp)$$

$$x_i = \frac{k_i^+}{P^+}$$

Invariant under boosts. Independent of P^μ

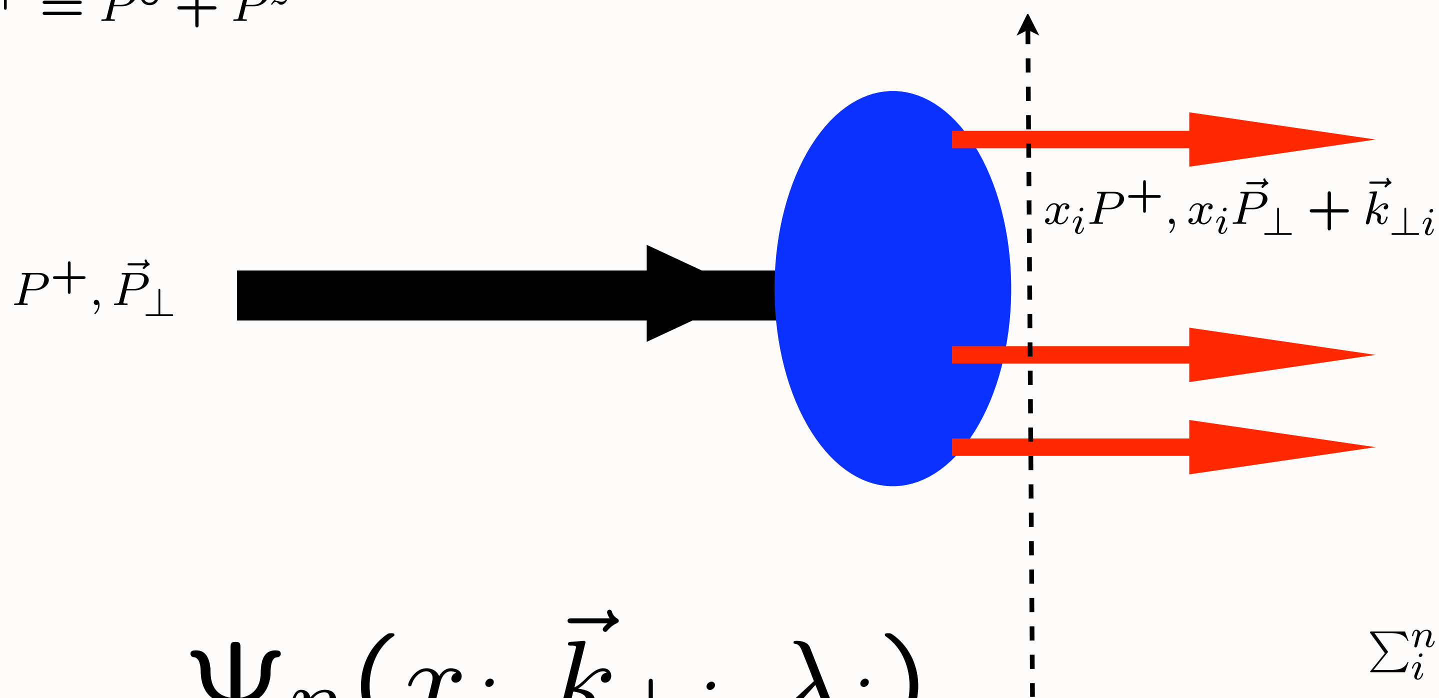
$$H_{LF}^{QCD}|\psi> = M^2|\psi>$$

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space

Light-Front Wavefunctions

Fixed $\tau = t + z/c$

$P^+ = P^0 + P^z$



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

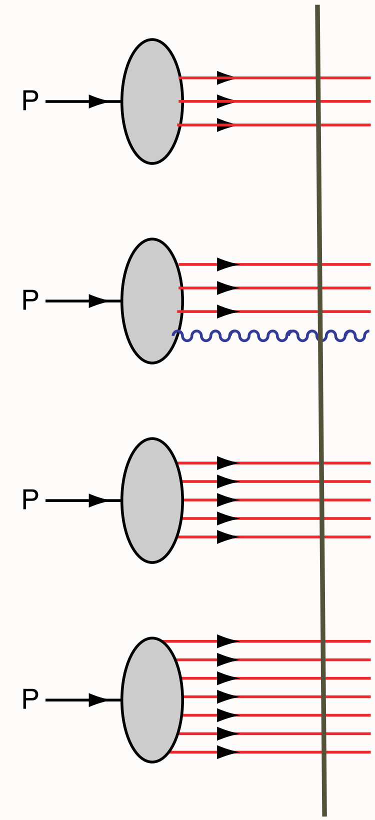
Invariant under boosts! Independent of P^μ

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$

$$\psi_n(x, k_\perp)$$

$$x_i = \frac{k_i^+}{P^+}$$



$$\mathrm{H}_{LF}^{QCD}|\psi> = M^2|\psi>$$

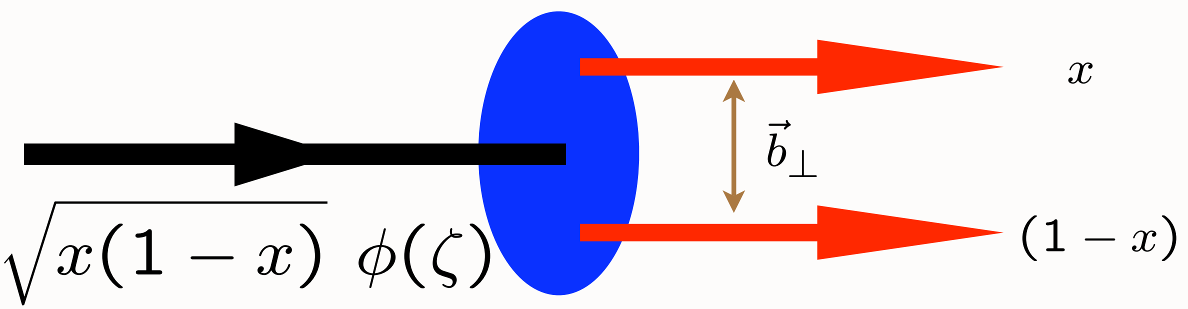
Intrinsic gluons, sea quarks, asymmetries

AdS/QCD

Mapping between LF(3+1) and AdS_5

LF(3+1)		AdS_5
$\psi(x, \vec{b}_\perp)$	⟷	$\phi(z)$
$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$	⟷	z

$$\psi(x, \vec{b}_\perp) = \sqrt{x(1-x)}\;\phi(\zeta)$$



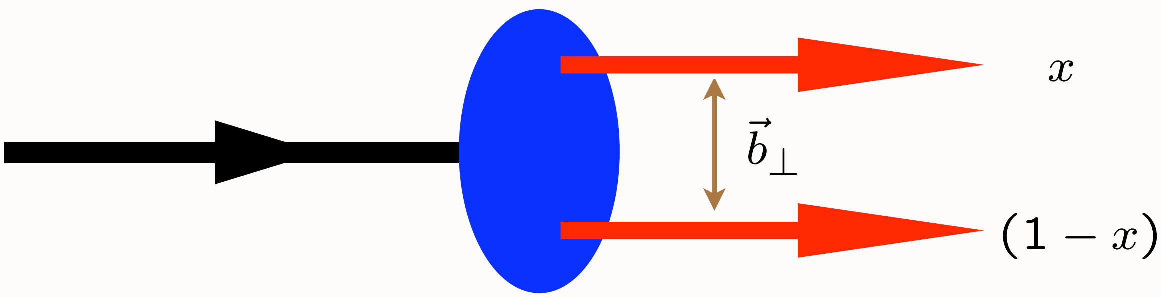
Holography: Map AdS/CFT to 3+1 LF Theory

Relativistic radial equation: *Frame Independent*

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta)\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta)$$

G. de Teramond, sjb

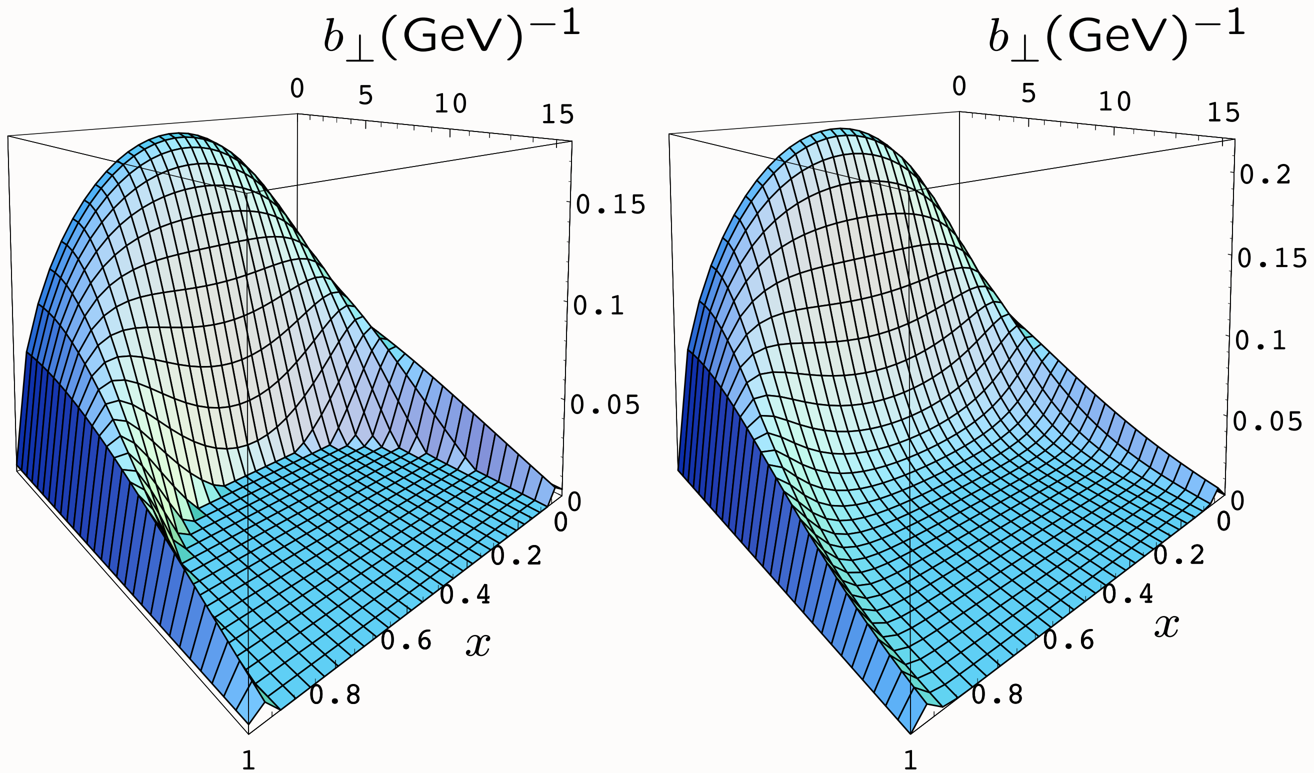
$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2.$$



