MSSM-Like Models with R-Parity from Strings

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Based on

`arXiv:0708.2691 [hep-th]`

Outline of my Talk

- Why we are not happy with the Standard Model . . .
- Hints at Physics beyond the Standard Model
- String theory as an ultraviolet completion of GUTS
- MSSM-like theories from strings
- Conclusions

The Standard Model

Gauge group:

$$\mathsf{SU}(3)_c \times \mathsf{SU}(2)_L \times \mathsf{U}(1)_Y$$

Particle content:

Q	$(\mathbf{3},\mathbf{2})_{1/3}$	L	$(\mathbf{1},\mathbf{2})_{\text{-}1}$	H	$(\mathbf{1},\mathbf{2})_{1}$
$\bar{u}$	$(\overline{\mathbf{3}},\mathbf{1})_{\text{-}4/3}$	$\bar{e}$	$(\mathbf{1},\mathbf{1})_{2}$	$\bar{H}$	$(\mathbf{1},\mathbf{2})_{\text{-}1}$
$\bar{d}$	$(\overline{\mathbf{3}},\mathbf{1})_{2/3}$	$\bar{\nu}$	$(\mathbf{1},\mathbf{1})_{0}$		

Why are we not happy with SM?

(i) Too many free parameters

Gauge sector: 3 couplings g', g, g_3	3
Quark sector: 6 masses, 3 mixing angles, 1 CP phase	10
Lepton sector: 6 masses, 3 mixing angles and 1-3 phases	10
Higgs sector: Quartic coupling λ and vev v	2
θ parameter of QCD	1
	26

Why are we not happy with SM?

(ii) Structure of gauge symmetry

Why the product structure SU$(\mathbf{3})_c \times$ SU$(\mathbf{2})_L \times$ U$(\mathbf{1})_Y$?
Why 3 different coupling constants g', g, g_3?

(iii) Structure of family multiplets

One family is

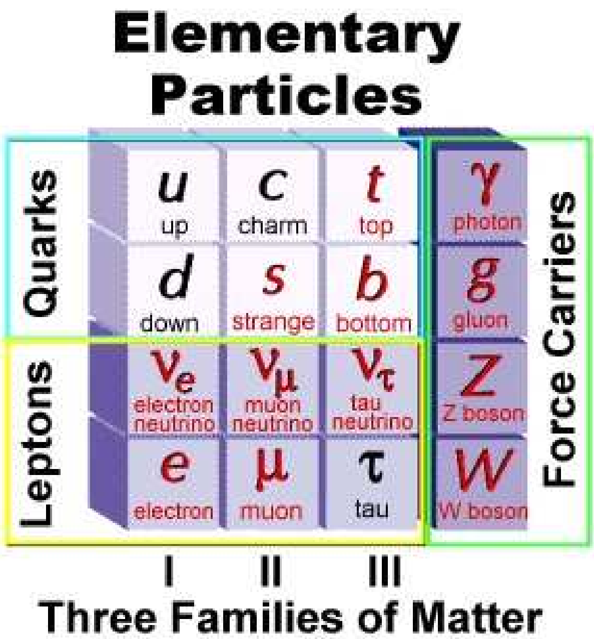
$$\begin{array}{cccccccccccc} (\mathbf{3},\mathbf{2})_{1/3} & + & (\overline{\mathbf{3}},\mathbf{1})_{\text{-}4/3} & + & (\mathbf{1},\mathbf{1})_{\text{-}2} & + & (\overline{\mathbf{3}},\mathbf{1})_{2/3} & + & (\mathbf{1},\mathbf{2})_{\text{-}1} & + & (\mathbf{1},\mathbf{1})_0 \\ Q & & \bar{u} & & \bar{e} & & \bar{d} & & L & & \bar{\nu} \end{array}$$

Can the particles be reorganized in a single representation?

Why are we not happy with SM?

(iv) Repetition of families

Earth, sun, stars, etc. are built from quarks and leptons of one generation. Why is this pattern for 1 generation replicated 3 times? Horizontal symmetries?



Why are we not happy with SM?

(v) Texture of Yukawa couplings

Minimal mixing in quark sector

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \simeq \begin{pmatrix} 0.97 & 0.22 & 0.00 \\ 0.22 & 0.97 & 0.04 \\ 0.00 & 0.04 & 0.99 \end{pmatrix}$$

Yukawa coupling of top-quark $\simeq 1$, and thus $m_t \simeq$ 200 GeV.
But why are the other quarks so light?

Why are we not happy with SM?

(vi) Texture of Yukawa couplings

Maximal mixing in lepton sector

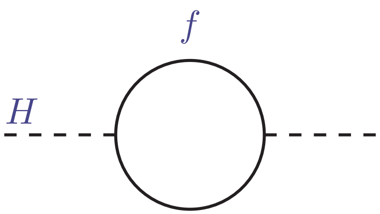
$$U_{\text{PMNS}} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \simeq \begin{pmatrix} 0.8 & 0.5 & 0.0 \\ -0.4 & 0.6 & 0.7 \\ 0.4 & -0.6 & 0.7 \end{pmatrix}$$

Why are neutrinos so light?

$$\Delta m_\nu^2 \sim 10^{-2} - 10^{-5}\ \text{eV}, \quad \sum m_\nu \lesssim 2\ \text{eV}$$

Why are we not happy with SM?

(vii) Hierarchy problem



$$\Delta m_H^2 = -\frac{|\lambda_f|^2}{8\pi^2}\Lambda_{\rm UV}^2 + \ldots$$

- Stability of the vacuum
- Requires incredible fine-tuning to set things right

Another way to put it: Why are there 2 fundamental scales at all?

Why are we not happy with SM?

(viii) Dark Matter

23% of our universe is made up of dark matter and the Standard Model offers no candidate particle . . .

Why are we not happy with SM?

(ix) Dark Energy

73% of our universe is made up of dark energy

Cosmological constant as calculated from QFT is the worst-predicted quantity in particle physics

Why are we not happy with SM?

