Brane Inflation in a Warped Throat

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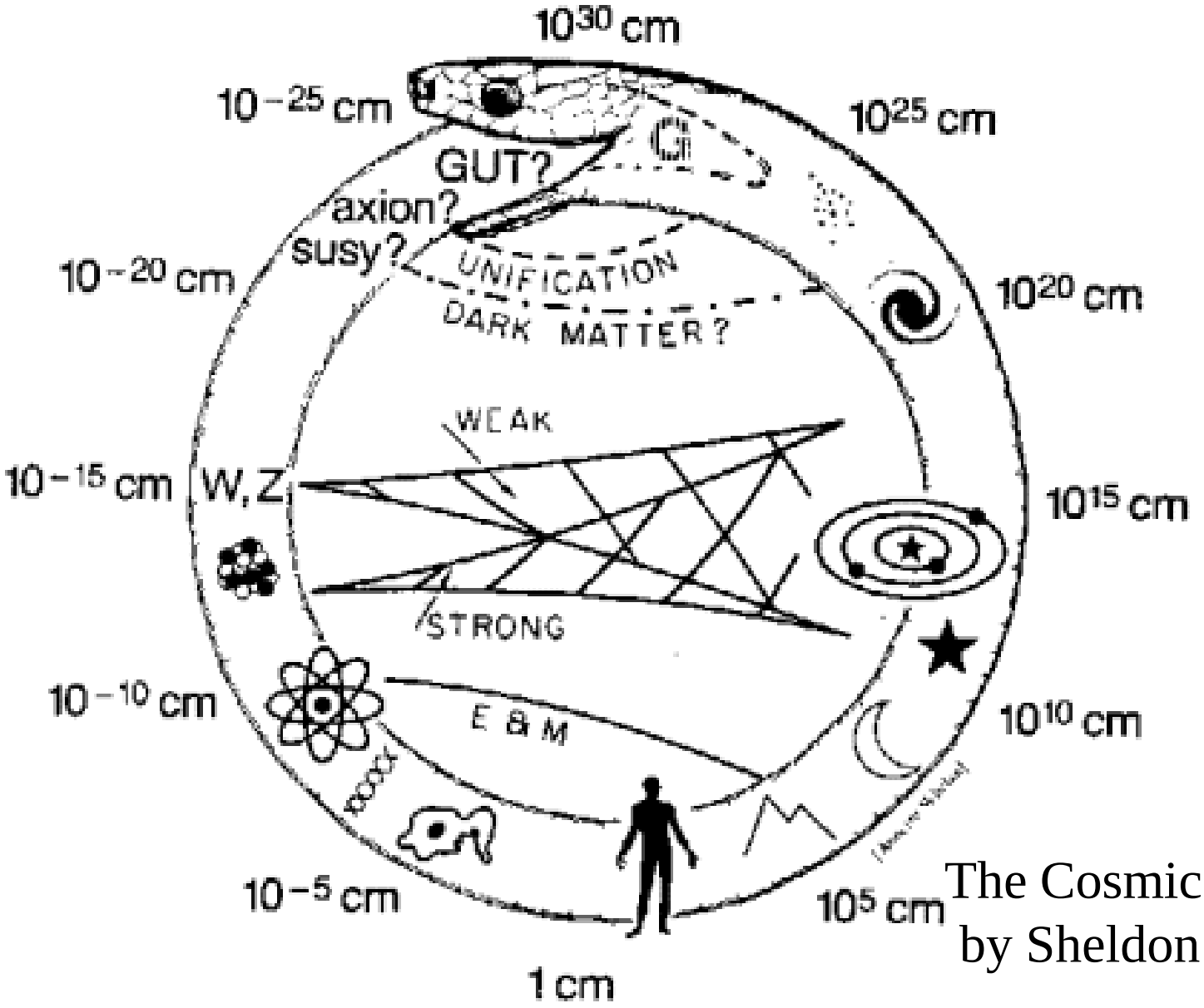
(University of Tokyo)

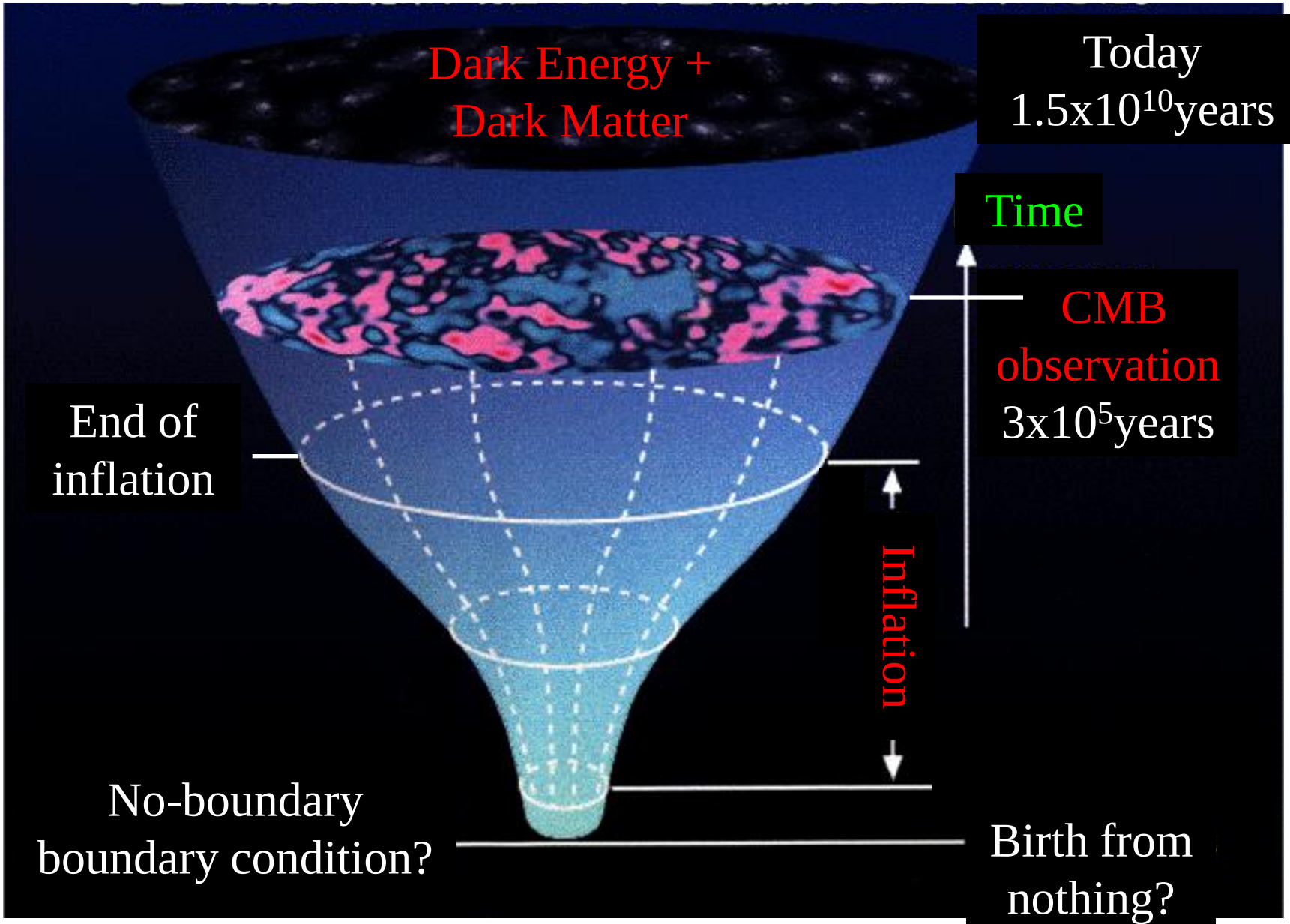
Based on recent works with
L.Kofman, T.Kobayashi and S.Kinoshita