Effective conformal potential:

$$V(\zeta) = -\frac{1-4L^2}{4\zeta^2}.$$

AdS/CFT Predictions for Meson LFWF $\psi(x, b_\perp)$

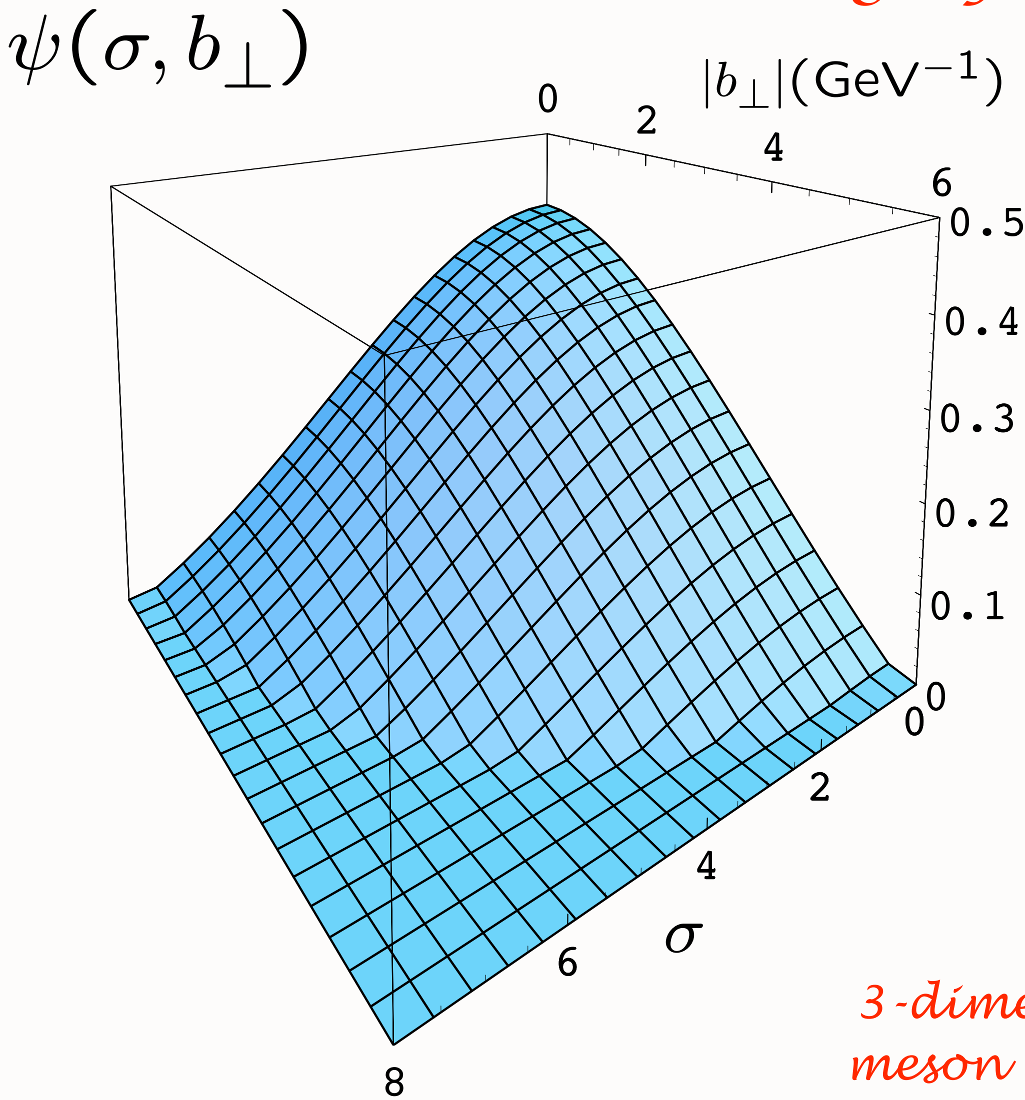


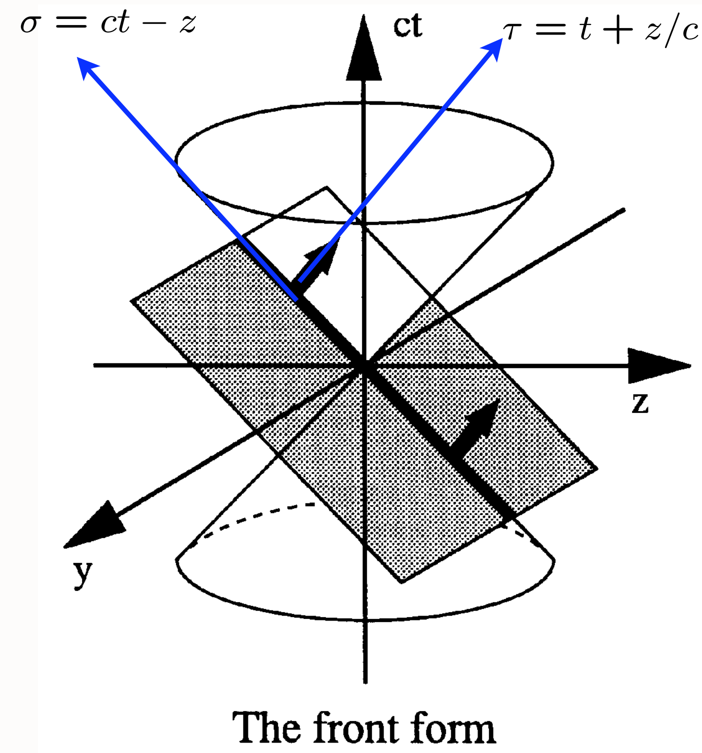
$\Lambda_{QCD} = 0.32 \text{ GeV}$ $\kappa = 0.76 \text{ GeV}$

Truncated Space ***Harmonic Oscillator***

AdS/CFT Holographic Model

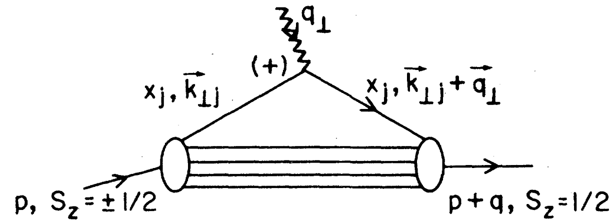
G. de Teramond
SJB





3-dimensional photograph:
meson LFWF at fixed LF Time

- Drell-Yan-West form factor



$$F(q^2) = \sum_q e_q \int_0^1 dx \int \frac{d^2\vec{k}_\perp}{16\pi^3}\, \psi^*_{P'}(x, \vec{k}_\perp - x\vec{q}_\perp)\, \psi_P(x, \vec{k}_\perp).$$

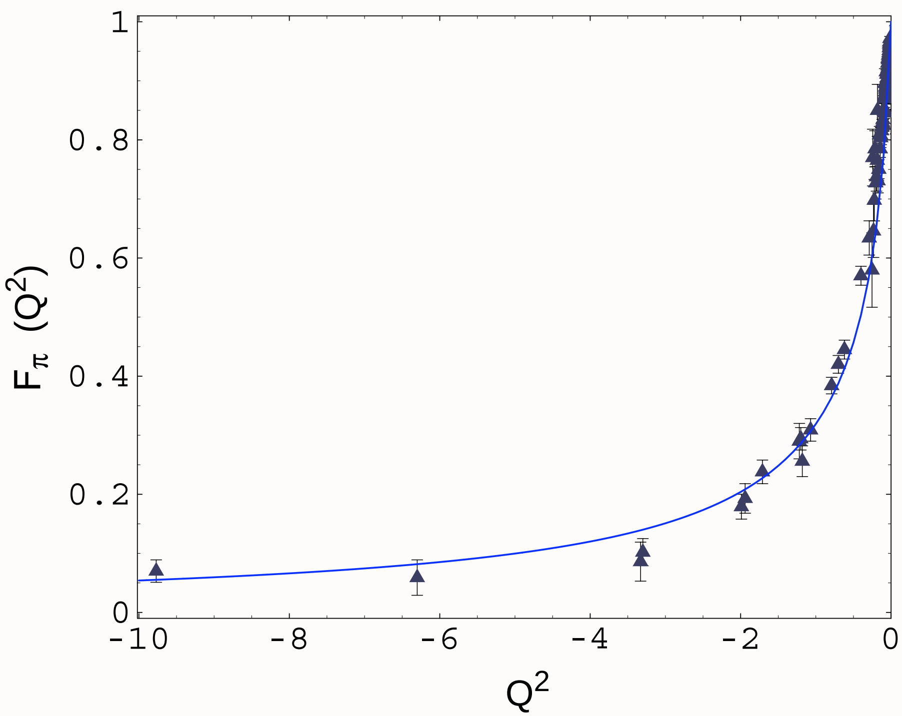
- Fourrier transform to impact parameter space $\vec{b}_\perp$

$$\psi(x, \vec{k}_\perp) = \sqrt{4\pi} \int d^2\vec{b}_\perp\, e^{i\vec{b}_\perp\cdot\vec{k}_\perp} \widetilde{\psi}(x, \vec{b}_\perp)$$

- Find ($b = |\vec{b}_\perp|$) :

$$\begin{aligned} F(q^2) &= \int_0^1 dx \int d^2\vec{b}_\perp\, e^{ix\vec{b}_\perp\cdot\vec{q}_\perp} \left|\widetilde{\psi}(x,b)\right|^2 \\ &= 2\pi \int_0^1 dx \int_0^\infty b\, db\, J_0(bqx)\, \left|\widetilde{\psi}(x,b)\right|^2, \end{aligned}$$

Soper



Space-like pion form factor in holographic model for $\Lambda_{QCD} = 0.2$ GeV.

Data Compilation from Baldini, Kloe and Volmer

Heuristic Argument for an IR Fixed Point

$$\alpha_s(Q^2) \simeq \text{const at small } Q^2$$

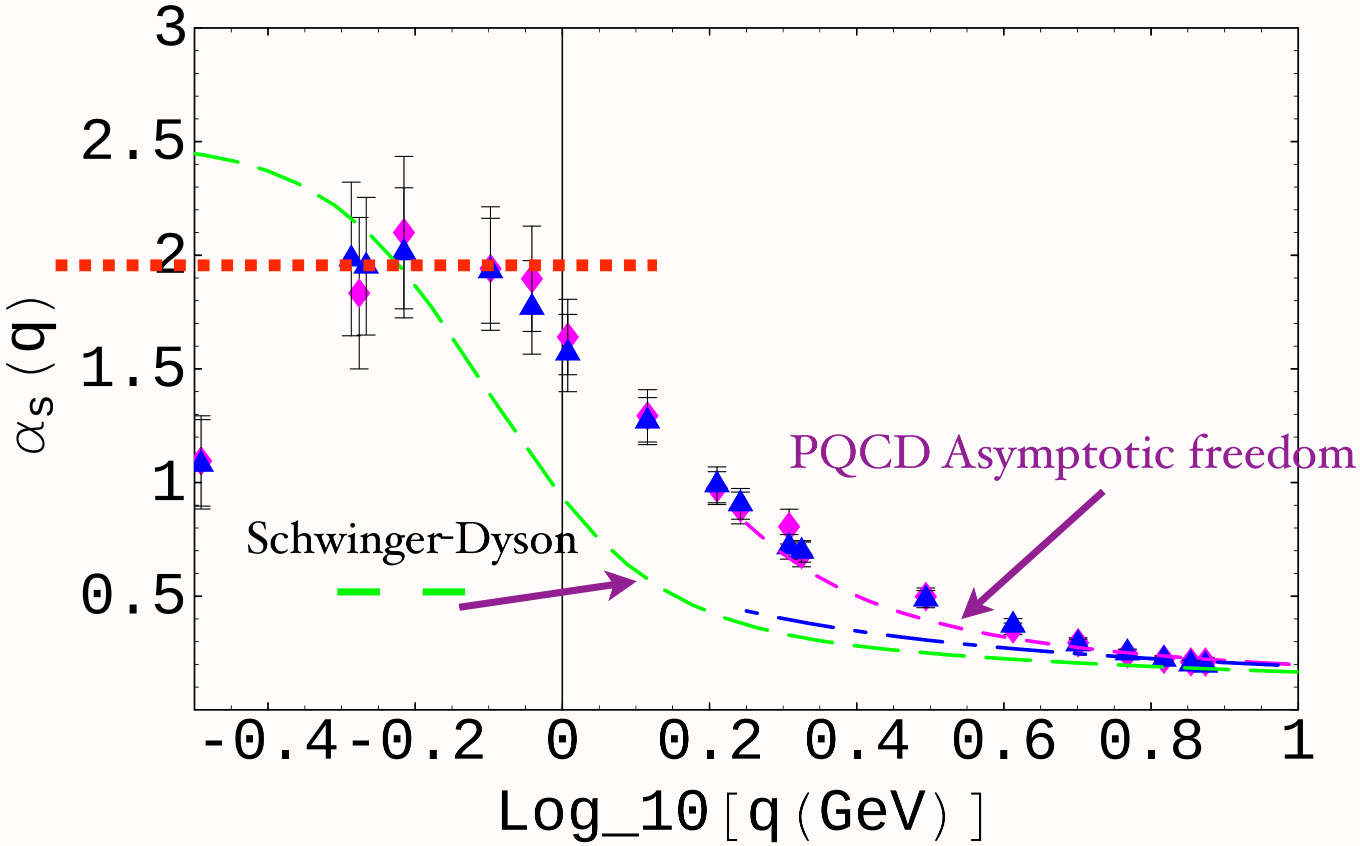
- Semi-Classical approximation to massless QCD
- No particle creation or absorption $\beta = 0$
- Conformal symmetry broken by confinement
- Effective gluon mass: vacuum polarization vanishes at small momentum transfer
- $\Pi(Q^2) \propto \frac{Q^2}{m_g^2}$ $\quad Q^2 << 4m_g^2$ $\quad \alpha_s(Q^2) \simeq \text{const}$

Analog of Serber-Uehling vacuum polarization in QED:

$$\Pi(Q^2) = \frac{\alpha}{15\pi} \frac{Q^2}{m_e^2} \qquad Q^2 << 4m_e^2$$

Decoupling of long wavelength gluonic interactions

Infrared-Finite QCD Coupling?