(x) Gravity

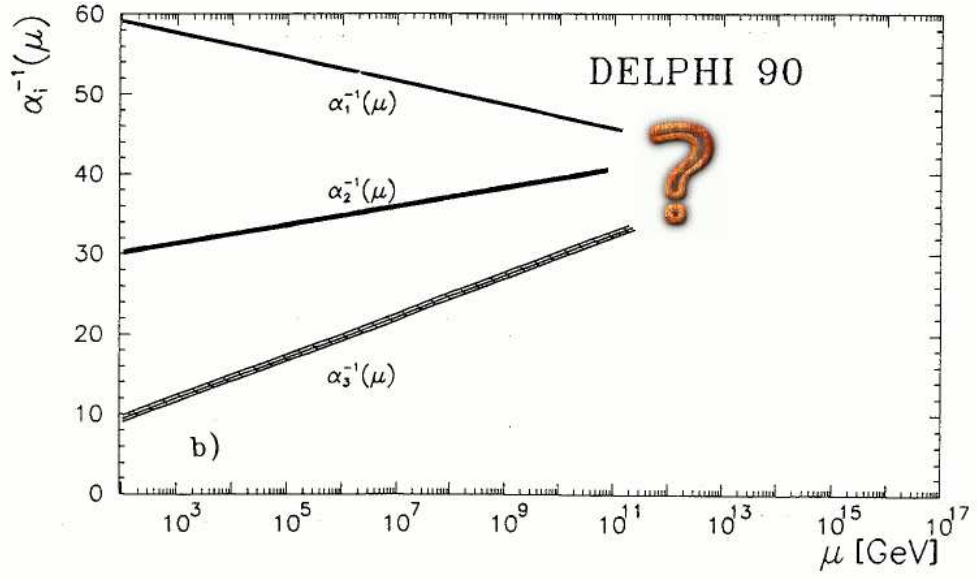
- Scales relevant in everyday life $\rightsquigarrow$ Newton's theory
- Satellites, solar system, etc. $\rightsquigarrow$ Still Newton's theory
- Cosmological scales $\rightsquigarrow$ Einstein's theory of GR
- Very small scales $\rightsquigarrow$ Need quantum theory of gravitation
- Don't know how to quantize gravity and how to unify with SM

(xi) Many other problems

Baryon asymmetry in the universe, charge quantization, . . .

Hints at Physics Beyond the SM?

H. Georgi and S. L. Glashow, “Unity of all elementary particle forces,” *Phys. Rev. Lett.* **32** (1974)



★ Running gauge couplings seem to meet at 10^{15} GeV ✔

⇝ Grand Unification: 1 gauge group, one 1 constant !

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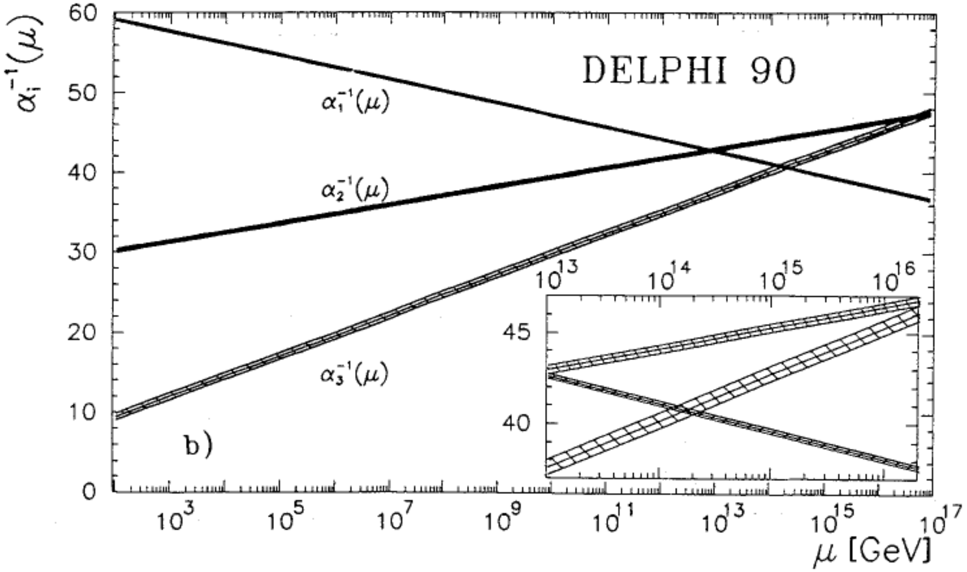
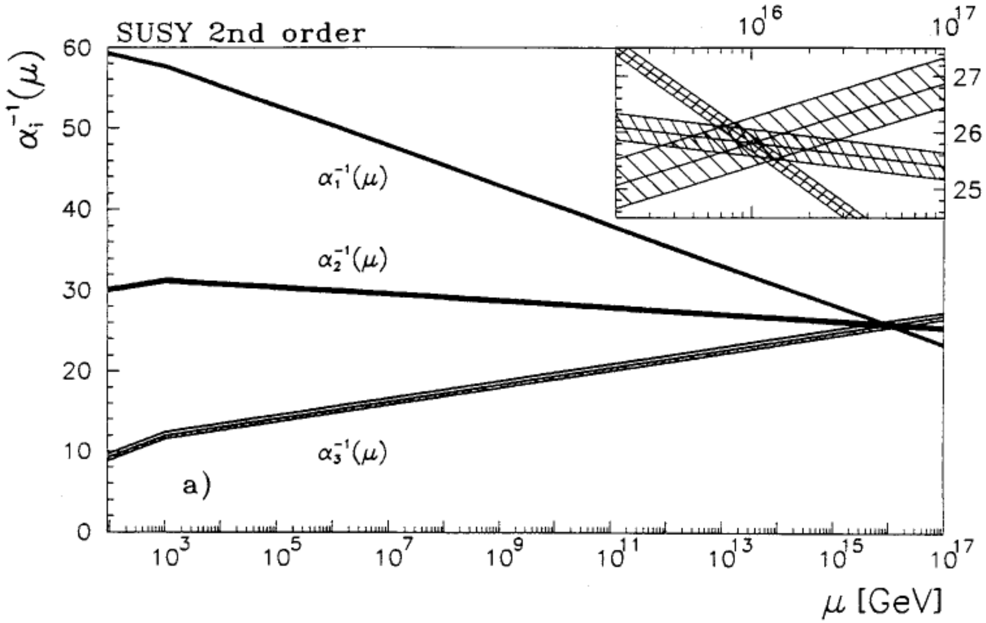


★ Looking more closely, couplings do not unify ✗

Are we missing something?

Hints at Physics Beyond the SM?

S. Dimopoulos, S. Raby, F. Wilczek, “Unification of couplings,” *Phys. Today.* **44N10** (1991) 25-33.

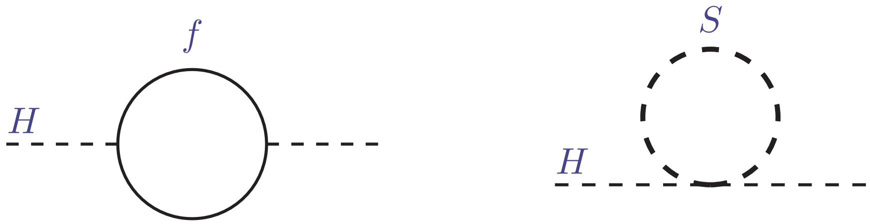


★ Supersymmetry helps: Unification at $\sim 3 \times 10^{16}$ GeV ✔

⇝ Strong motivation for GUTS *and* SUSY!