The Cosmic Uroboros

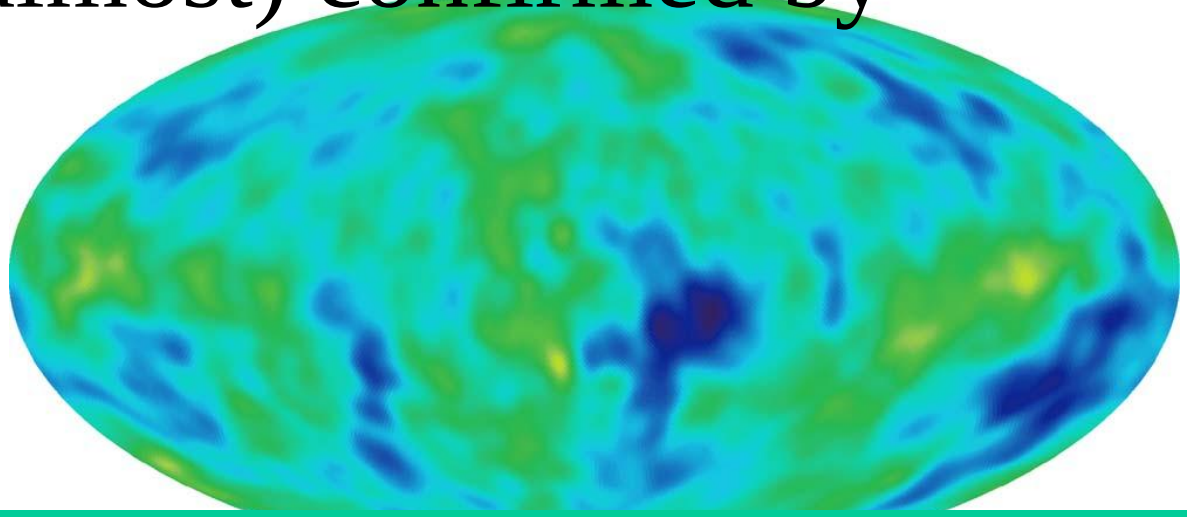


The Cosmic Uroboros
by Sheldon Glashow

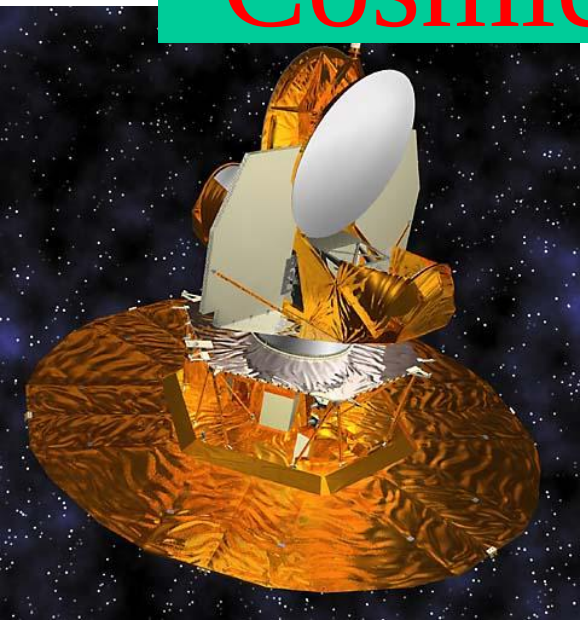
History of the Universe

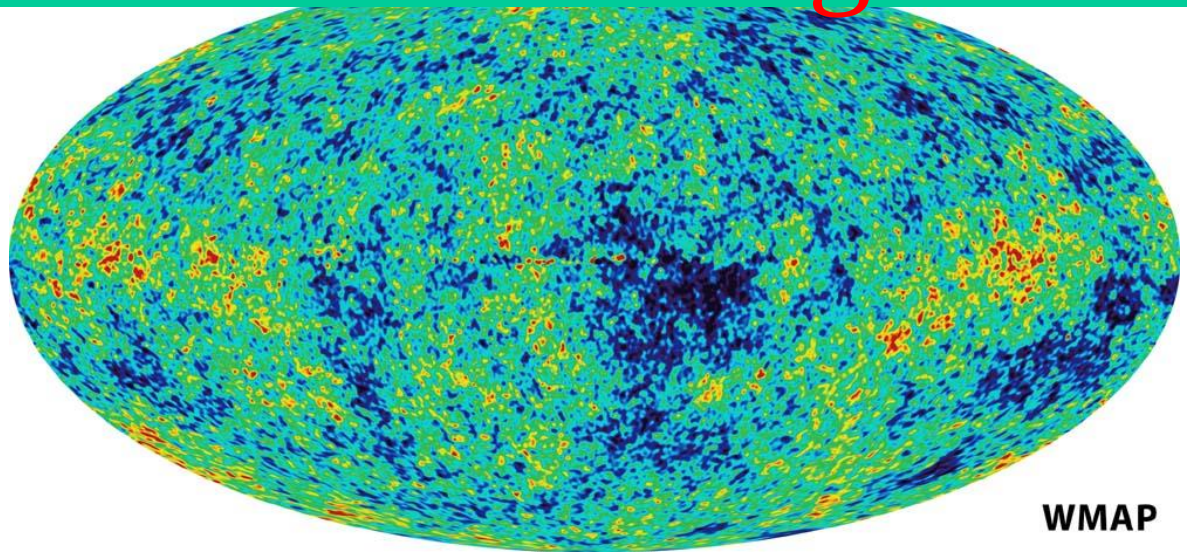


Inflation, dark energy and dark matter are (almost) confirmed by



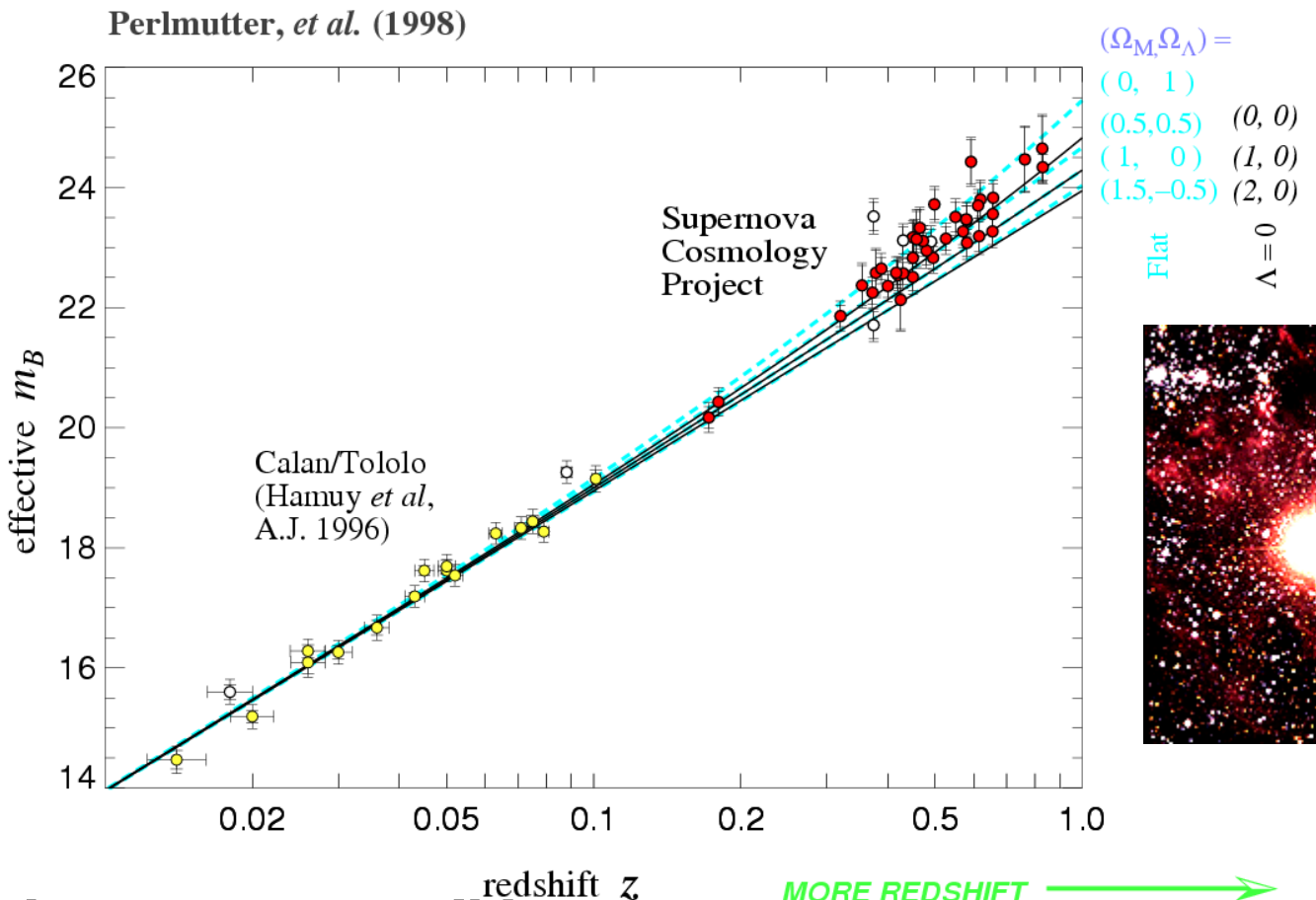
Cosmic microwave background





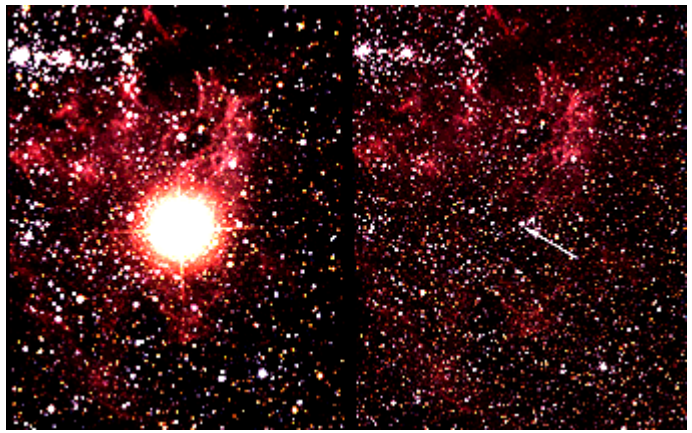
& Supernova observation

FAINTER
(Farther)
(Further back in time)



$(\Omega_M, \Omega_\Lambda)$ =
(0, 1)
(0.5,0.5) *(0, 0)*
(1, 0) *(1, 0)*
(1.5,–0.5) *(2, 0)*