Shirkov
Gribov
Dokshitser
Siminov
Maxwell

Lattice simulation (MILC)

Furui, Nakajima

DSE: Alkofer, Fischer, von Smekal et al.

Define QCD Coupling from Observable

Grunberg
Neubert
Maxwell
Kataev

$$R_{e^+e^-\to X}(s) \equiv 3\Sigma_q e_q^2 \ [1 + \frac{\alpha_R(s)}{\pi}]$$

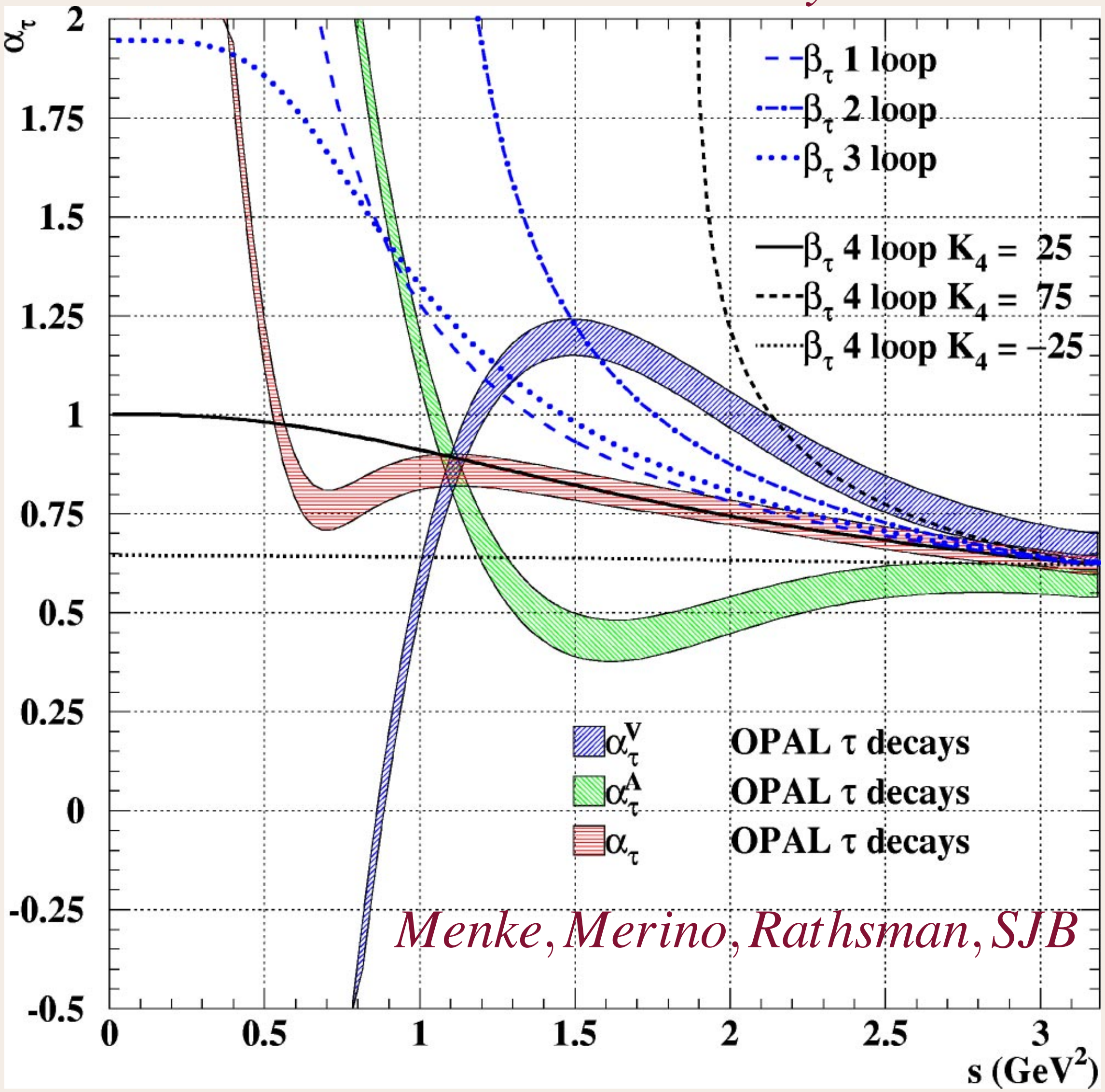
$$\Gamma(\tau \to X e\nu)(m_\tau^2) \equiv \Gamma_0(\tau \to u\bar{d}e\nu)\times[1+\frac{\alpha_\tau(m_\tau^2)}{\pi}]$$

Commensurate scale relations: Relate observable to observable at commensurate scales

A. Kataev, Lu, Rathsman, sjb

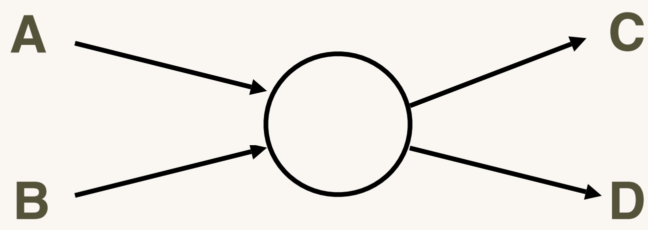
Effective Charges: analytic at quark mass thresholds, finite at small momenta

QCD Effective Coupling from
hadronic τ decay



AdS/QCD

Constituent Counting Rules



$$\frac{d\sigma}{dt}(s,t) = \frac{F(\theta_{\rm cm})}{s^{[n_{\rm tot}-2]}} \qquad s = E^2_{\rm cm}$$

$$F_H(Q^2) \sim [\frac{1}{Q^2}]^{n_H-1} \qquad -t = Q^2$$

Farrar & sjb; Matveev et al

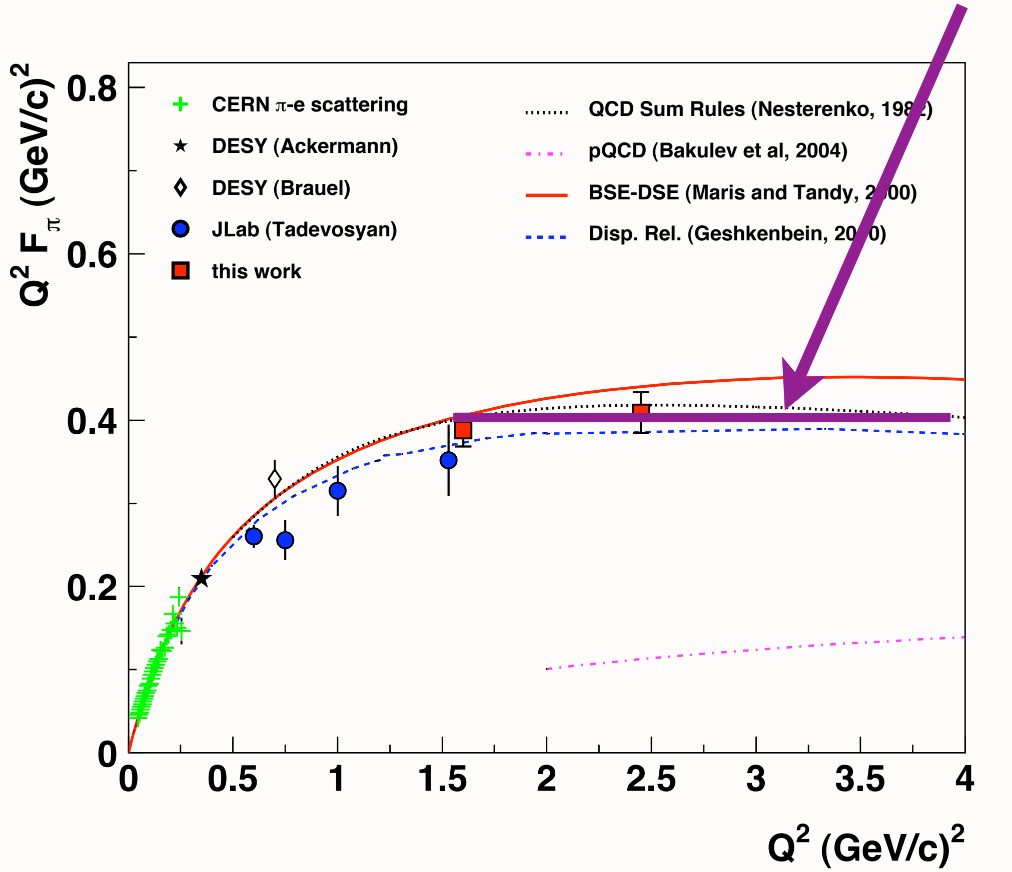
Conformal symmetry and PQCD predicts leading-twist power behavior

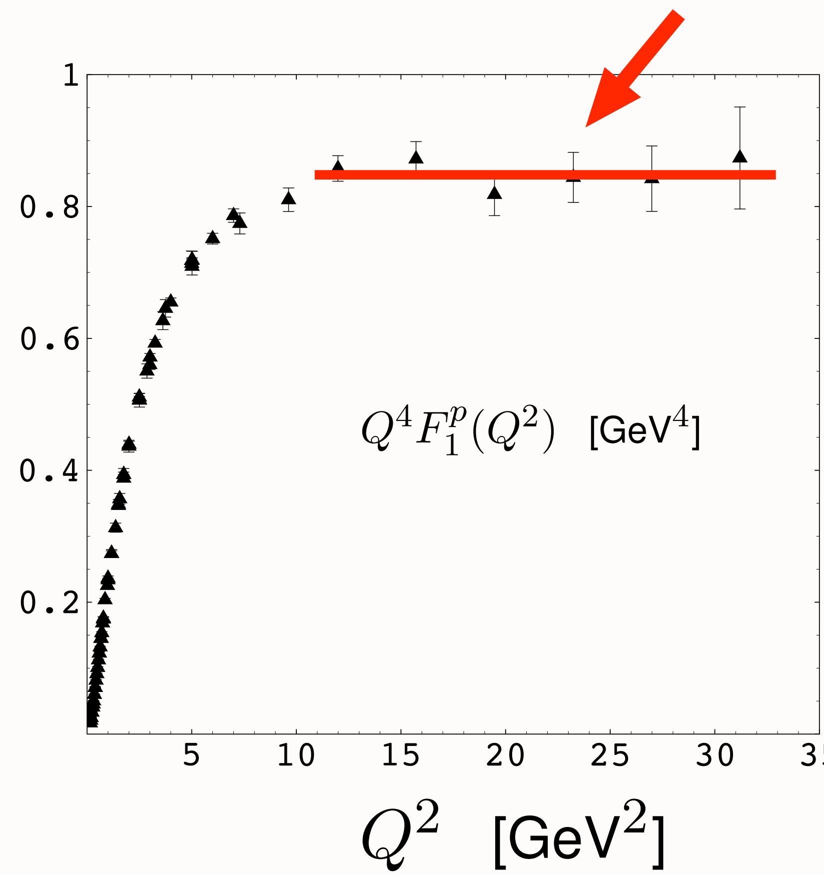
Characteristic scale of QCD: 300 MeV

New J-PARC, GSI, J-Lab, Belle, Babar tests

Conformal behavior: $Q^2 F_\pi(Q^2) \to \text{const}$

$Q^4 F_1(Q^2) \to \text{const}$





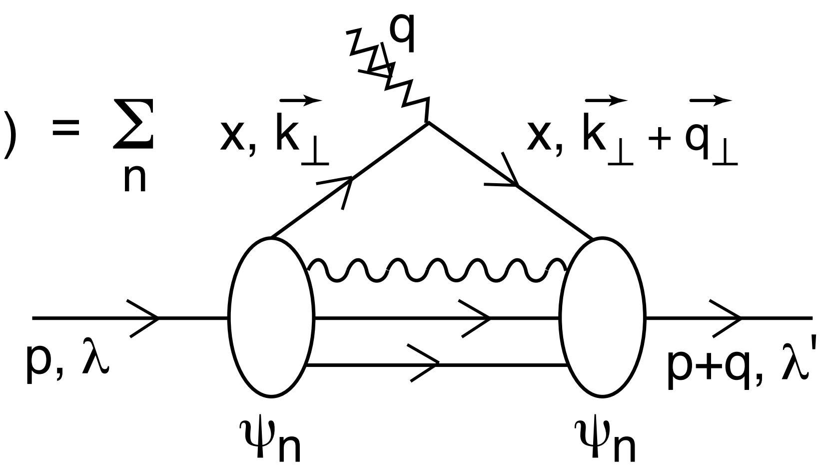
Determination of the Charged Pion Form Factor at Q2=1.60 and 2.45 (GeV/c)2.
By Fpi2 Collaboration (T. Horn *et al.*). Jul 2006. 4pp.
e-Print Archive: **nucl-ex/0607005**