Another Motivation for SUSY

- Remember the hierarchy problem



(a) (b)

For fermions : $\Delta m_H^2 = -\frac{|\lambda_f|^2}{8\pi^2}\Lambda_{\rm UV}^2 + \ldots$

For scalars : $\Delta m_H^2 = \frac{\lambda_S}{16\pi^2}\left[\Lambda_{\rm UV}^2 - 2m_S^2 \ln(\Lambda_{\rm UV}/m_S) + \ldots\right]$

- SUSY correlates 1 fermion to 2 real scalars
  $\rightsquigarrow$ Quadratic divergencies cancel !

- “Technical” solution to hierarchy problem:
  Still 2 mass scales in theory, but energy dependence ’mild’

Grand Unification

Assume some grand unified gauge group at 3×10^{16}:

- Big enough to include SU$(\mathbf{3})_c \times$ SU$(\mathbf{2})_L \times$ U$(\mathbf{1})_Y$
- "Minimal" otherwise: Rank 4

All Lie algebras: classical and exceptional

SU$(n-\mathbf{1})$	E$_{\mathbf{6}}$	G$_{\mathbf{2}}$
Sp$(\mathbf{2}n)$	E$_{\mathbf{7}}$	F$_{\mathbf{4}}$
SO$(\mathbf{2}n+\mathbf{1})$	E$_{\mathbf{8}}$	
SO$(\mathbf{2}n)$		

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Now look at the minimal ones:

SU$(\mathbf{2})^4$, SO$(\mathbf{5})^2$, SU$(\mathbf{3})^2$, $(\mathrm{G}_\mathbf{2})^2$, SO$(\mathbf{8})$, SO$(\mathbf{9})$, Sp$(\mathbf{8})$, $\mathrm{F}_\mathbf{4}$, SU$(\mathbf{5})$

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- Do not contain SU$(\mathbf{3})$ as a subset

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- Cannot define sensible charge operator

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Grand Unification

Assume some grand unified gauge group at 3×10^{16}:

- Big enough to include $\text{SU}(\mathbf{3})_c \times \text{SU}(\mathbf{2})_L \times \text{U}(\mathbf{1})_Y$
- "Minimal" otherwise: Rank 4

Now look at the minimal ones:

$\cancel{\text{SU}(\mathbf{2})^4}$, $\cancel{\text{SO}(\mathbf{5})^2}$, $\cancel{\text{SU}(\mathbf{3})^2}$, $\cancel{(\text{G}_\mathbf{2})^2}$, $\cancel{\text{SO}(\mathbf{8})}$, $\cancel{\text{SO}(\mathbf{9})}$, $\cancel{\text{Sp}(\mathbf{8})}$, $\cancel{\text{F}_\mathbf{4}}$, $\text{SU}(\mathbf{5})$

- Do not contain $\text{SU}(\mathbf{3})$ as a subset
- Cannot define sensible charge operator
- No complex representations
- $\text{SU}(\mathbf{5})$ is unique candidate !!!

Grand Unification

Minimal SU($\mathbf{5}$) very predictive, but also very constrained ...
$\rightsquigarrow$ Suffers from some problems (see later in talk)

We need to introduce some "extra degrees of freedom" into the theory

- Consider next-to-minimal algebras
- Prefer simple Lie algebras, i.e. no direct products
- Must have complex representations for particles and anti-particles

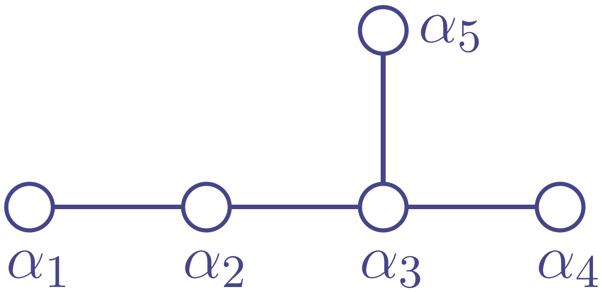
Candidates:

$$\text{SU}(\boldsymbol{n}) \text{ for } n \geq 5, \qquad \text{SO}(\mathbf{4}\boldsymbol{n}+\mathbf{2}) \text{ for } n \geq 2, \qquad \text{E}_{\mathbf{6}}$$

Supersymmetric Grand Unification

Assume SO$(\mathbf{10})$ at fundamental scale $\sim 10^{16}$ GeV

H. Fritzsch and P. Minkowski, “Unified interactions of leptons and hadrons,” *Ann. Phys.* **93** (1975)



SO$(\mathbf{10})$

Supersymmetric Grand Unification

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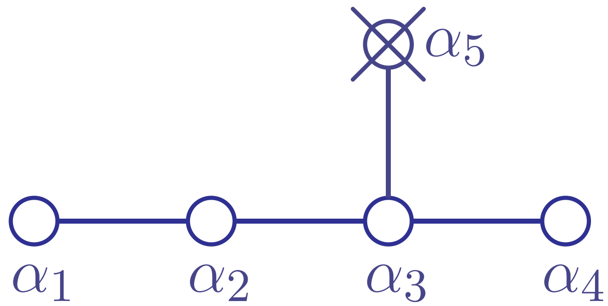

SO$(\mathbf{10})$ $\quad\rightarrow\quad$ SU$(\mathbf{5})\times$U$(\mathbf{1})_X$

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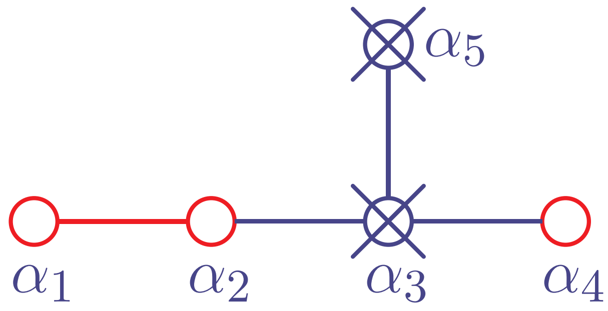

$$\mathrm{SO}(\mathbf{10}) \quad \rightarrow \quad \mathrm{SU}(\mathbf{5})\times\mathrm{U}(\mathbf{1})_X \quad \rightarrow \quad \mathrm{SU}(\mathbf{3})\times\mathrm{SU}(\mathbf{2})\times\mathrm{U}(\mathbf{1})_Y\times\mathrm{U}(\mathbf{1})_{\text{B-L}}$$

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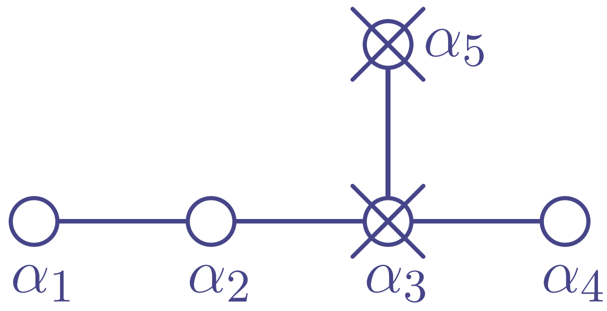