Flat $\Lambda = 0$



http://supernova.lbl.gov/

MORE REDSHIFT
(More total expansion of universe
since the supernova explosion)

Three major mysteries in modern cosmlogy

- Early Universe
  Inflation
- Universe Today
  Dark Energy & Dark Matter

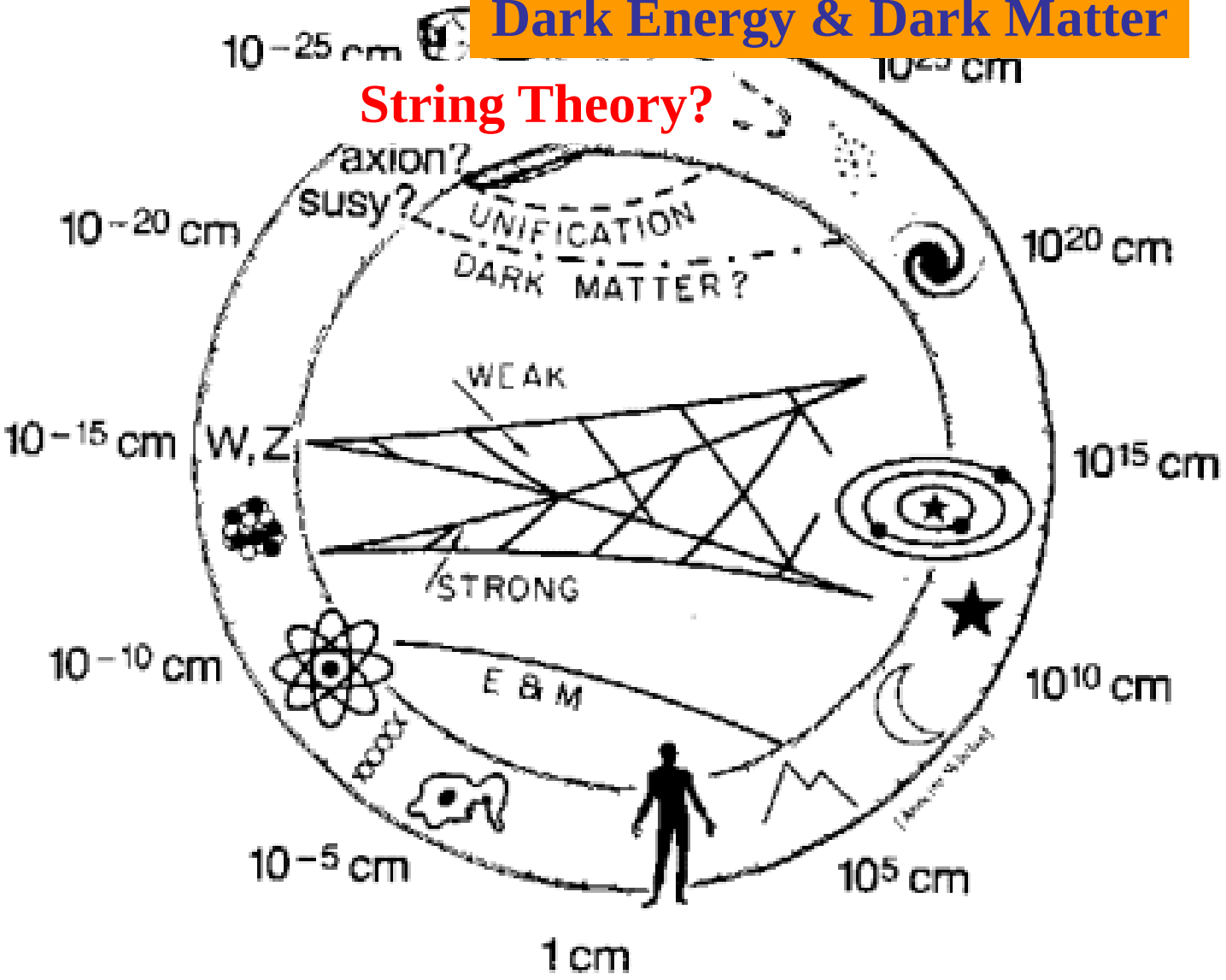
Two major (quasi-) de Sitter phases

We know they are (or were) there...

But, we don't know what they are.

Three mysteries: Inflation, Dark Energy & Dark Matter

String Theory?



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Unified Theory (Candidate): String Theory

Good things

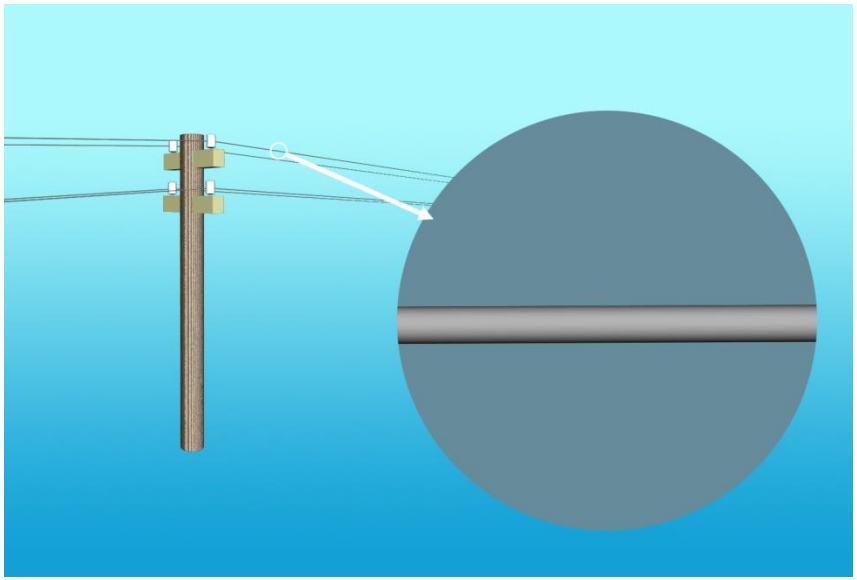
- Different particles = different oscillation modes of a string: possibility to explain complicated and diverse phenomena by LESS ELEMENTS.
- Unified theory candidate including GRAVITY
- GOOD CONTROL of quantum corrections (at least perturbatively, partly non-perturbatively)

String theory

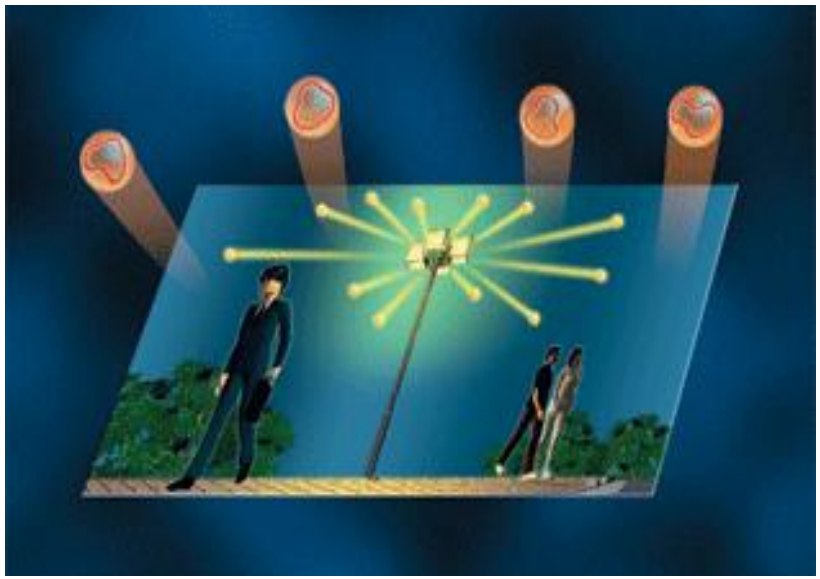
Something unusual

- Spacetime is 10 or 11 dimensional
- But, we know how to make those extra 6 or 7 dimensions invisible at low energy