Generalized parton distributions from nucleon form-factor da
M. Diehl (**DESY**) , **Th. Feldmann** (**CERN**) , **R. Jakob**, **P. Kroll** (**W**
DESY-04-146, CERN-PH-04-154, WUB-04-08, Aug 2004. 68pp.
Published in Eur.Phys.J.C39:1-39,2005
e-Print Archive: **hep-ph/0408173**

G. Huber

Form Factors $\ell p \to \ell' p'$ $\langle p' \lambda' | J^+(0) | p \lambda \rangle$

$F_{\lambda\lambda'}(Q^2) = \sum_n$

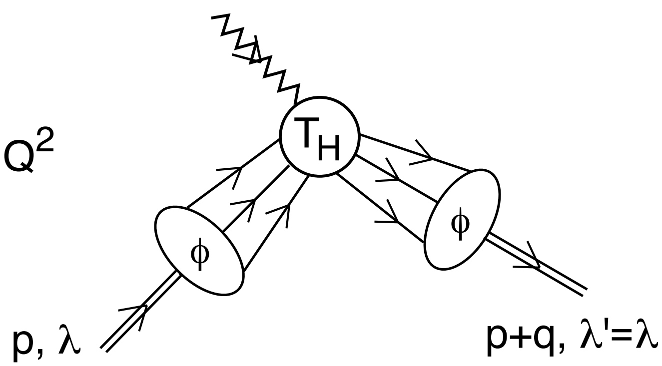


Lepage, Sjb
Efremov
Radyushkin

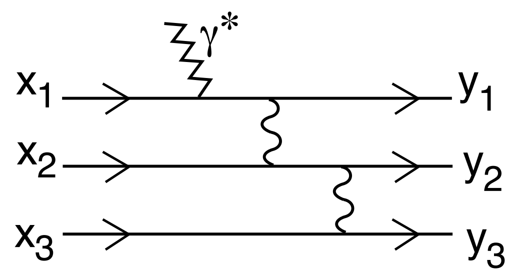
QCD Factorization

Scaling Laws from PQCD or AdS/CFT

⇨ Large Q^2



$T_H = \sum$



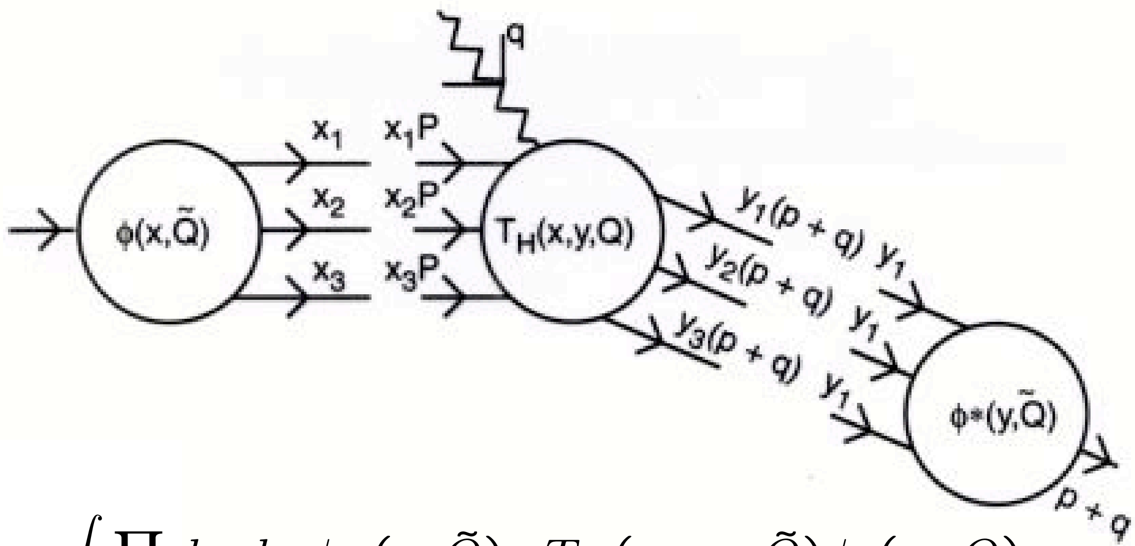
$+ \ \dots$

$$= \frac{\alpha_s^2}{Q^4} f(x_i, y_i,)$$

AdS/QCD

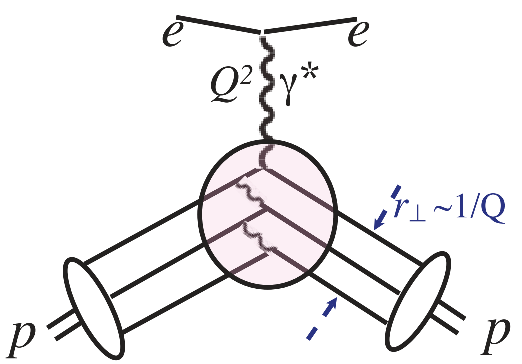
Leading-Twist PQCD Factorization

Lepage, sjb



$$M = \int \prod dx_i dy_i \phi_F(x, \tilde{Q}) \times T_H(x_i, y_i, \tilde{Q}) \phi_I(y_i, Q)$$

Exclusive



If $\alpha_s(\tilde{Q}^2) \simeq$ constant

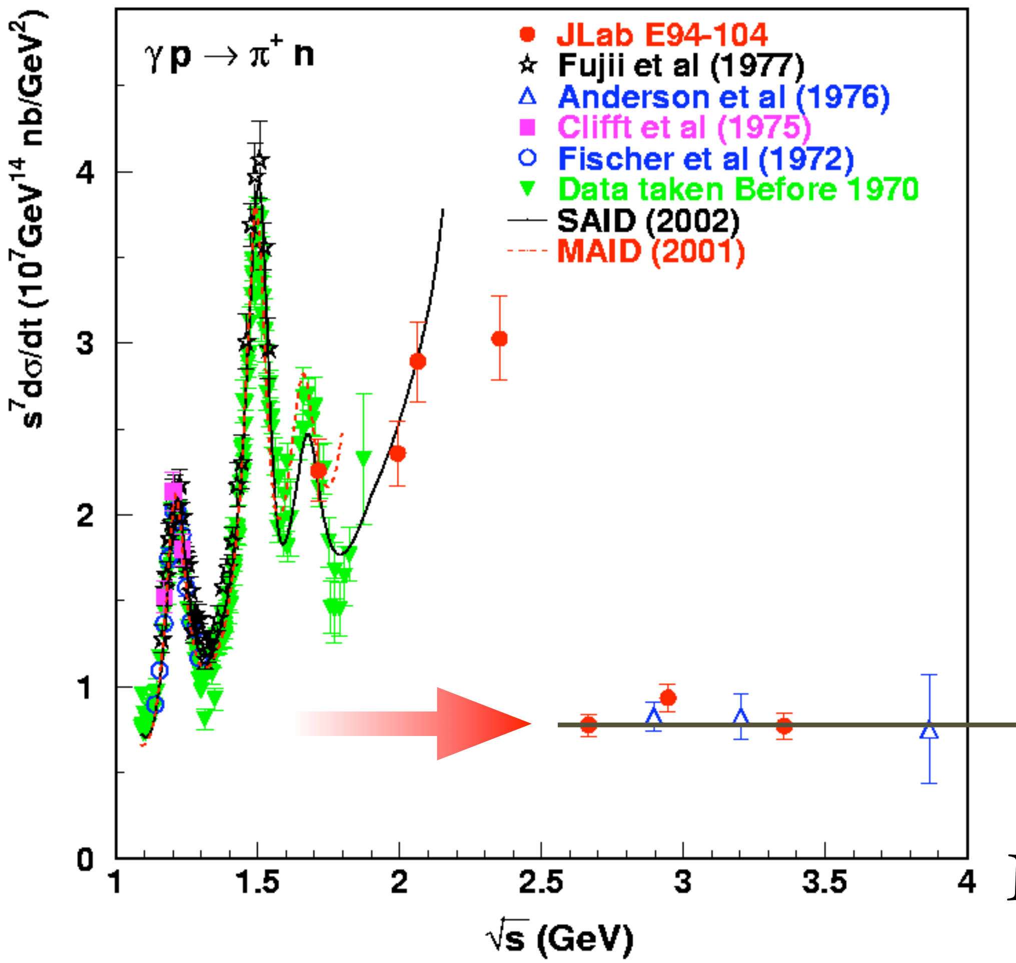
$Q^4 F_1(Q^2) \simeq$ constant

AdS/QCD

Test of PQCD Scaling

Constituent counting rules

Farrar, sjb; Muradyan, Matveev, Taveklidze



$s^7 d\sigma/dt(\gamma p \to \pi^+ n) \sim const$
fixed θ_{CM} scaling

PQCD and AdS/CFT:

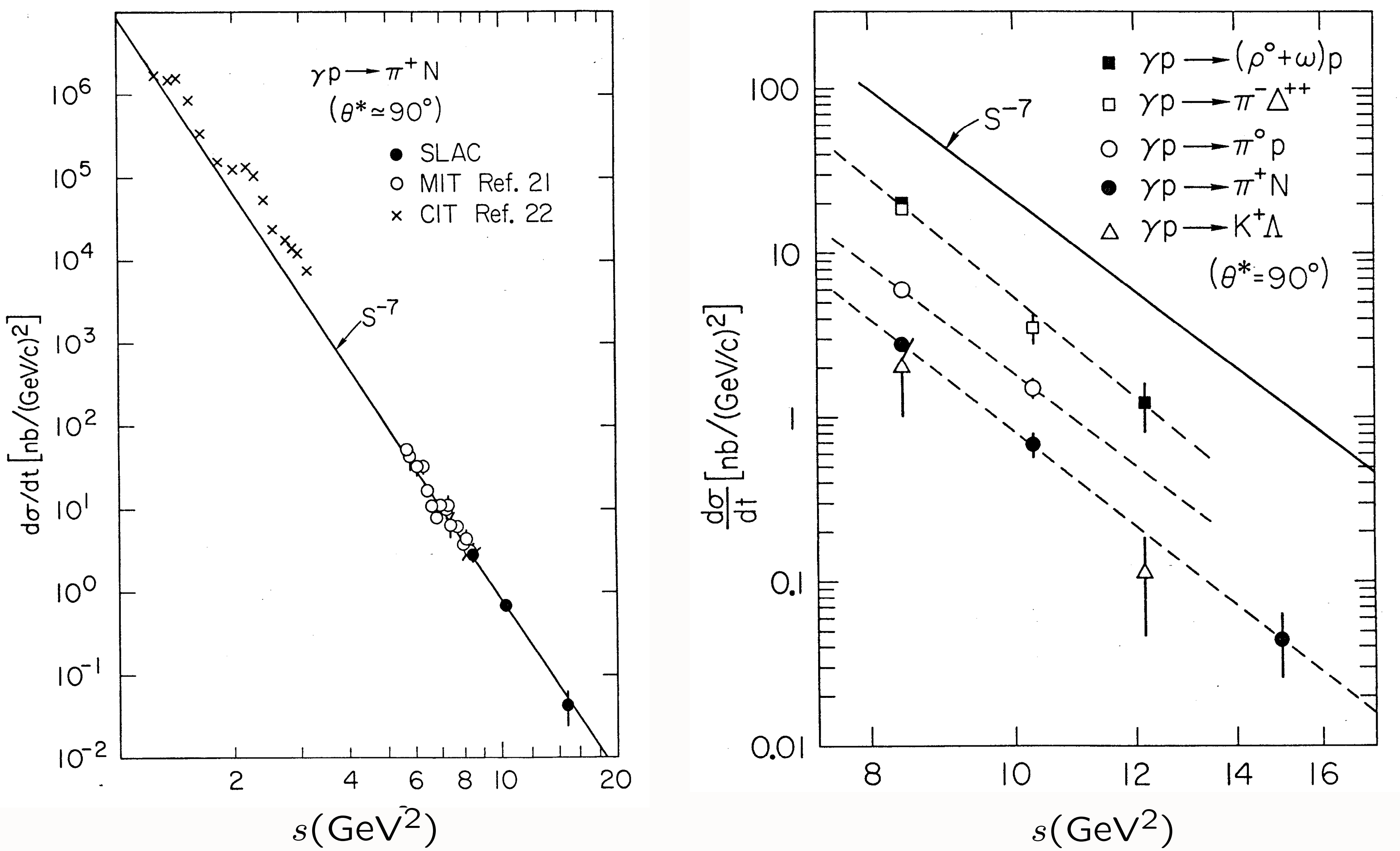
$s^{n_{tot}-2}\frac{d\sigma}{dt}(A+B \to C+D) =$
$F_{A+B\to C+D}(\theta_{CM})$

$s^7\frac{d\sigma}{dt}(\gamma p \to \pi^+ n) = F(\theta_{CM})$
$n_{tot} = 1+3+2+3 = 9$

No sign of running coupling

Conformal invariance at high momentum transfer!