SO$(\mathbf{10})$ $\rightarrow$ SU$(\mathbf{5})\times$U$(\mathbf{1})_X$ $\rightarrow$ SU$(\mathbf{3})\times$SU$(\mathbf{2})\times$U$(\mathbf{1})_Y\times$U$(\mathbf{1})_{\text{B-L}}$

$\mathbf{16}$

Supersymmetric Grand Unification

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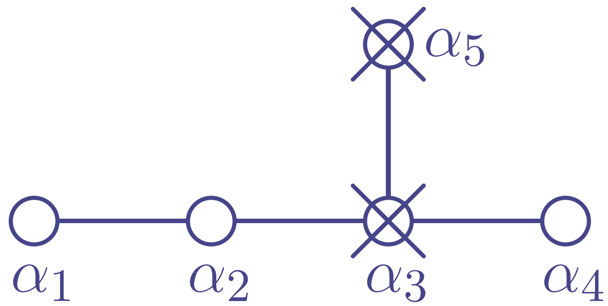

$$\text{SO}(\mathbf{10}) \quad \rightarrow \quad \text{SU}(\mathbf{5})\times\text{U}(\mathbf{1})_X \quad \rightarrow \quad \text{SU}(\mathbf{3})\times\text{SU}(\mathbf{2})\times\text{U}(\mathbf{1})_Y\times\text{U}(\mathbf{1})_{\text{B-L}}$$

$$\mathbf{16} \quad \rightarrow \quad \mathbf{10}+\overline{\mathbf{5}}+\mathbf{1}$$

Supersymmetric Grand Unification

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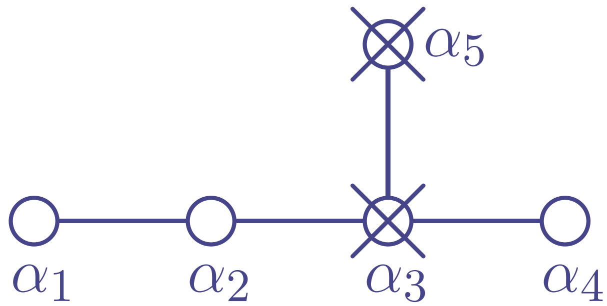


$$\text{SO}(\mathbf{10}) \;\rightarrow\; \text{SU}(\mathbf{5})\times\text{U}(\mathbf{1})_X \;\rightarrow\; \text{SU}(\mathbf{3})\times\text{SU}(\mathbf{2})\times\text{U}(\mathbf{1})_Y\times\text{U}(\mathbf{1})_{\text{B-L}}$$

$$\mathbf{16} \;\rightarrow\; \mathbf{10}+\overline{\mathbf{5}}+\mathbf{1}$$

$$\rightarrow (\mathbf{3},\mathbf{2})_{1/3} + (\overline{\mathbf{3}},\mathbf{1})_{-4/3} + (\mathbf{1},\mathbf{1})_{-2} + (\overline{\mathbf{3}},\mathbf{1})_{2/3} + (\mathbf{1},\mathbf{2})_{-1} + (\mathbf{1},\mathbf{1})_{0}$$

$$Q \qquad \bar{u} \qquad \bar{e} \qquad \bar{d} \qquad L \qquad \bar{\nu}$$

Predictions from Grand Unification

	$SU(5)$	$SO(10)$
• Gauge coupling unification	👍	👍
• 1 family of quarks and leptons in 1 irrep	👎	👍
• Quantization of electric charge	👍	☞
• $\sin^2\theta_w$ in agreement w/experiment	👍	☞
• $\frac{m_b}{m_\tau}$ ratio	☞	☞
• Right-handed neutrino	👎	👍
• Smallness of neutrino masses	☞	👍

Problems of Grand Unification

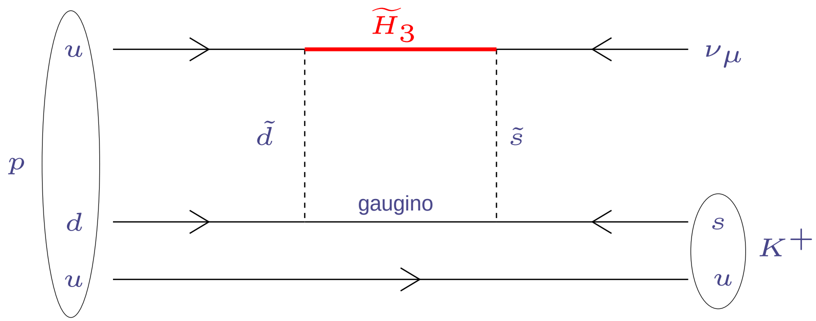
- Large representations required:

  $\mathbf{24}$ to break SU(5) $\rightsquigarrow$ Many more particles

  $\mathbf{45}$ for realistic mass matrices $\rightsquigarrow$ Even more particles

- Doublet-triplet splitting problem

$$\mathbf{10} \rightarrow \mathbf{5} + \overline{\mathbf{5}} \rightarrow (\mathbf{1},\mathbf{2}) + (\mathbf{3},\mathbf{1}) + (\mathbf{1},\mathbf{2}) + (\overline{\mathbf{3}},\mathbf{1})$$

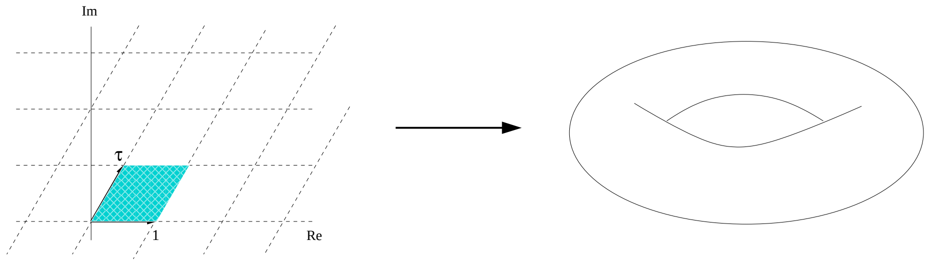


- Both problems can be avoided by extra dimensions !