Compactification



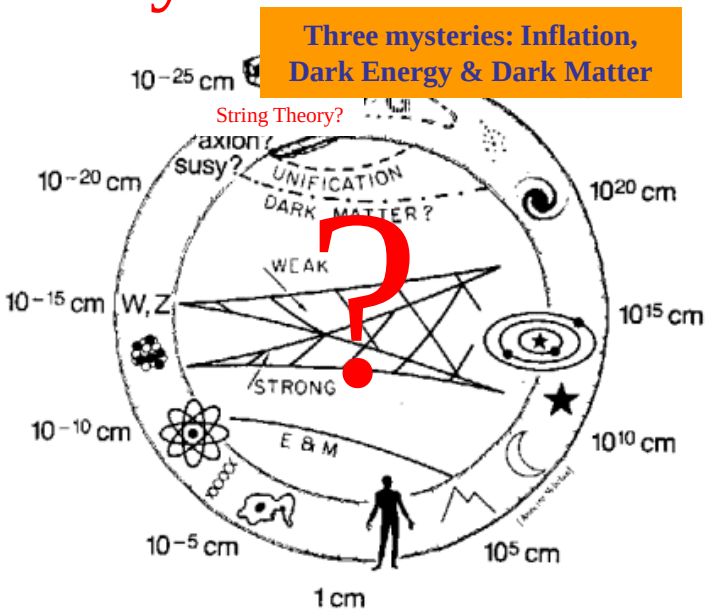
Brane world



String theory until 2002

Bad thing

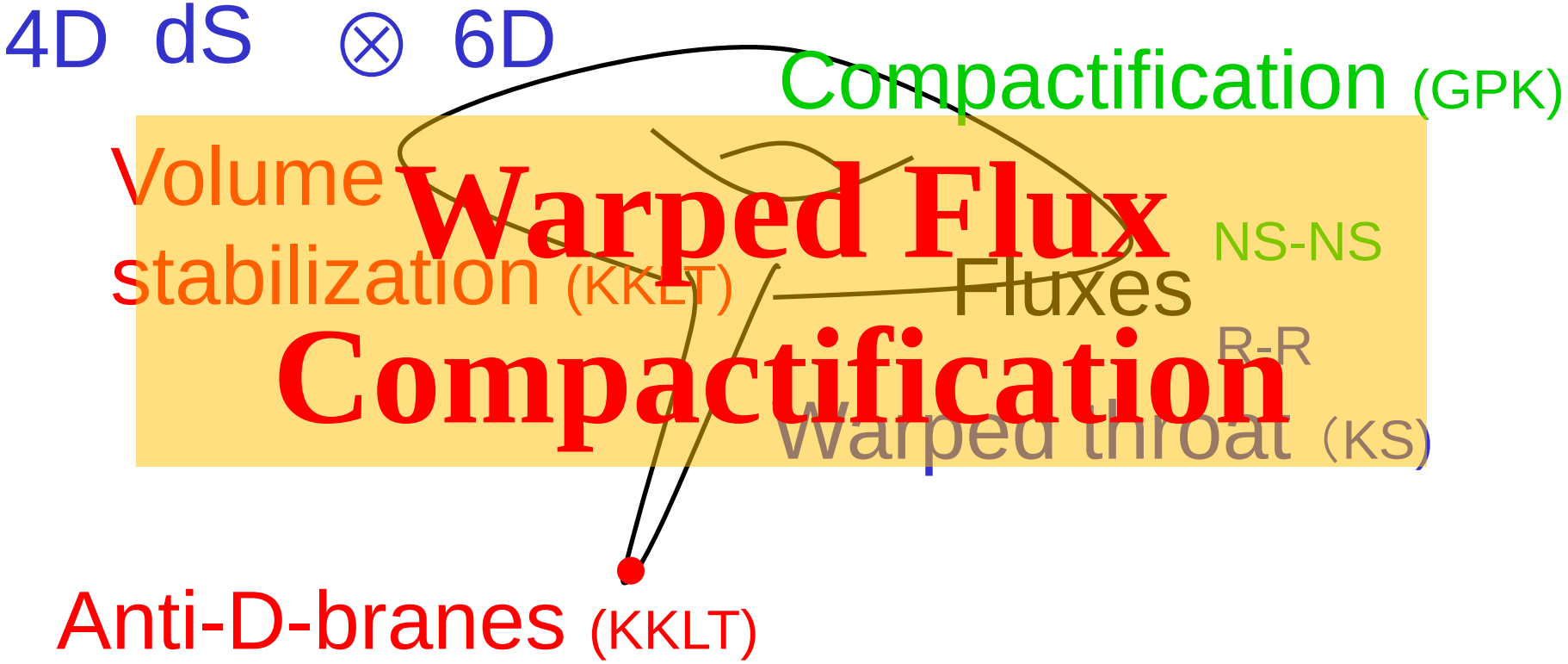
- No 4-dimensional de Sitter solution with stabilized moduli.
- No-go theorem!
- Contradict with inflation and dark energy?
- No way to reconcile with cosmology???



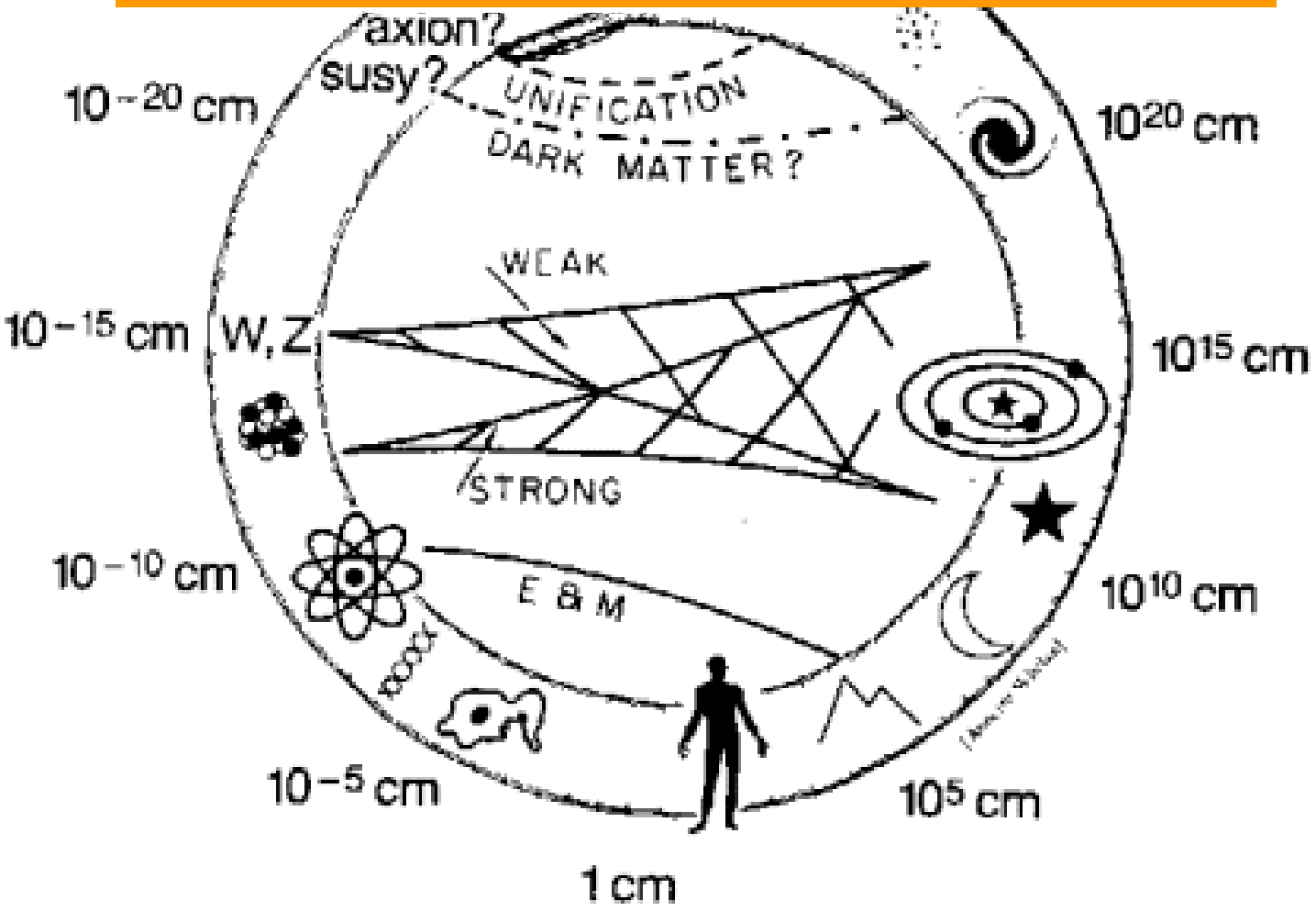
The Cosmic Uroboros does not close?

Recent Progress

- In 2003, a 4-dimensional de Sitter solution was finally found! Kachru, Kallosh, Linde and Trivedi (KKLT)
- In the previous no-go theorem, non-perturbative effects and branes were not taken into account.



Cosmology with Wraped Flux Compactification



The Cosmic Uroboros by Sheldon Glashow

3 models of stringy brane inflation

- **Wrapped DBI inflation**
  arXiv:0708.4285 [hep-th] with
  T.Kobayashi and S.Kinoshita
- **Chaotic brane inflation**
  work in progress with L.Kofman
- **Conformal rapid-roll inflation**
  arXiv:0709.1952 [hep-th] with L.Kofman