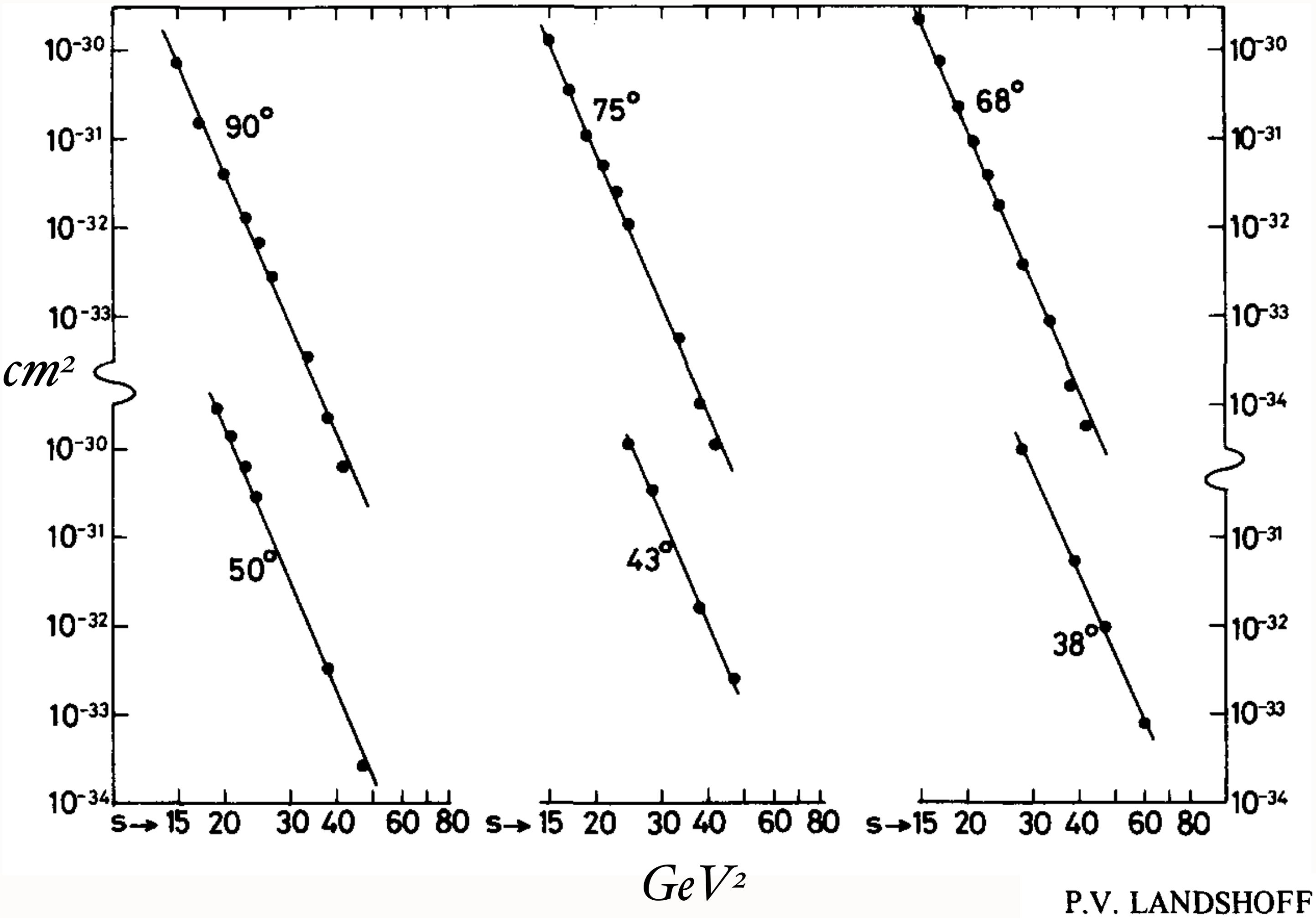
AdS/QCD



Conformal Invariance:

$$\frac{d\sigma}{dt}(\gamma p \to MB) = \frac{F(\theta_{cm})}{s^7}$$

Quark-Counting : $\frac{d\sigma}{dt}(pp \to pp) = \frac{F(\theta_{CM})}{s^{10}}$ $n = 4 \times 3 - 2 = 10$



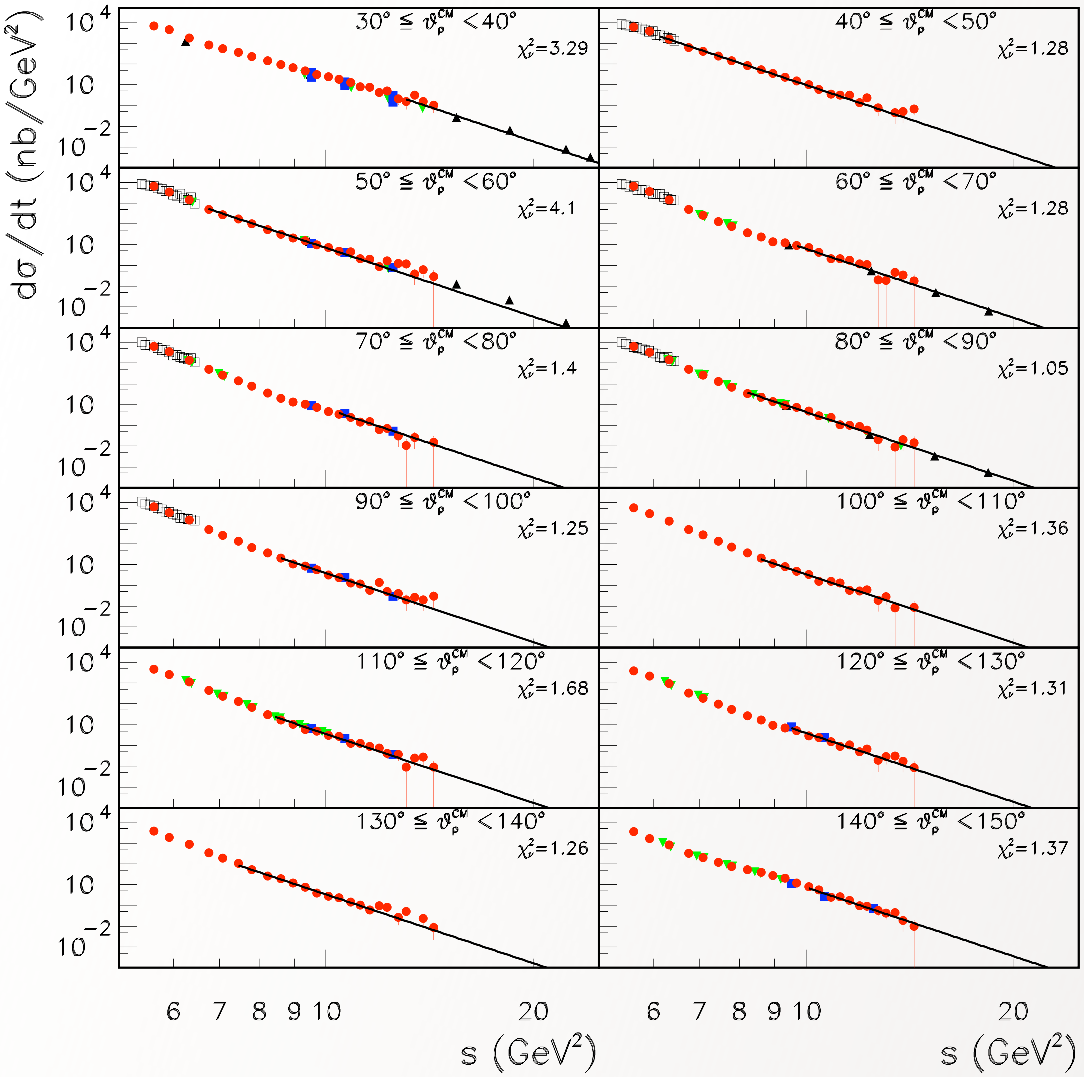
P.V. LANDSHOFF and J.C. POLKINGHORNE

Best Fit

$n = 9.7 \pm 0.5$

Reflects underlying conformal scale-free interactions

Deuteron Photodisintegration



J-Lab

PQCD and AdS/CFT:

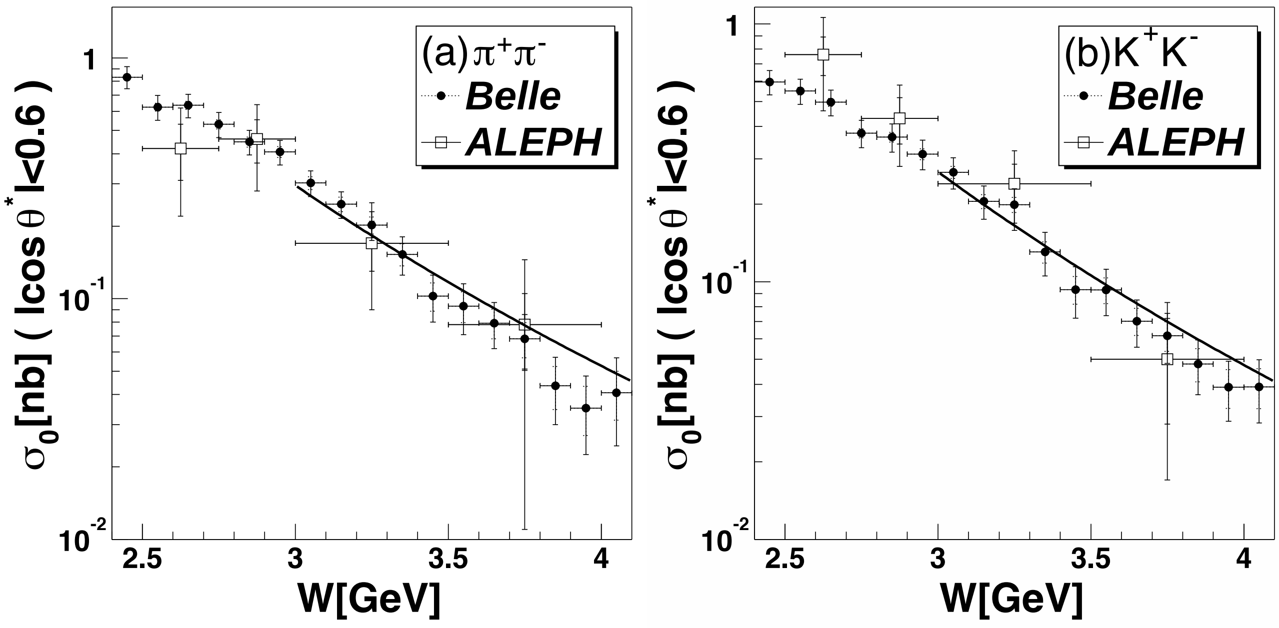
$$s^{n_{tot}-2}\frac{d\sigma}{dt}(A+B\to C+D) = F_{A+B\to C+D}(\theta_{CM})$$

$$s^{11}\frac{d\sigma}{dt}(\gamma d\to np) = F(\theta_{CM})$$

$n_{tot}-2 =$
$(1 + 6 + 3 + 3) - 2 = 11$

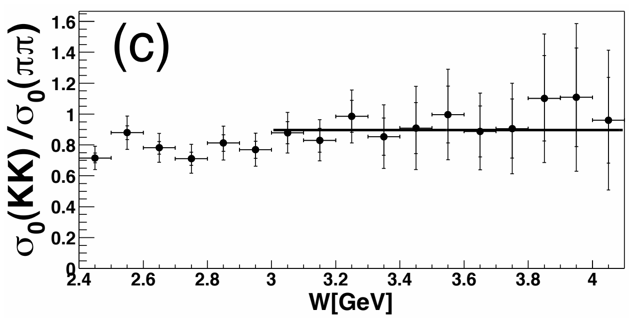
Conformal invariance
at high momentum transfers!