The Idea of an Orbifold

Consider QFT in 6 dimensions: 4 large and 2 small

- Compactify 2 dimensions on torus



The Idea of an Orbifold

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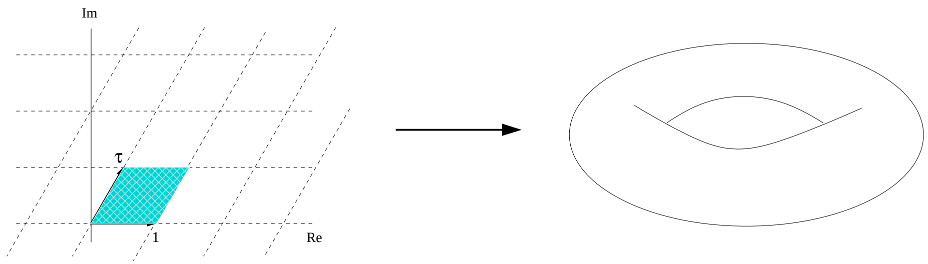
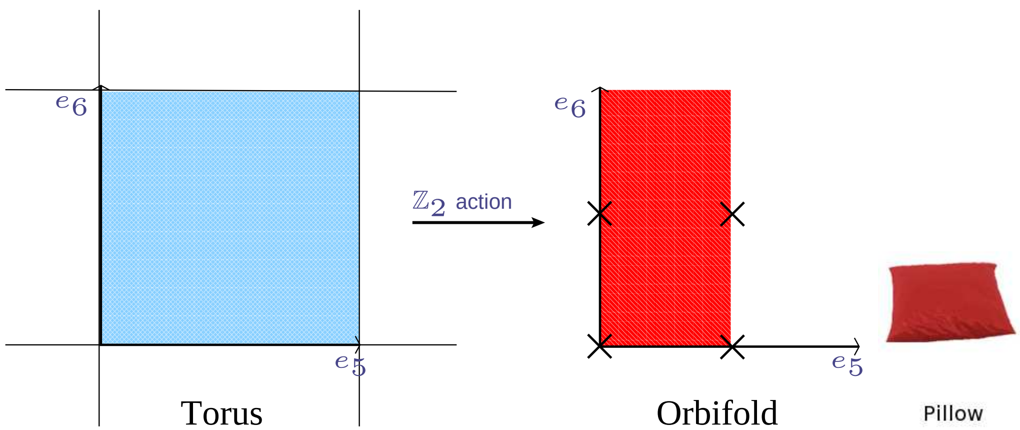
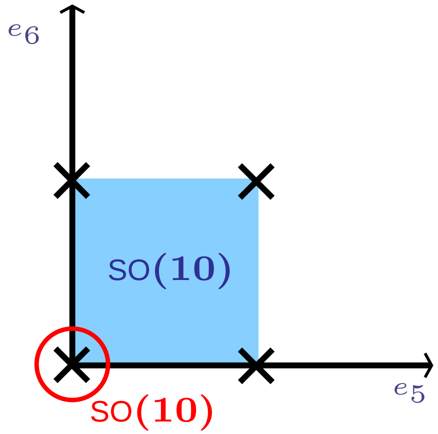


- Impose e.g. $\mathbb{Z}_2$ symmetry: $\mathcal{O} = T^2/P, \quad P = \{\mathbb{1}, \theta\}$



Breaking the Symmetry



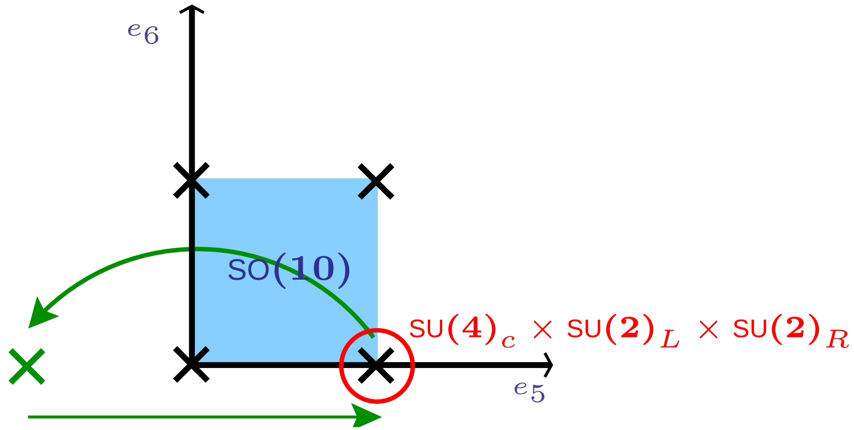
- Rotation + translation in spacetime $\sim$ phase in gauge d.o.f.
  $\hookrightarrow$ Convenient in field theory orbifolds
  $\hookrightarrow$ Mandatory in string orbifolds
- Consider fixed f_1 point in origin

$$\theta f_1 = f_1 \qquad \rightsquigarrow \qquad |\alpha_i\rangle \longrightarrow e^{2\pi i V\cdot\alpha_i}\,|\alpha_i\rangle$$

State survives if and only if “parity” is +1 !!!

- In this model, $V \equiv 0$ $\quad\rightsquigarrow\quad$ Full symmetry survives !!!

Breaking the Symmetry



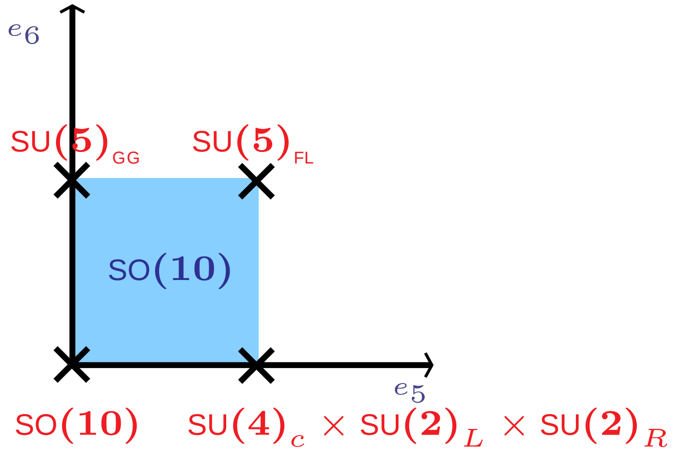
- Consider fixed f_2 point in origin

$$\theta f_2 + e_5 = f_2 \qquad \rightsquigarrow \qquad |\alpha_i\rangle \longrightarrow e^{2\pi i (V + A_5)\cdot\alpha_i}\,|\alpha_i\rangle$$

- Choose $A_5 = (*****)$:

$$A_5 \cdot \alpha_i \stackrel{!}{=} 0 \quad \longrightarrow \quad 21 \;\; \alpha_i \text{ survive} \quad \sim \quad SU(4)_c \times SU(2)_L \times SU(2)_R$$

Breaking the Symmetry



- 4 gauge groups at fixed points in extra dimension
- In 4 dimensions, we see intersection of these gauge symmetries:

$$\mathrm{SO}(10) \cap \mathrm{SU}(5) \cap \mathrm{PS} = \mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$$

- Judicious placement of fields $\rightsquigarrow$ Semi-realistic fermion masses
- Doublet-triplet splitting $\rightsquigarrow$ proton stability