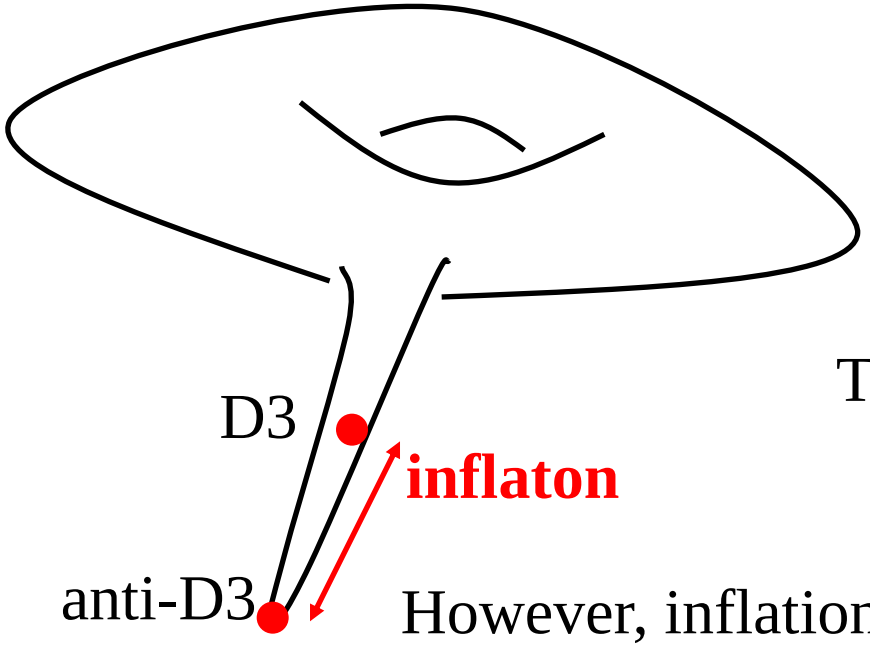
Model I:
Wrapped DBI inflation

arXiv:0708.4285 [hep-th] with
T.Kobayashi and S.Kinoshita

The KKLMMT model

Kachru, Kallosh, Linde, Maldacena, McAllister, and Trivedi 2003

= brane inflation (Dvali&Tye 1998) in KKLT



$$K = -3\log\left(\rho + \bar{\rho} - \phi\bar{\phi}\right)$$

$$W = W_0 + Ae^{-a\rho}$$

The inflaton gets the mass

$$m_\phi^2 = 2H^2$$

However, inflation with almost scale-invariant spectrum of perturbations requires $m_\phi^2 \sim 10^{-2} H^2$

This can be achieved by considering ϕ-dependent W and fine-tuning it.

KKLMMT fine-tuning

- **$m_\phi^2 = 2H^2$ would stop inflation.**
- **This is based on dynamics of a scalar with canonical kinetic term $-\partial^\mu\phi\partial_\mu\phi/2$.**
- However, the brane position is described by **nonlinear DBI kinetic action.**
- We should take it into account!
  [Silberstein&Tong 2003]

DBI inflation: model description

- Mobile D3-brane with relativistic speed
- Action

$$S=\int d^4\xi\sqrt{-g^{(4)}}\left[-T(\phi)\sqrt{1+\partial^\mu\phi\partial_\mu\phi/T(\phi)}+T(\phi)-V(\phi)\right]$$

DBI part

$$d\phi = T_3^{1/2}d\rho$$
$$T(\phi)=T_3h^4$$

ρ : radial position of the brane

- Energy density & Pressure

$$\rho = T(\phi)(\gamma-1)+V(\phi)$$
$$p = T(\phi)(1-\gamma^{-1})-V(\phi)$$

$$\gamma=\frac{1}{\sqrt{1-\dot\phi^2/T(\phi)}}$$

DBI inflation: good things

- A kind of k-inflation with general sound speed:
  a new model of stringy inflation!
- No need for slow-roll:
  a remedy to the η problem (KKLMMT fine-tuning)?
- Large non-Gaussianity:
  signature of stringy inflation?

$$f_{NL} \approx \frac{1}{3}(\gamma^2 - 1)$$

DBI inflation: bad thing

Baumann&Mcllister 2006; Lidsey&Huston 2007

- (UV) DBI inflation with large non-Gaussianity seems **inconsistent with WMAP data.**
- Can be consistent only in the limit when it **goes back to the slow-roll KKLMMT inflation.**
- The reason:

large but not too large $|f_{NL}|$ (say, $20<|f_{NL}|<300$)

➡ large r ➡ large $\Delta\phi/M_{Pl}$

i) **large $\Delta\phi$ ➡ long throat** ➡ large V_6

ii) not large M_{Pl} ➡ not large V_6 **confliction**

Useful equations to derive constraints

$$1 - n_s = \frac{r}{4}\sqrt{1+3f_{\rm NL}} - \frac{2\tilde{s}}{3f_{NL}} + \frac{\dot{T}}{TH}$$

$$\left(\frac{\Delta\phi}{M_p}\right)^2 \simeq \frac{r}{8}(\Delta\mathcal{N})^2$$

$$\frac{\pi^2}{16} r^2 P_s \left(1 + \frac{1}{3f_{\rm NL}}\right) = \frac{T(\phi)}{M_p^4}$$

$$\tilde{\epsilon} \equiv \frac{2M_p^2}{\gamma}\left(\frac{H'}{H}\right)^2$$

$$\tilde{\eta} \equiv \frac{2M_p^2 H''}{\gamma H}$$

$$\tilde{s} \equiv \frac{2M_p^2 \gamma' H'}{\gamma^2 H}$$

$$P_s = \frac{1}{8\pi^2 M_p^2}\frac{H^2}{c_s\tilde{\epsilon}} \qquad P_t = \frac{2}{\pi^2}\frac{H^2}{M_p^2} \qquad n_s - 1 = 2\tilde{\eta} - 4\tilde{\epsilon} - 2\tilde{s}$$

$$n_t = -2\tilde{\epsilon} \qquad r = 16 c_s \tilde{\epsilon} \qquad f_{\rm NL} = \frac{1}{3}\left(\frac{1}{c_s^2} - 1\right) \qquad c_s = \frac{1}{\gamma}$$

Our attempt: wrapped DBI

Kobayashi, Mukohyama and Kinoshita 2007

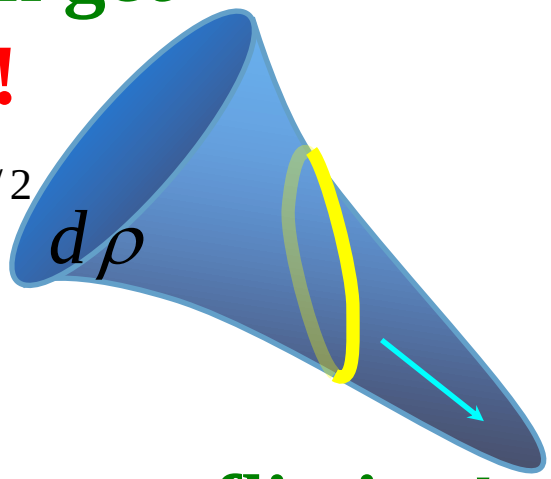
- The essential reason for the inconsistency of DBI inflation with WMAP data:

  large $\Delta\phi$ ➡ long throat

- For D3, this is inevitable: $d\phi = T_3^{1/2} d\rho$
- **For a wrapped D5 or D7, we can get larger $\Delta\phi$ from the same throat!**

$$d\phi = T_{3+2n}^{1/2} \left\{ \int d^{2n}\xi \sqrt{\det(G_{kl} - B_{kl})} \right\}^{1/2} d\rho$$

Large factor!



- **This significantly ameliorates the confliction!**

More stringent bound on wrapped DBI inflation

- Wrapped DBI inflation with large non-Gaussianity still requires a long throat. (Not as long as for D3 but still long.)
- **Known Calabi-Yau manifold cannot sustain such a long throat, (let alone for D3).**

long throat ➡ large background charge

➡ **large Euler number of CY**,

exceeding the known maximal value

$\chi = 1820448$ [Klemm,Lian,Roan&Yau 1997]

Summary of wrapped DBI inflation

- **Wrapping D5 or D7 over a cycle changes the relation between the brane position and the inflaton field.**
- This significantly ameliorates the confliction between (UV) DBI inflation and WMAP data.
- However, **successful wrapped (UV) DBI inflation requires Euler number larger than the known maximal value.**

Model II:
Chaotic Brane Inflation

Work in progress with L.Kofman

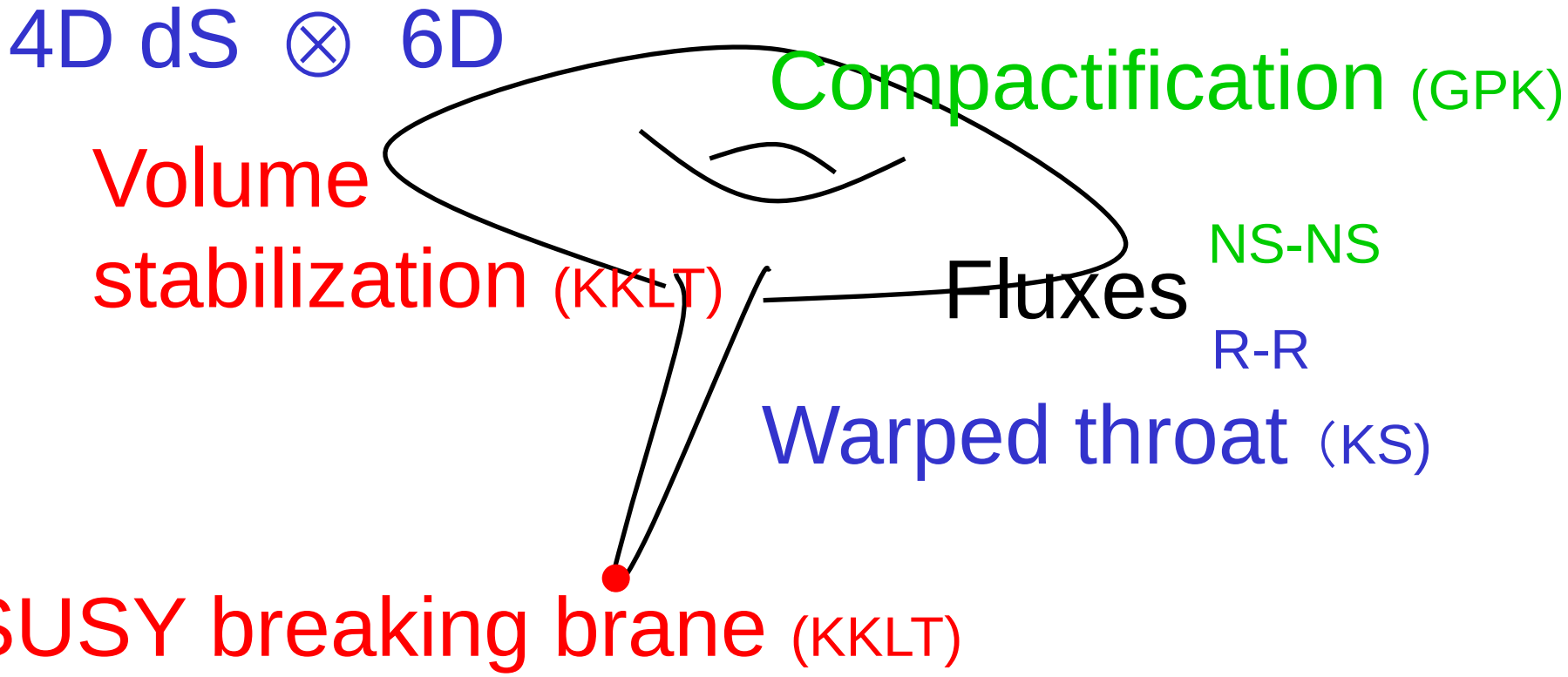
KKLT 4-dimensional de Sitter "solution"