AdS/QCD



Two-Photon Reactions

Hard Exclusive Processes: Fixed angle



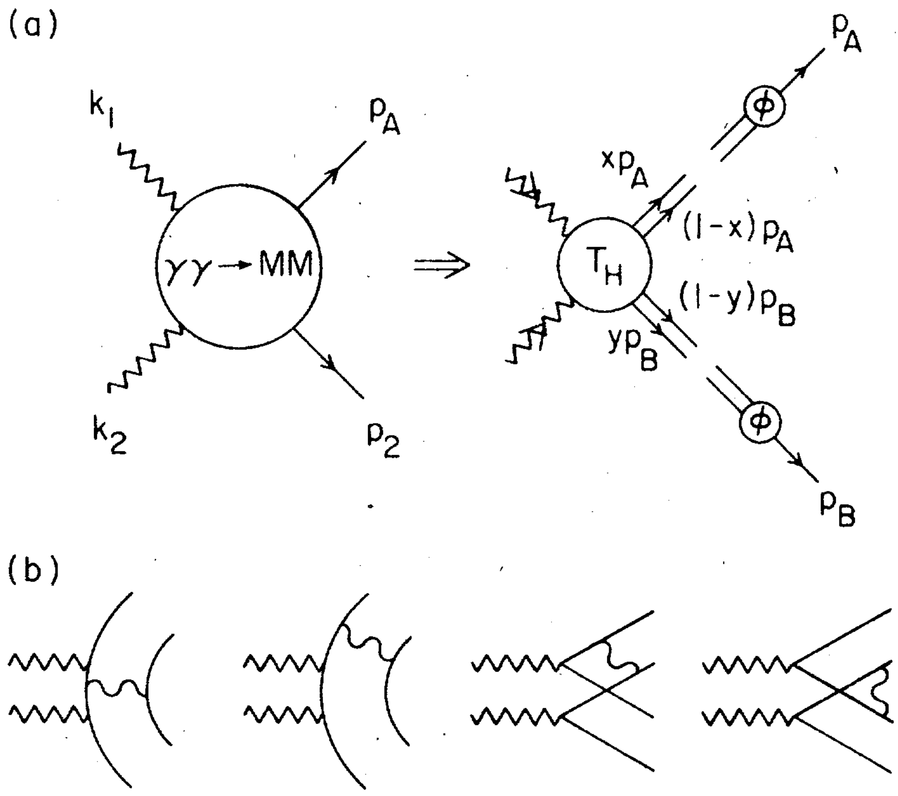
PQCD, AdS/CFT:

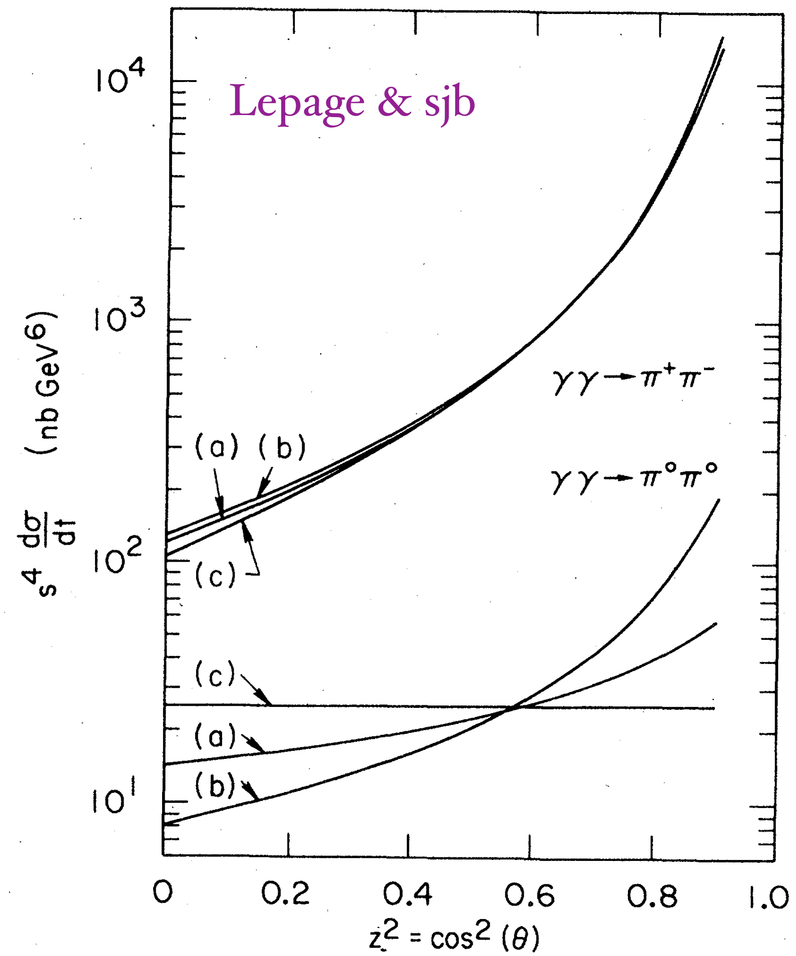
$$\Delta\sigma(\gamma\gamma \to \pi^+\pi^-, K^+, K^-) \sim 1/W^6$$

$$|\cos(\theta_{CM})| < 0.6$$

Conformal invariance at high momentum transfers!

Fig. 5. Cross section for (a) $\gamma\gamma\to\pi^+\pi^-$, (b) $\gamma\gamma\to K^+K^-$ in the c.m. angular region $|\cos\theta^*| < 0.6$ together with a W^{-6} dependence line derived from the fit of $s|R_M|$. (c) shows the cross section ratio. The solid line is the result of the fit for the data above 3 GeV. The errors indicated by short ticks are statistical only.





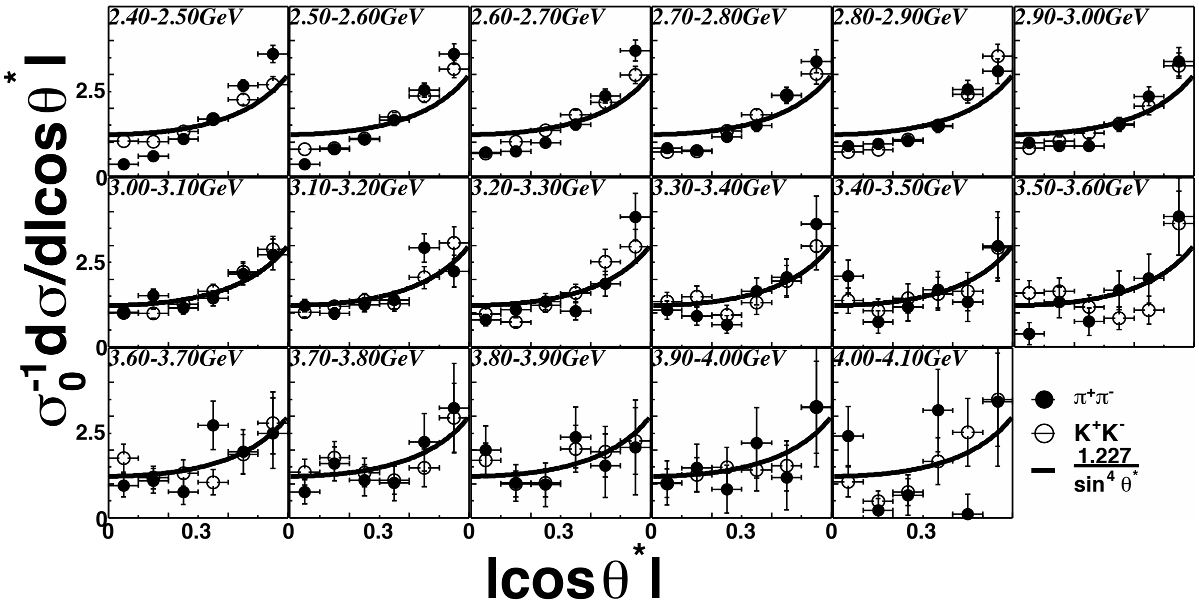
$$\frac{\frac{d\sigma}{dt}(\gamma\gamma\to\pi^+\pi^-)}{\frac{d\sigma}{dt}(\gamma\gamma\to\mu^+\mu^-)} \sim \frac{4\,|F_\pi(s)|^2}{1-\cos^4\theta_{c.m.}}$$

(a): $\phi_\pi(x) \propto x(1-x)$

(b): $\phi_\pi(x) \propto [x(1-x)]^{1/4}$

(c): $\phi_\pi(x) \propto \delta(x-1/2)$

PQCD: $$\frac{d\sigma}{d|\cos\theta^*|}(\gamma\gamma \to M^+M^-) \approx \frac{16\pi\alpha^2}{s}\frac{|F_M(s)|^2}{\sin^4\theta^*},$$



4. Angular dependence of the cross section, $\sigma_0^{-1}d\sigma/d|\cos\theta^*|$, for the $\pi^+\pi^-$(closed circles) and K^+K^-(open circles) processes. The curves are $1.227 \times \sin^{-4}\theta^*$. The errors are statistical only.

Measurement of the $\gamma\gamma \to \pi^+\pi^-$ and $\gamma\gamma \to K^+K^-$ processes at energies of 2.4–4.1 GeV

Belle Collaboration

QCD Prediction for Deuteron Form Factor

$$F_d(Q^2) = \left[\frac{\alpha_s(Q^2)}{Q^2}\right]^5 \sum_{m,n} d_{mn} \left(\ln \frac{Q^2}{\Lambda^2}\right)^{-\gamma_n^d - \gamma_m^d} \left[1 + O\left(\alpha_s(Q^2), \frac{m}{Q}\right)\right]$$

Define "Reduced" Form Factor

$$f_d(Q^2) \equiv \frac{F_d(Q^2)}{F_N{}^2(Q^2/4)}.$$

Same large momentum transfer behavior as pion form factor

$$f_d(Q^2) \sim \frac{\alpha_s(Q^2)}{Q^2} \left(\ln \frac{Q^2}{\Lambda^2}\right)^{-(2/5)\, C_F/\beta}$$