Taking the Step to 10 Dimensions

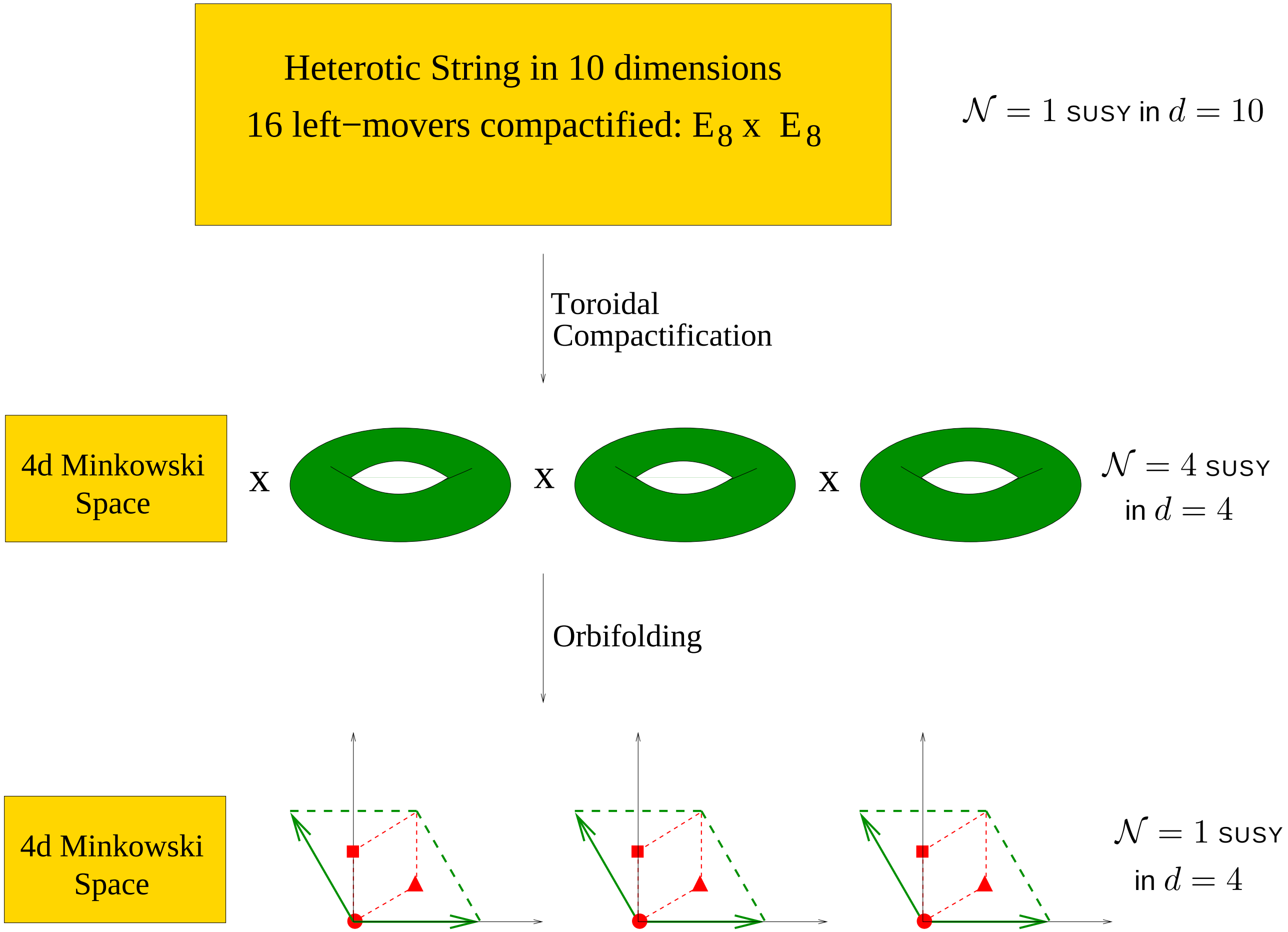
Some shortcomings of field theory orbifolds:

- Non-renormalizable and as such UV-divergent
- Number of dimensions arbitrary
- Symmetry breaking pattern arbitrary
- Placement of fields arbitrary
- In short: No organizing principle

Advantages of striny orbifolds:

- Finite and does not need renormalization
- Predicts 10 spacetime dimensions
- Symmetry in 10 dimensions ("bulk") is $E_8 \times E_8$
- Spectrum and localization of particles predicted
- Includes quantum theory of gravity

Orbifolds in 10 Dimensions



How to choose the Compactification Lattice

- Symmetry must be automorphism of lattice defining T^6
- In 3 dimensions: 219 crystallographic space groups
- In 6 dimensions: 28,927,922 crystallographic space groups
- Demand $P \subset$ SU(3) $\subset$ SO(6) for $\mathcal{N} = 1$ SUSY
- Restrict to abelian P for simplicity of construction

How to choose the Compactification Lattice

Point Group	Twist v
$\mathbb{Z}_3$	$(1/3,\ 1/3,\ -2/3)$
$\mathbb{Z}_4$	$(1/4,\ 1/4,\ -1/2)$
$\mathbb{Z}_6$-I	$(1/6,\ 1/6,\ -1/3)$
$\mathbb{Z}_6$-II	$(1/6,\ 1/3,\ -1/2)$
$\mathbb{Z}_7$	$(1/7,\ 2/7,\ -3/7)$
$\mathbb{Z}_8$-I	$(1/8,\ 1/4,\ -3/8)$
$\mathbb{Z}_8$-II	$(1/8,\ 3/8,\ -1/2)$
$\mathbb{Z}_{12}$-I	$(1/12,\ 1/3,\ -5/12)$
$\mathbb{Z}_{12}$-II	$(1/12,\ 5/12,\ -1/2)$

Point Group	Twist v_1	Twist v_2
$\mathbb{Z}_2 \times \mathbb{Z}_2$	$(1/2,\ 0,\ -1/2)$	$(0,\ 1/2,\ -1/2)$
$\mathbb{Z}_3 \times \mathbb{Z}_3$	$(1/3,\ 0,\ -1/3)$	$(0,\ 1/3,\ -1/3)$
$\mathbb{Z}_2 \times \mathbb{Z}_4$	$(1/2,\ 0,\ -1/2)$	$(0,\ 1/4,\ -1/4)$
$\mathbb{Z}_4 \times \mathbb{Z}_4$	$(1/4,\ 0,\ -1/4)$	$(0,\ 1/4,\ -1/4)$
$\mathbb{Z}_2 \times \mathbb{Z}_6$-I	$(1/2,\ 0,\ -1/2)$	$(0,\ 1/6,\ -1/6)$
$\mathbb{Z}_2 \times \mathbb{Z}_6$-II	$(1/2,\ 0,\ -1/2)$	$(1/6,\ 1/6,\ -1/3)$
$\mathbb{Z}_3 \times \mathbb{Z}_6$	$(1/3,\ 0,\ -1/3)$	$(0,\ 1/6,\ -1/6)$
$\mathbb{Z}_6 \times \mathbb{Z}_6$	$(1/6,\ 0,\ -1/6)$	$(0,\ 1/6,\ -1/6)$