- After stabilizing all moduli, anti-D-branes were introduced.
- Anti-D-branes or other SUSY breaking branes are indispensable!
- Without them, 4-dimensional cosmological constant would be negative and completely contradicts with cosmology.

SUSY breaking branes as Dark Matter

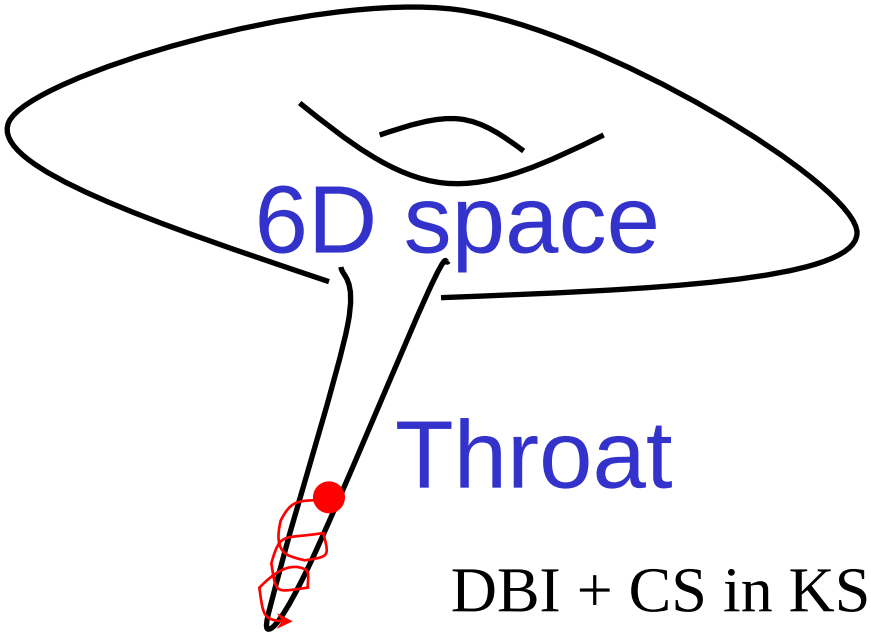
S.Mukohyama, hep-th/0505042

- What happens if SUSY breaking branes move in the extra 6 dimensions?



SUSY breaking branes as Dark Matter

- Falls toward the bottom of the throat, with rotation in the extra 5 dimensions.
- Behaves as DARK MATTER, from 4-dimensional viewpoint.



$\otimes$ 4D universe

DBI + CS in KS

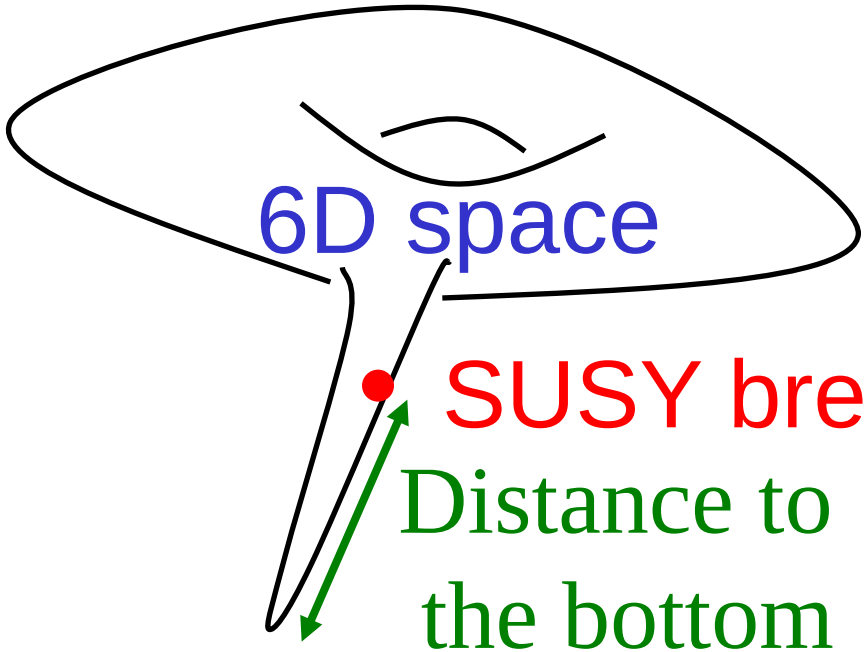
$\rho \propto a^{-3}$

brane motion = Dark Matter

Chaotic Inflation driven by brane motion

in progress, with L.Kofman

- Large motion of SUSY breaking brane
- In 4D, $V \sim \lambda\phi^4$

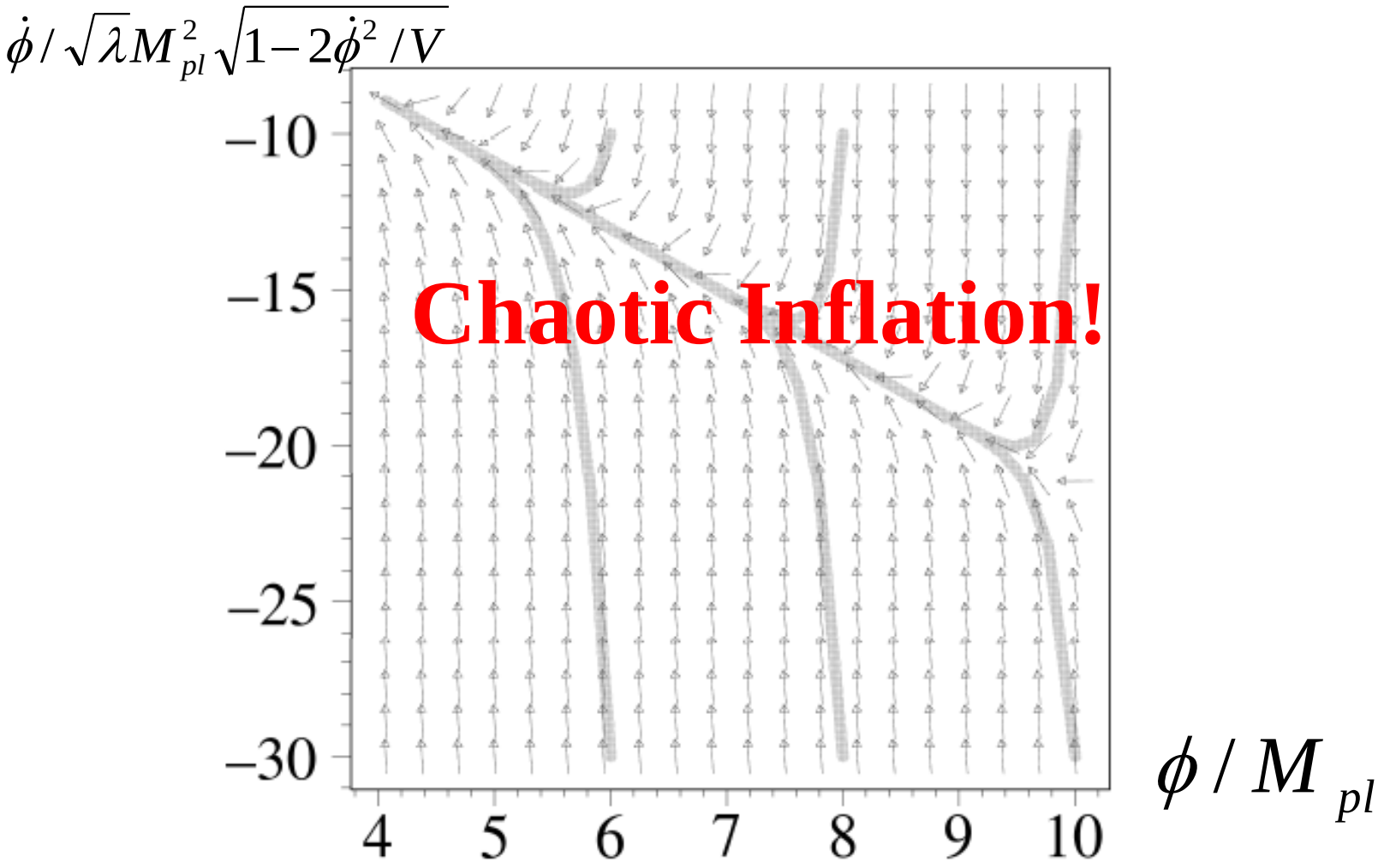


$\otimes$ 4D universe

SUSY breaking brane

Distance to the bottom = Inflaton

Phase portrait for an anti-D3-brane without non-rel. approximation



Length of the throat

- Can we have $\varphi > M_{Pl}$?
- For stack of anti-D3s, the answer is NO.