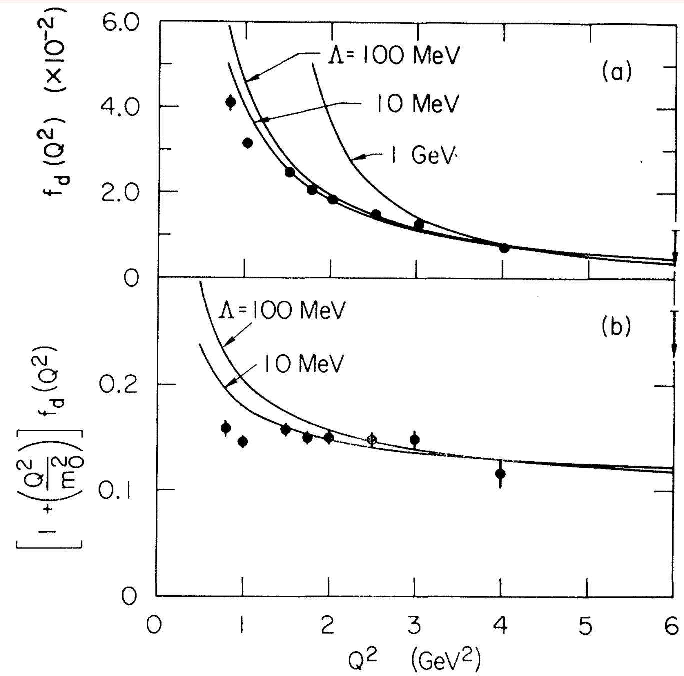
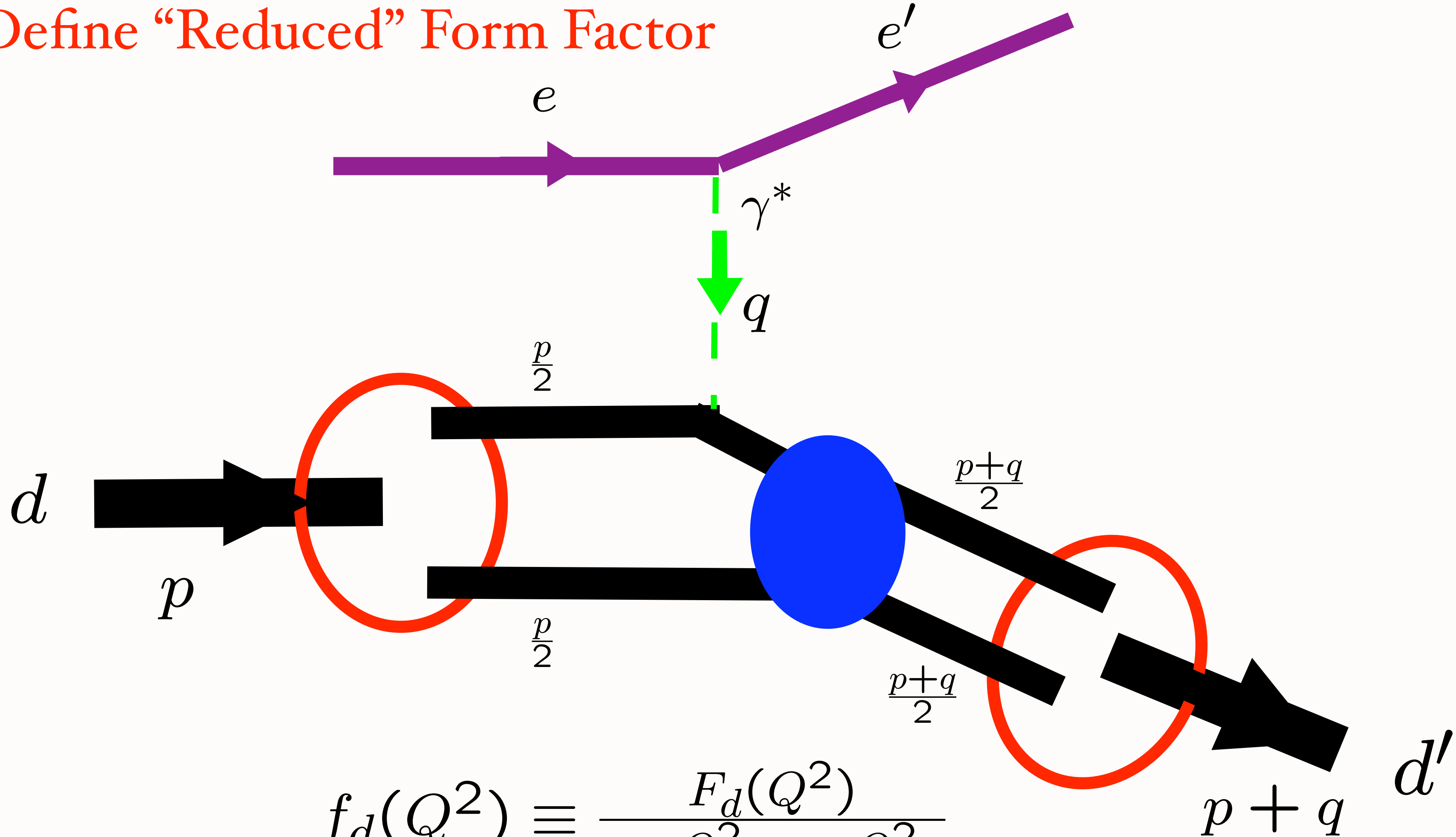


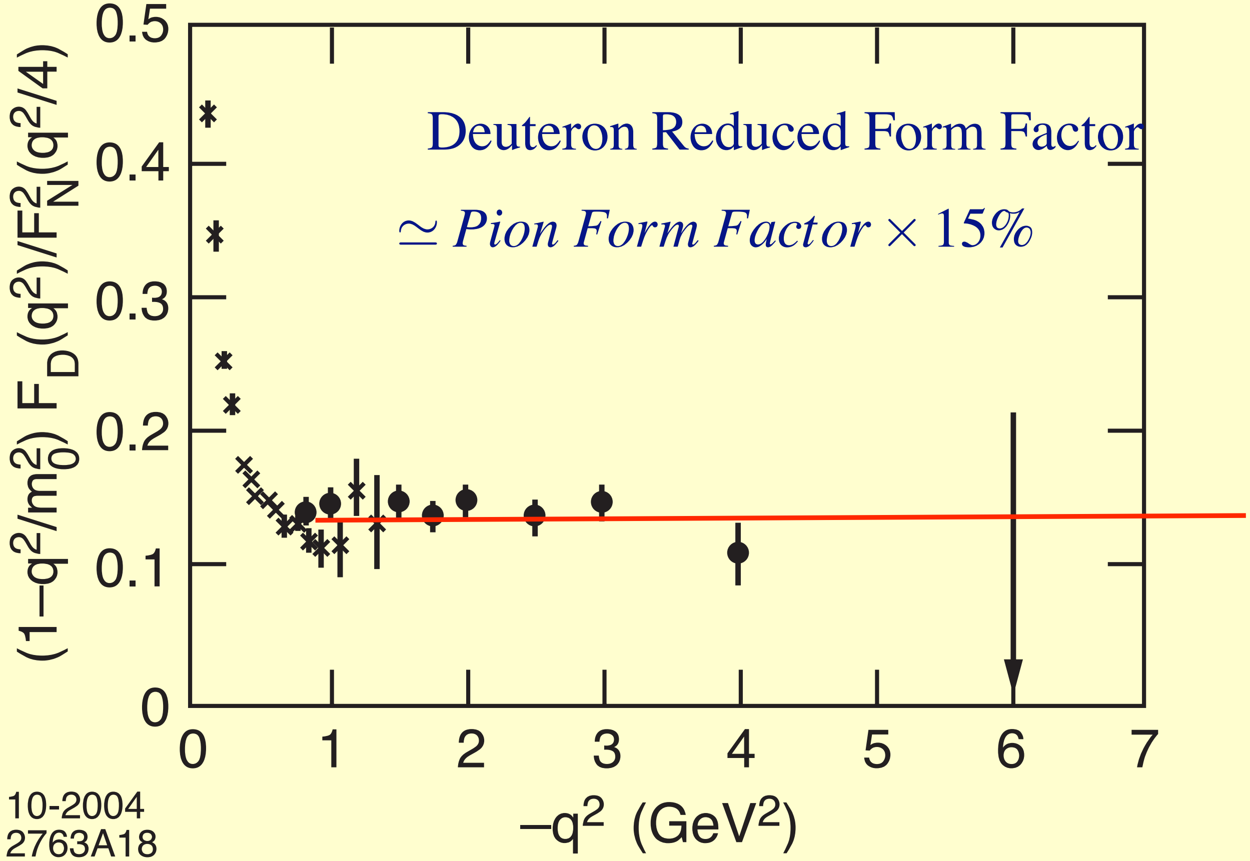
FIG. 2. (a) Comparison of the asymptotic QCD prediction $f_d(Q^2) \propto (1/Q^2)[\ln(Q^2/\Lambda^2)]^{-1-(2/5)C_F/\beta}$ with final data of Ref. 10 for the reduced deuteron form factor, where $F_N(Q^2) = [1 + Q^2/(0.71\ GeV^2)]^{-2}$. The normalization is fixed at the $Q^2 = 4\ GeV^2$ data point. (b) Comparison of the prediction $[1 + (Q^2/m_0{}^2)] f_d(Q^2) \propto [\ln(Q^2/\Lambda^2)]^{-1-(2/5)C_F/\beta}$ with the above data. The value $m_0{}^2 = 0.28\ GeV^2$ is used (Ref. 8).

Define "Reduced" Form Factor



$$f_d(Q^2) \equiv \frac{F_d(Q^2)}{F_p(\frac{Q^2}{4})F_n(\frac{Q^2}{4})}$$

Elastic electron-deuteron scattering



- Evidence for Hidden Color in the Deuteron

Why do dimensional counting rules work so well?

- **PQCD predicts log corrections from powers of α_s, logs, pinch contributions** Lepage, sjb; Efremov, Radyushkin

- **DSE: QCD coupling (mom scheme) has IR Fixed point!** Alkofer, Fischer, von Smekal et al.

- **Lattice results show similar flat behavior** Furui, Nakajima

- **PQCD exclusive amplitudes dominated by integration regime where α_s is large and flat**

Conformal symmetry: Template for QCD

- Take conformal symmetry as initial approximation; then correct for non-zero beta function and quark masses

- Eigensolutions of ERBL evolution equation for distribution amplitudes

  V. Braun et al;
  Frishman, Lepage, Sachrajda, sjb

- Commensurate scale relations: relate observables at corresponding scales: Generalized Crewther Relation

- Use AdS/CFT

String Theory

↓

AdS/CFT

Mapping of Poincare' and Conformal SO(4,2) symmetries of 3+1 space to AdS5 space

↓

Goal: First Approximant to QCD

Counting rules for Hard Exclusive Scattering
Regge Trajectories
QCD at the Amplitude Level

AdS/QCD

Conformal behavior at short distances
+ Confinement at large distance

↓

Semi-Classical QCD / Wave Equations

↓

Holography

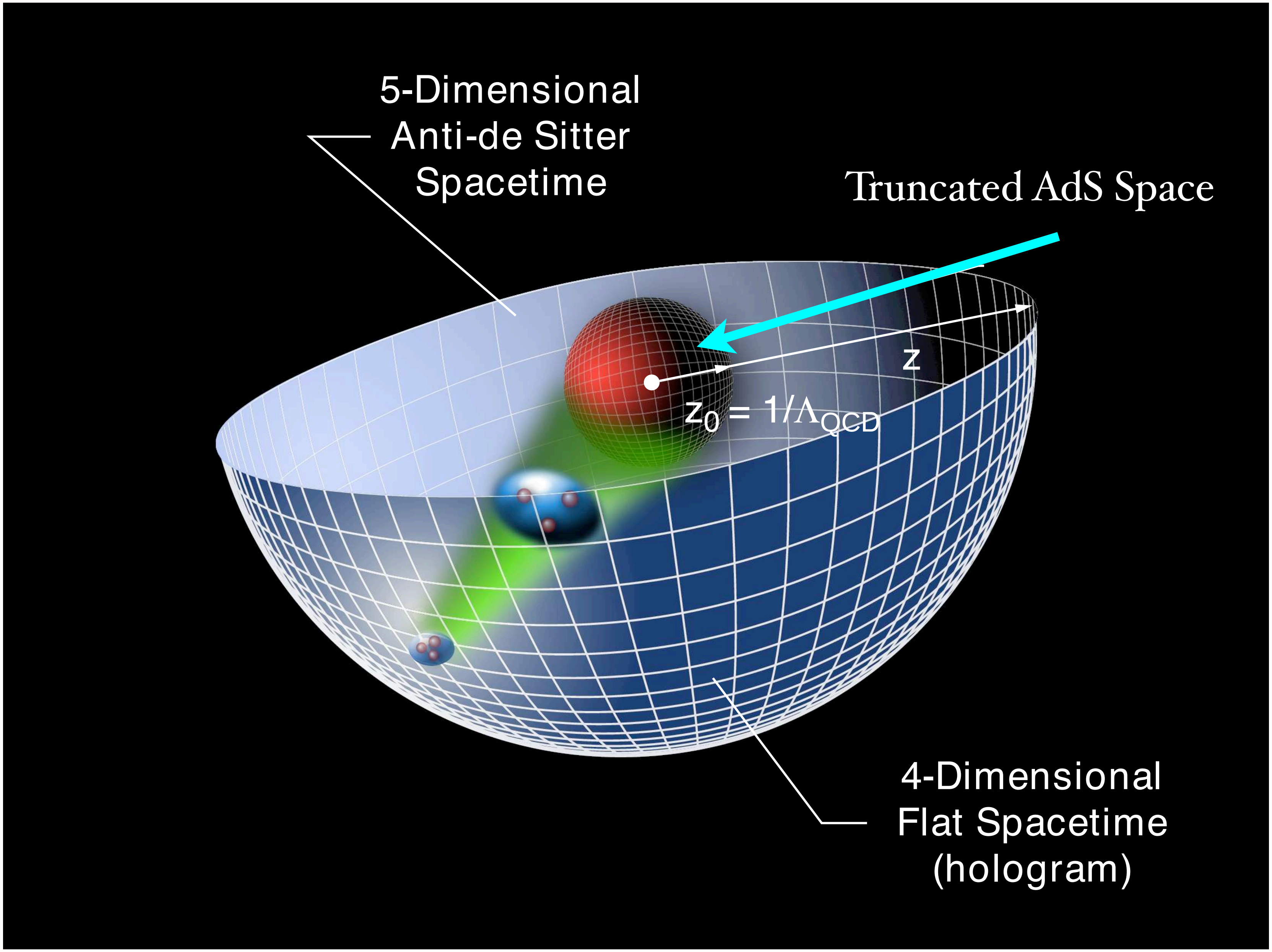
Boost Invariant 3+1 Light-Front Wave Equations

↓

J =0,1,1/2,3/2 plus L

Integrable!