Abelian point groups with $\mathcal{N} = 1$ SUSY in 4 dimensions

How to choose the Compactification Lattice

Point Group	Twist v
$\mathbb{Z}_3$	$(1/3,\ 1/3,\ \text{-}2/3)$
$\mathbb{Z}_4$	$(1/4,\ 1/4,\ \text{-}1/2)$
$\mathbb{Z}_6$-I	$(1/6,\ 1/6,\ \text{-}1/3)$
$\mathbb{Z}_6$-II	$(1/6,\ 1/3,\ \text{-}1/2)$
$\mathbb{Z}_7$	$(1/7,\ 2/7,\ \text{-}3/7)$
$\mathbb{Z}_8$-I	$(1/8,\ 1/4,\ \text{-}3/8)$
$\mathbb{Z}_8$-II	$(1/8,\ 3/8,\ \text{-}1/2)$
$\mathbb{Z}_{12}$-I	$(1/12,\ 1/3,\ \text{-}5/12)$
$\mathbb{Z}_{12}$-II	$(1/12,\ 5/12,\ \text{-}1/2)$

Point Group	Twist v_1	Twist v_2
$\mathbb{Z}_2 \times \mathbb{Z}_2$	$(1/2,\ 0,\ \text{-}1/2)$	$(0,\ 1/2,\ \text{-}1/2)$
$\mathbb{Z}_3 \times \mathbb{Z}_3$	$(1/3,\ 0,\ \text{-}1/3)$	$(0,\ 1/3,\ \text{-}1/3)$
$\mathbb{Z}_2 \times \mathbb{Z}_4$	$(1/2,\ 0,\ \text{-}1/2)$	$(0,\ 1/4,\ \text{-}1/4)$
$\mathbb{Z}_4 \times \mathbb{Z}_4$	$(1/4,\ 0,\ \text{-}1/4)$	$(0,\ 1/4,\ \text{-}1/4)$
$\mathbb{Z}_2 \times \mathbb{Z}_6$-I	$(1/2,\ 0,\ \text{-}1/2)$	$(0,\ 1/6,\ \text{-}1/6)$
$\mathbb{Z}_2 \times \mathbb{Z}_6$-II	$(1/2,\ 0,\ \text{-}1/2)$	$(1/6,\ 1/6,\ \text{-}1/3)$
$\mathbb{Z}_3 \times \mathbb{Z}_6$	$(1/3,\ 0,\ \text{-}1/3)$	$(0,\ 1/6,\ \text{-}1/6)$
$\mathbb{Z}_6 \times \mathbb{Z}_6$	$(1/6,\ 0,\ \text{-}1/6)$	$(0,\ 1/6,\ \text{-}1/6)$

Abelian point groups with $\mathcal{N} = 1$ SUSY in 4 dimensions

Ibanez / Kim / Nilles / Quevedo (1987)
and many others

How to choose the Compactification Lattice

Point Group	Twist v
$\mathbb{Z}_3$	(1/3, 1/3, -2/3)
$\mathbb{Z}_4$	(1/4, 1/4, -1/2)
$\mathbb{Z}_6$-I	(1/6, 1/6, -1/3)
$\mathbb{Z}_6$-II	(1/6, 1/3, -1/2)
$\mathbb{Z}_7$	(1/7, 2/7, -3/7)
$\mathbb{Z}_8$-I	(1/8, 1/4, -3/8)
$\mathbb{Z}_8$-II	(1/8, 3/8, -1/2)
$\mathbb{Z}_{12}$-I	(1/12, 1/3, -5/12)
$\mathbb{Z}_{12}$-II	(1/12, 5/12, -1/2)

Point Group	Twist v_1	Twist v_2
$\mathbb{Z}_2 \times \mathbb{Z}_2$	(1/2, 0, -1/2)	(0, 1/2, -1/2)
$\mathbb{Z}_3 \times \mathbb{Z}_3$	(1/3, 0, -1/3)	(0, 1/3, -1/3)
$\mathbb{Z}_2 \times \mathbb{Z}_4$	(1/2, 0, -1/2)	(0, 1/4, -1/4)
$\mathbb{Z}_4 \times \mathbb{Z}_4$	(1/4, 0, -1/4)	(0, 1/4, -1/4)
$\mathbb{Z}_2 \times \mathbb{Z}_6$-I	(1/2, 0, -1/2)	(0, 1/6, -1/6)
$\mathbb{Z}_2 \times \mathbb{Z}_6$-II	(1/2, 0, -1/2)	(1/6, 1/6, -1/3)
$\mathbb{Z}_3 \times \mathbb{Z}_6$	(1/3, 0, -1/3)	(0, 1/6, -1/6)
$\mathbb{Z}_6 \times \mathbb{Z}_6$	(1/6, 0, -1/6)	(0, 1/6, -1/6)

Abelian point groups with $\mathcal{N} = 1$ SUSY in 4 dimensions

K.S. Choi / Groot Nibbelink / Trapletti (2004)
Nilles / Ramos-Sanchez / Vaudrevange / A. W. (2006)

How to choose the Compactification Lattice

Point Group	Twist v
$\mathbb{Z}_3$	$(1/3,\ 1/3,\ \text{-}2/3)$
$\mathbb{Z}_4$	$(1/4,\ 1/4,\ \text{-}1/2)$
$\mathbb{Z}_6$-I	$(1/6,\ 1/6,\ \text{-}1/3)$
$\mathbb{Z}_6$-II	$(1/6,\ 1/3,\ \text{-}1/2)$
$\mathbb{Z}_7$	$(1/7,\ 2/7,\ \text{-}3/7)$
$\mathbb{Z}_8$-I	$(1/8,\ 1/4,\ \text{-}3/8)$
$\mathbb{Z}_8$-II	$(1/8,\ 3/8,\ \text{-}1/2)$
$\mathbb{Z}_{12}$-I	$(1/12,\ 1/3,\ \text{-}5/12)$
$\mathbb{Z}_{12}$-II	$(1/12,\ 5/12,\ \text{-}1/2)$

Point Group	Twist v_1	Twist v_2
$\mathbb{Z}_2 \times \mathbb{Z}_2$	$(1/2,\ 0,\ \text{-}1/2)$	$(0,\ 1/2,\ \text{-}1/2)$
$\mathbb{Z}_3 \times \mathbb{Z}_3$	$(1/3,\ 0,\ \text{-}1/3)$	$(0,\ 1/3,\ \text{-}1/3)$
$\mathbb{Z}_2 \times \mathbb{Z}_4$	$(1/2,\ 0,\ \text{-}1/2)$	$(0,\ 1/4,\ \text{-}1/4)$
$\mathbb{Z}_4 \times \mathbb{Z}_4$	$(1/4,\ 0,\ \text{-}1/4)$	$(0,\ 1/4,\ \text{-}1/4)$
$\mathbb{Z}_2 \times \mathbb{Z}_6$-I	$(1/2,\ 0,\ \text{-}1/2)$	$(0,\ 1/6,\ \text{-}1/6)$
$\mathbb{Z}_2 \times \mathbb{Z}_6$-II	$(1/2,\ 0,\ \text{-}1/2)$	$(1/6,\ 1/6,\ \text{-}1/3)$
$\mathbb{Z}_3 \times \mathbb{Z}_6$	$(1/3,\ 0,\ \text{-}1/3)$	$(0,\ 1/6,\ \text{-}1/6)$
$\mathbb{Z}_6 \times \mathbb{Z}_6$	$(1/6,\ 0,\ \text{-}1/6)$	$(0,\ 1/6,\ \text{-}1/6)$