$$\frac{\varphi^2_{\max}}{M^2_{Pl}} \sim \frac{4N_{\bar{D}3}}{MK}$$

- Better for stack of wrapped D5s and D7s.

$$\frac{\varphi^2_{\max}}{M^2_{Pl}} \sim \frac{4N_{D5}}{M} \qquad \frac{\varphi^2_{\max}}{M^2_{Pl}} \sim \frac{N_{D7}K}{M}$$

Open issues for chaotic brane inflation

- Effects of volume moduli stabilization
- Coupling to curvature
- Backreaction to the KS geometry
- e.t.c.

still work in progress, with L.Kofman

If successful, this would be the first realization of chaotic inflation in string theory!

Model III:
Conformal Rapid-roll Inflation

arXiv:0709.1952 [hep-th] with L.Kofman

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Kachru, Kallosh, Linde, Maldacena, McAllister, and Trivedi 2003

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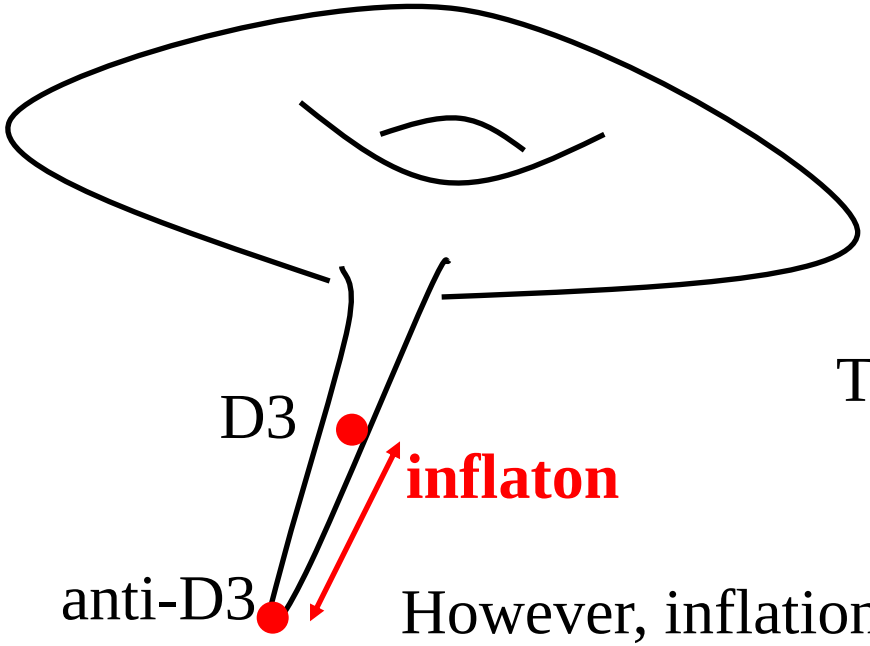

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This can be achieved by considering ϕ-dependent W and fine-tuning it.

KKLMMT fine-tuning

- **$m_\phi^2 = 2H^2$ would stop inflation.**
- **This is due to the conformal coupling $-R\phi^2/12$.**
- However, people have not yet looked at **modification of Einstein equation.**
- We should take it into account!
  [Kofman&Mukohyama 2007]

Scalar field with non-minimal coupling to curvature

$$I = \int d^4x\sqrt{-g}\left[\frac{R}{2\kappa^2} - \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - V(\phi) - \frac{\xi}{2}R\phi^2\right]$$

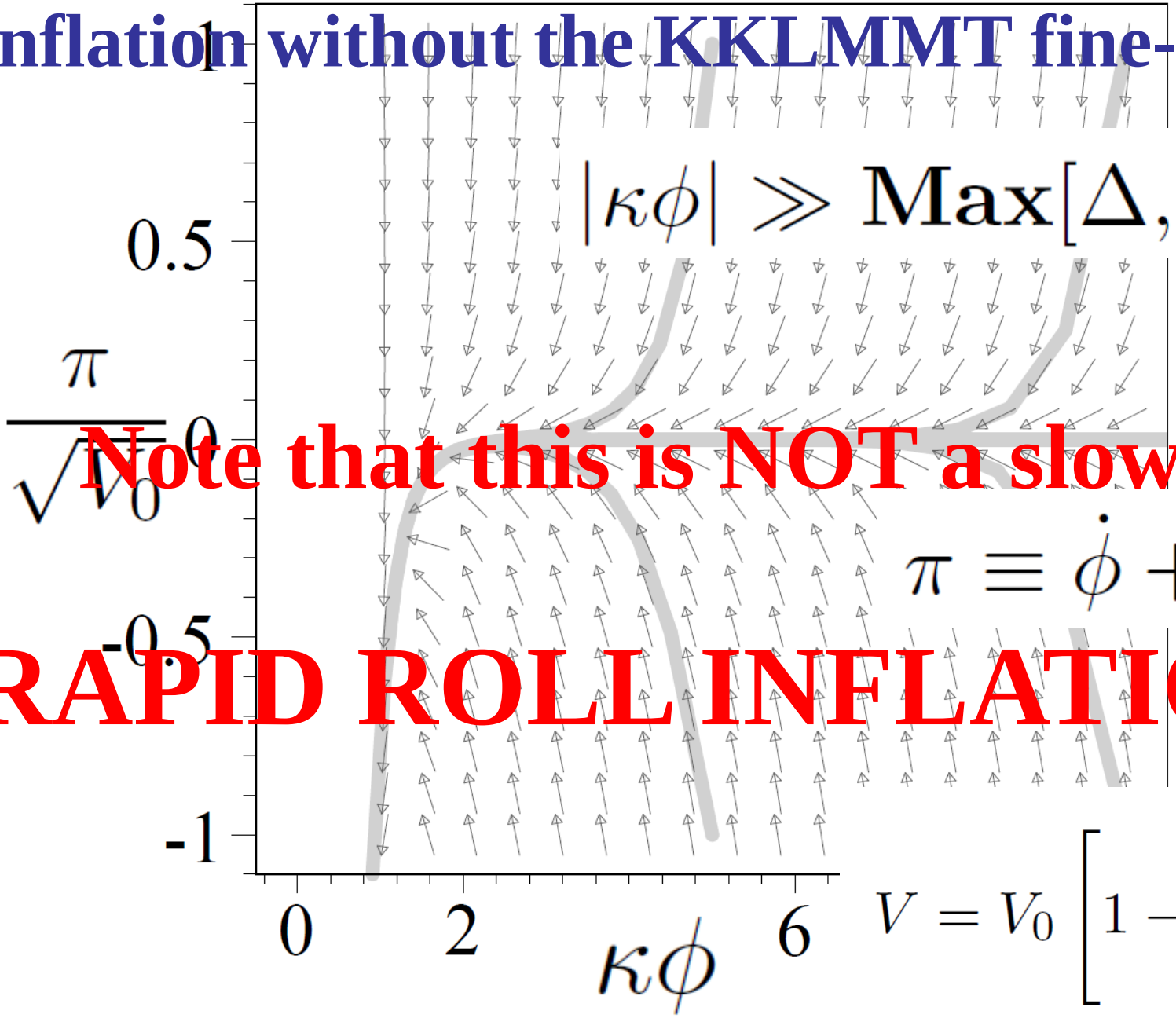
$$3\left(\kappa^{-2} - \xi\phi^2\right)H^2 = \frac{1}{2}\dot{\phi}^2 + 6\xi H\phi\dot{\phi} + V(\phi)$$

$$\ddot{\phi} + 3H\dot{\phi} + 6\xi(\dot{H} + 2H^2)\phi + V'(\phi) = 0$$

Looks like an additional mass term…

Actually, there are more terms!