Hadron Spectra, Wavefunctions, Dynamics


5-Dimensional
Anti-de Sitter
Spacetime
Truncated AdS Space
z
$z_0 = 1/\Lambda_{QCD}$
4-Dimensional
Flat Spacetime
(hologram)

Scale Transformations

- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2),$$

invariant measure

$x^\mu \to \lambda x^\mu,\ z \to \lambda z$, maps scale transformations into the holographic coordinate z.

- AdS mode in z is the extension of the hadron wf into the fifth dimension.

- Different values of z correspond to different scales at which the hadron is examined.

$$x^2 \to \lambda^2 x^2, \quad z \to \lambda z.$$

$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \to 0$ correspond to the $Q \to \infty$, UV zero separation limit.

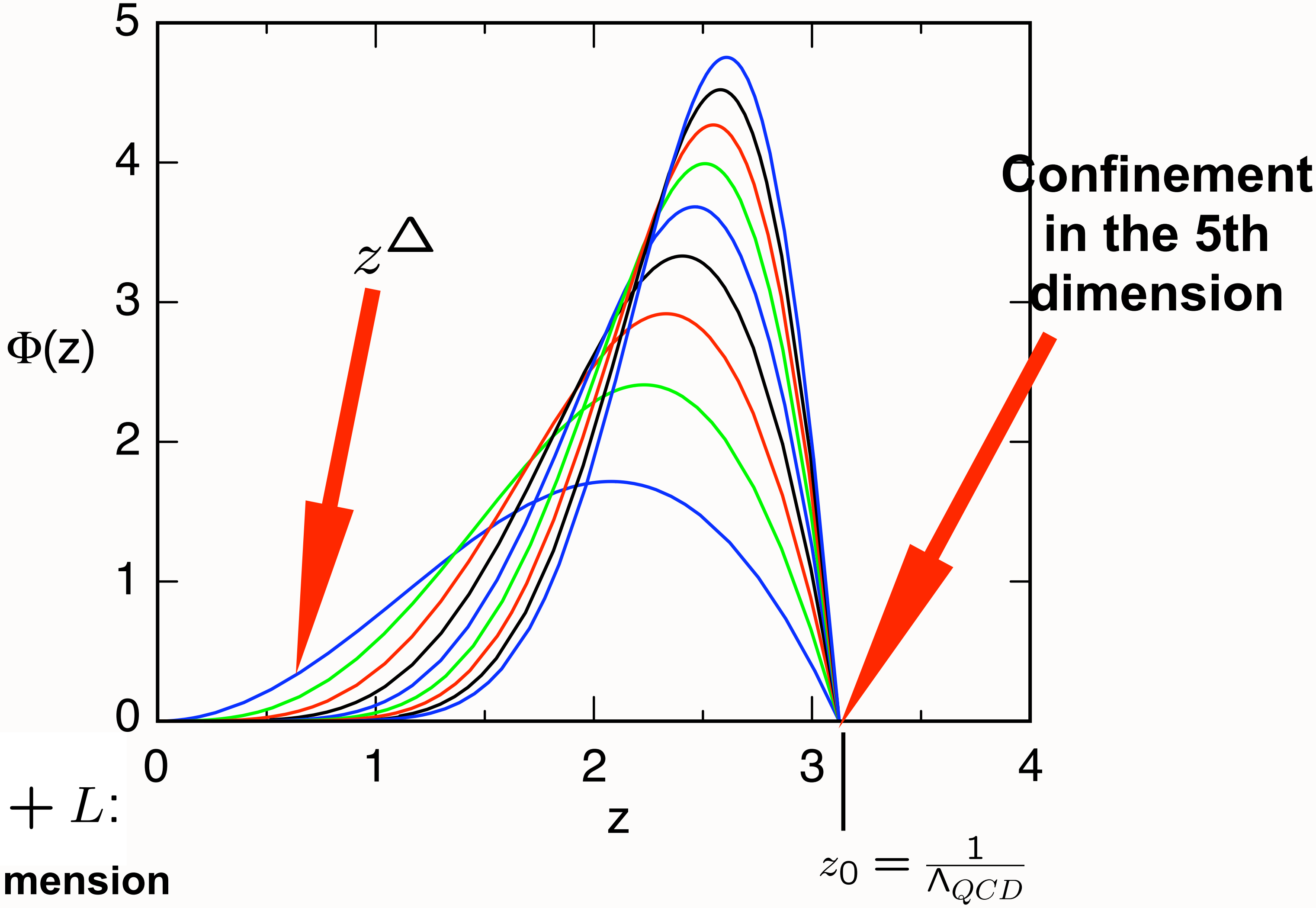
Guy de Teramond
SJB

AdS/CFT

- Use mapping of conformal group SO(4,2) to AdS_5
- Scale Transformations represented by wavefunction $\psi(z)$ in 5th dimension $x_\mu^2 \to \lambda^2 x_\mu^2$ $\quad z \to \lambda z$
- Holographic model: Confinement at large distances and conformal symmetry in interior $0 < z < z_0$
- Match solutions at small z to conformal dimension of hadron wavefunction at short distances $\psi(z) \sim z^\Delta$ at $z \to 0$
- Truncated space simulates "bag" boundary conditions

$$\psi(z_0) = 0 \qquad z_0 = \frac{1}{\Lambda_{QCD}}$$

Identify hadron by its interpolating operator at z --> 0



$\Delta = 3 + L$:

Twist dimension of baryon

de Teramond, sjb

Action for scalar field in AdS_5

$$S[\Phi] = \kappa' \int d^4x dz \sqrt{g} \left[g^{\ell m} \partial_\ell \Phi^* \partial_m \Phi - \mu^2 \Phi^* \Phi \right]$$

where $[\kappa'] = L^{-2}$ $\quad g^{\ell m} = \frac{z^2}{R^2} \eta^{\ell m}$ $\quad \sqrt{g} = R^5/z^5$

Action is invariant under scale transformations

$$x^\mu \to \lambda x^\mu, \quad z \to \lambda z.$$

$$\Phi(x^\ell) = \Phi(\lambda x^\ell)$$

Variation wrt Φ

$$\frac{1}{\sqrt{g}} \frac{\partial}{\partial x^\ell} \left(\sqrt{g}\, g^{\ell m} \frac{\partial}{\partial x^m} \Phi \right) + \mu^2 \Phi = 0$$

Solutions of form: $\Phi(x,z) = e^{-iP\cdot x} f(z)$ $\quad P_\mu P^\mu = \mathcal{M}^2$

$$S = -\kappa R^3 \int \frac{dz}{z^3} \left[(\partial_z f)^2 - \mathcal{M}^2 f^2 + \frac{(\mu R)^2}{z^2} f^2 \right]$$

Variation of S wrt f :

$$z^5 \partial_z \left(\frac{1}{z^3} \partial_z f \right) + z^2 \mathcal{M}^2 f - (\mu R)^2 f = 0.$$

$$\left[z^2 \partial_z^2 - 3z\partial_z + z^2 \mathcal{M}^2 - (\mu R)^2 \right] f = 0,$$

Introduce confinement, break conformal invariance

P-S Boundary Condition $\quad f(z = \frac{1}{\Lambda_{QCD}}) = 0$

Normalization in truncated space $\quad R^3 \int_0^{\Lambda_{\rm QCD}^{-1}} \frac{dz}{z^3} \, f^2(z) = 1$

Identify Orbital Angular Momentum $(\mu R)^2 = -4 + L^2$

- Wave equation in AdS for bound state of two scalar partons with conformal dimension $\Delta = 2 + L$

$$\left[z^2\partial_z^2 - 3z\,\partial_z + z^2\,\mathcal{M}^2 - L^2 + 4\right]\Phi(z) = 0,$$

with solution

$$\Phi(z) = Ce^{-iP\cdot x}z^2 J_L(z\mathcal{M}).$$

- For spin-carrying constituents: $\Delta \to \tau = \Delta - \sigma, \;\; \sigma = \sum_{i=1}^{n}\sigma_i.$
- The twist τ is equal to the number of partons $\tau = n$.

Introduce confinement, break conformal invariance

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Introduce confinement, break conformal invariance

$$f(z = \frac{1}{\Lambda_{QCD}}) = 0$$

Match fall-off at small z to Conformal Dimension of hadron state at short distances

- Pseudoscalar mesons: $\mathcal{O}_{3+L} = \overline{\psi}\gamma_5 D_{\{\ell_1} \dots D_{\ell_m\}}\psi$ ($\Phi_\mu = 0$ gauge).
- 4-d mass spectrum from boundary conditions on the normalizable string modes at $z = z_0$, $\Phi(x, z_o) = 0$, given by the zeros of Bessel functions $\beta_{\alpha,k}$: $\mathcal{M}_{\alpha,k} = \beta_{\alpha,k}\Lambda_{QCD}$
- Normalizable AdS modes $\Phi(z)$

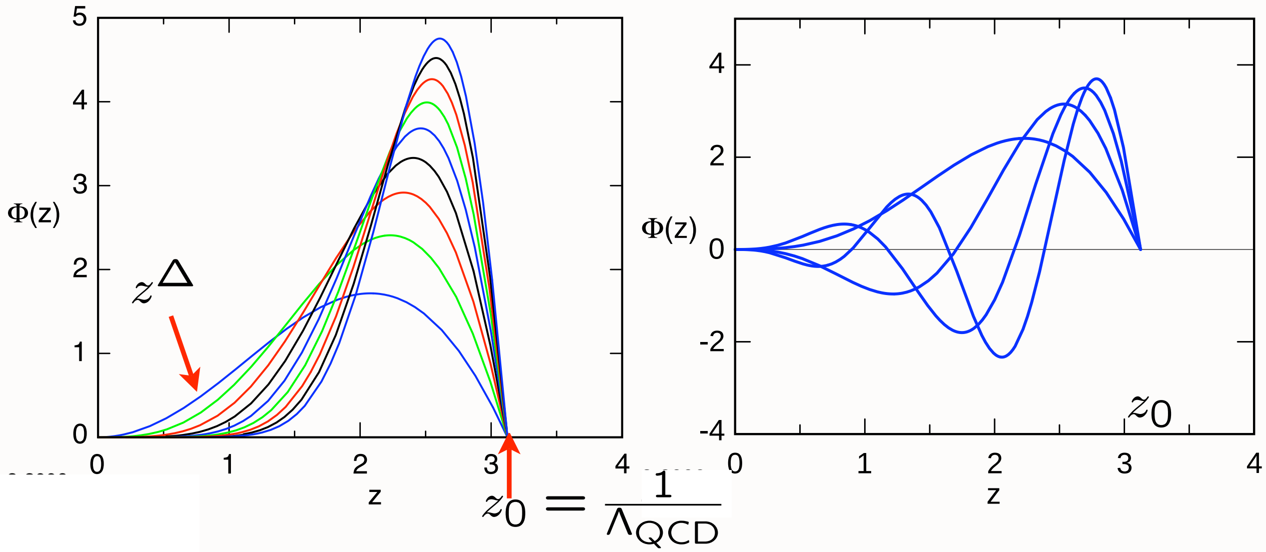

Fig: Meson orbital and radial AdS modes for $\Lambda_{QCD} = 0.32$ GeV.