Inflation without the KKLMMT fine-tuning!



$$|\kappa\phi| \gg \mathrm{Max}[\Delta, \Delta^{2/3}]$$

Note that this is NOT a slow roll!

$$\pi \equiv \dot{\phi} + H\phi$$

RAPID ROLL INFLATION!!!

$$V = V_0 \left[1 - \left(\frac{\Delta}{\kappa\phi}\right)^4\right]$$

Attractor behavior

With $\xi = 1/6$ and $V(\phi) = V_0$ $\quad \phi = \dfrac{\varphi}{a}$ $\quad H_0 \equiv \sqrt{\dfrac{\kappa^2 V_0}{3}}$ $\quad dt = a d\eta$

$$a'^2 = H_0^2 \left(a^4 + \frac{\varphi'^2}{2V_0} \right) \qquad \varphi'' = 0 \qquad a_0^4 \equiv \frac{v^2}{2V_0}$$

$$a'^2 = H_0^2 \left(a^4 + a_0^4 \right) \qquad \varphi = \varphi_* + v\eta$$

$$H_0(\eta_0 - \eta) = \int_a^\infty \frac{dx}{\sqrt{x^4 + a_0^4}} \simeq \frac{1}{a} \qquad a \gg a_0$$

$$a \simeq \frac{1}{H_0(\eta_0 - \eta)}$$

de Sitter attractor !!!

Condition for inflation with conformal coupling

- **Usual slow roll condition**

$$\epsilon \ll 1, \quad |\eta| \ll 1 \qquad \epsilon \equiv \frac{1}{2\kappa^2}\left(\frac{V'}{V}\right)^2 \quad \eta \equiv \frac{V''}{\kappa^2 V}$$

+ additional conditions

$$|\tilde{\epsilon}| \ll 1, \quad |\tilde{\eta}| \ll 1 \qquad \tilde{\epsilon} \equiv \frac{V'\phi}{2V} \quad \tilde{\eta} \equiv \frac{V''\phi}{V'} + c$$

- The 3rd condition is not satisfied by power-law potentials.
- **The D/anti-D potential satisfies it!**

Inflation with V(ϕ)

Starting with:

$$H^2 = \frac{\kappa^2}{3}\left(\frac{1}{2}\pi^2 + V\right) \quad (1)$$

$$-\frac{\dot{H}}{H^2} = \frac{\pi^2 + V'\phi/2}{\pi^2/2 + V} \quad (2)$$

$$\dot{\pi} + 2H\pi + V' = 0 \quad (3)$$

$$\pi \equiv \dot{\phi} + H\phi$$

Goal:

$$H^2 \simeq \frac{\kappa^2}{3}V \quad (1')$$

$$|\dot{H}/H^2| \ll 1 \quad (2')$$

$$\tilde{c}H\pi \simeq -V' \quad (3')$$

$$\tilde{c} = 2 + c$$

Definitions:

$$\epsilon \equiv \frac{1}{2\kappa^2}\left(\frac{V'}{V}\right)^2 \qquad \eta \equiv \frac{V''}{\kappa^2 V}$$

$$\tilde{\epsilon} \equiv \frac{V'\phi}{2V} \qquad \tilde{\eta} \equiv \frac{V''\phi}{V'} + c$$

Consistency conditions

(1) to (1')

$$\frac{\pi^2}{V} = \frac{(\tilde{c}H\pi)^2}{\tilde{c}^2H^2V} \simeq \frac{6}{\tilde{c}^2}\epsilon \quad \Rightarrow \quad \epsilon \ll 1$$

(2) to (2')

$$-\frac{\dot{H}}{H^2} = \frac{\pi^2/V + \tilde{\epsilon}}{\pi^2/2V + 1} \simeq \frac{6\epsilon/\tilde{c}^2 + \tilde{\epsilon}}{1 + 3\epsilon/\tilde{c}^2} \quad \Rightarrow \quad |\tilde{\epsilon}| \ll 1$$

(3) to (3')

$$\begin{aligned}\frac{\dot{\pi} - (\tilde{c}-2)H\pi}{\tilde{c}H\pi} &\simeq -\frac{V''}{\tilde{c}^2H^2} - \frac{\dot{H}}{\tilde{c}H^2} + \frac{V''\phi}{\tilde{c}^2H\pi} - \frac{\tilde{c}-2}{\tilde{c}} \\ &\simeq -\frac{3\eta}{\tilde{c}^2} - \frac{1}{\tilde{c}}\tilde{\eta} - \frac{\dot{H}}{\tilde{c}H^2} \quad \Rightarrow \quad |\eta| \ll 1 \quad |\tilde{\eta}| \ll 1\end{aligned}$$

e-foldings & mass hierarchy

- e-foldings:

  $a\,\phi \sim \text{const.}$

  $\mathbf{N \sim \ln (\phi_i / \phi_e)}$

- Mass hierarchy a la Randall-Sundrum :

  $\mathbf{M / M_{pl} \sim e^{-N}}$

- **Enough inflation vs TeV gravity:**

  $N \sim 62 + \ln (M / 10^{16}\text{GeV})$

  $M \sim \text{TeV}$

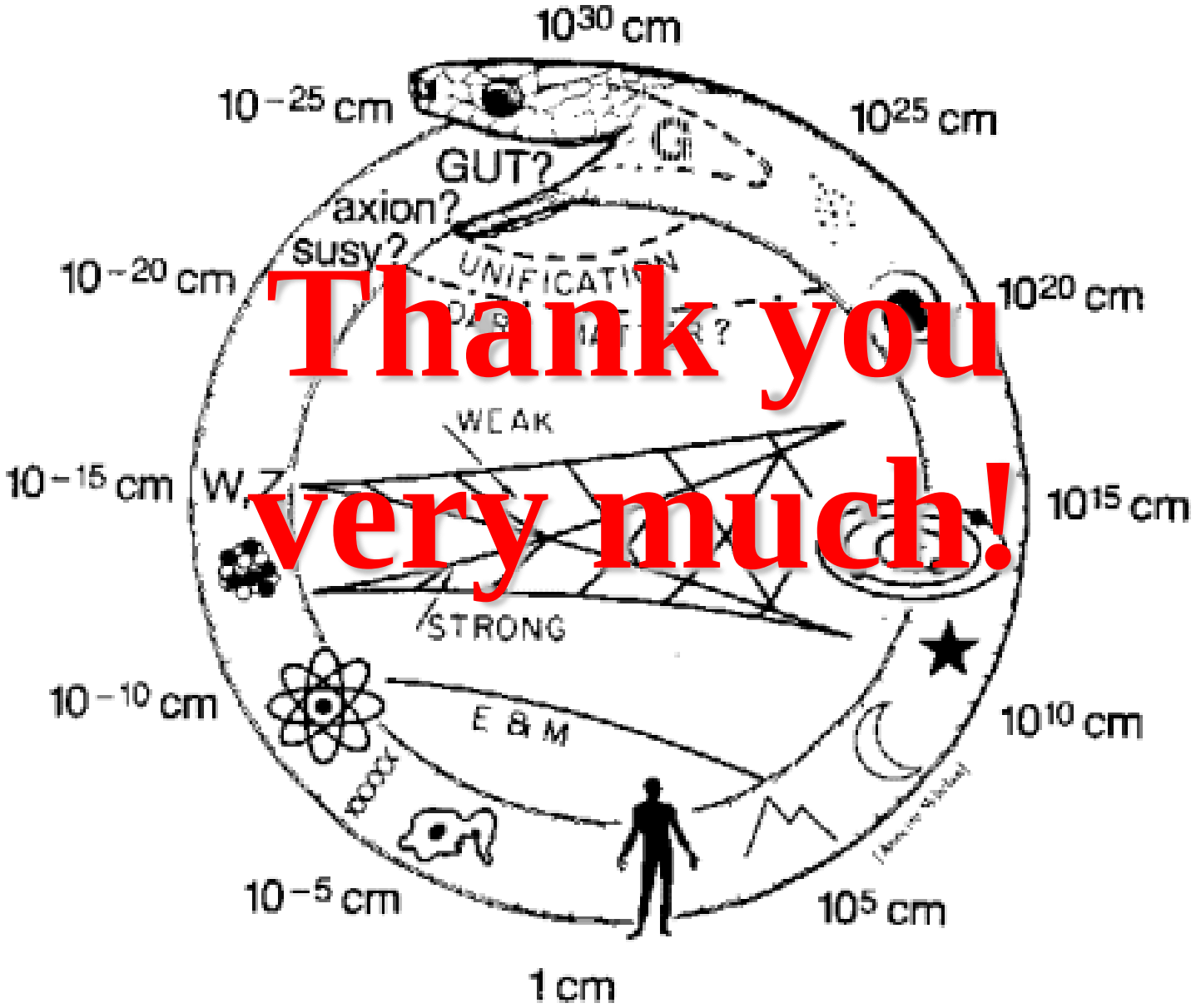
  These conditions are **equivalent!**

Summary of conformal rapid-roll inflation

- **Conformal coupling does NOT necessarily spoil inflation.**
- **Brane / anti-brane inflation may work without severe fine-tuning.**
- **E-foldings & mass hierarchy are related.**
- **Modulated reheating can generate scale-invariant density perturbation.**

Summary of this talk

- **It seems that we can really enjoy cosmology in the framework of string theory.**
- **Model I: Wrapping D5 or D7 over a cycle ameliorates the confliction between the DBI inflation and WMAP data.**
- **Model II: Chaotic brane inflation might be possible. If successful, this would be the first realization of chaotic inflation in string theory.**
- **Model III: Conformal rapid-roll inflation is possible without the KKLLMT fine-tuning. The mass hierarchy and e-foldings are related!**
- **A lot of interesting subjects are still remaining!**



Thank you very much!

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