Abelian point groups with $\mathcal{N} = 1$ SUSY in 4 dimensions

Kobayashi / Raby / Zhang (2004)
Buchmuller / Hamaguchi / Lebedev / Ratz (2005/6)
Lebedev / Nilles / Raby / Ramos-Sanchez / Ratz / Vaudrevange / A.W. (2006/7)

How to choose the Compactification Lattice

Point Group	Twist v
$\mathbb{Z}_3$	$(1/3,\ 1/3,\ \text{-}2/3)$
$\mathbb{Z}_4$	$(1/4,\ 1/4,\ \text{-}1/2)$
$\mathbb{Z}_6$-I	$(1/6,\ 1/6,\ \text{-}1/3)$
$\mathbb{Z}_6$-II	$(1/6,\ 1/3,\ \text{-}1/2)$
$\mathbb{Z}_7$	$(1/7,\ 2/7,\ \text{-}3/7)$
$\mathbb{Z}_8$-I	$(1/8,\ 1/4,\ \text{-}3/8)$
$\mathbb{Z}_8$-II	$(1/8,\ 3/8,\ \text{-}1/2)$
$\mathbb{Z}_{12}$-I	$(1/12,\ 1/3,\ \text{-}5/12)$
$\mathbb{Z}_{12}$-II	$(1/12,\ 5/12,\ \text{-}1/2)$

Point Group	Twist v_1	Twist v_2
$\mathbb{Z}_2 \times \mathbb{Z}_2$	$(1/2,\ 0,\ \text{-}1/2)$	$(0,\ 1/2,\ \text{-}1/2)$
$\mathbb{Z}_3 \times \mathbb{Z}_3$	$(1/3,\ 0,\ \text{-}1/3)$	$(0,\ 1/3,\ \text{-}1/3)$
$\mathbb{Z}_2 \times \mathbb{Z}_4$	$(1/2,\ 0,\ \text{-}1/2)$	$(0,\ 1/4,\ \text{-}1/4)$
$\mathbb{Z}_4 \times \mathbb{Z}_4$	$(1/4,\ 0,\ \text{-}1/4)$	$(0,\ 1/4,\ \text{-}1/4)$
$\mathbb{Z}_2 \times \mathbb{Z}_6$-I	$(1/2,\ 0,\ \text{-}1/2)$	$(0,\ 1/6,\ \text{-}1/6)$
$\mathbb{Z}_2 \times \mathbb{Z}_6$-II	$(1/2,\ 0,\ \text{-}1/2)$	$(1/6,\ 1/6,\ \text{-}1/3)$
$\mathbb{Z}_3 \times \mathbb{Z}_6$	$(1/3,\ 0,\ \text{-}1/3)$	$(0,\ 1/6,\ \text{-}1/6)$
$\mathbb{Z}_6 \times \mathbb{Z}_6$	$(1/6,\ 0,\ \text{-}1/6)$	$(0,\ 1/6,\ \text{-}1/6)$

Abelian point groups with $\mathcal{N} = 1$ SUSY in 4 dimensions

Förste / Nilles/ Vaudrevange / A. W. (2004)

How to choose the Compactification Lattice

Point Group	Twist v
$\mathbb{Z}_3$	$(1/3,\ 1/3,\ \text{-}2/3)$
$\mathbb{Z}_4$	$(1/4,\ 1/4,\ \text{-}1/2)$
$\mathbb{Z}_6$-I	$(1/6,\ 1/6,\ \text{-}1/3)$
$\mathbb{Z}_6$-II	$(1/6,\ 1/3,\ \text{-}1/2)$
$\mathbb{Z}_7$	$(1/7,\ 2/7,\ \text{-}3/7)$
$\mathbb{Z}_8$-I	$(1/8,\ 1/4,\ \text{-}3/8)$
$\mathbb{Z}_8$-II	$(1/8,\ 3/8,\ \text{-}1/2)$
$\mathbb{Z}_{12}$-I	$(1/12,\ 1/3,\ \text{-}5/12)$
$\mathbb{Z}_{12}$-II	$(1/12,\ 5/12,\ \text{-}1/2)$

Point Group	Twist v_1	Twist v_2
$\mathbb{Z}_2 \times \mathbb{Z}_2$	$(1/2,\ 0,\ \text{-}1/2)$	$(0,\ 1/2,\ \text{-}1/2)$
$\mathbb{Z}_3 \times \mathbb{Z}_3$	$(1/3,\ 0,\ \text{-}1/3)$	$(0,\ 1/3,\ \text{-}1/3)$
$\mathbb{Z}_2 \times \mathbb{Z}_4$	$(1/2,\ 0,\ \text{-}1/2)$	$(0,\ 1/4,\ \text{-}1/4)$
$\mathbb{Z}_4 \times \mathbb{Z}_4$	$(1/4,\ 0,\ \text{-}1/4)$	$(0,\ 1/4,\ \text{-}1/4)$
$\mathbb{Z}_2 \times \mathbb{Z}_6$-I	$(1/2,\ 0,\ \text{-}1/2)$	$(0,\ 1/6,\ \text{-}1/6)$
$\mathbb{Z}_2 \times \mathbb{Z}_6$-II	$(1/2,\ 0,\ \text{-}1/2)$	$(1/6,\ 1/6,\ \text{-}1/3)$
$\mathbb{Z}_3 \times \mathbb{Z}_6$	$(1/3,\ 0,\ \text{-}1/3)$	$(0,\ 1/6,\ \text{-}1/6)$
$\mathbb{Z}_6 \times \mathbb{Z}_6$	$(1/6,\ 0,\ \text{-}1/6)$	$(0,\ 1/6,\ \text{-}1/6)$

Abelian point groups with $\mathcal{N} = 1$ SUSY in 4 dimensions

Raby / A. W. *work in progress*

How to choose the Compactification Lattice

Point Group	Twist v
$\mathbb{Z}_3$	$(1/3,\ 1/3,\ \text{-}2/3)$
$\mathbb{Z}_4$	$(1/4,\ 1/4,\ \text{-}1/2)$
$\mathbb{Z}_6$-I	$(1/6,\ 1/6,\ \text{-}1/3)$
$\mathbb{Z}_6$-II	$(1/6,\ 1/3,\ \text{-}1/2)$
$\mathbb{Z}_7$	$(1/7,\ 2/7,\ \text{-}3/7)$
$\mathbb{Z}_8$-I	$(1/8,\ 1/4,\ \text{-}3/8)$
$\mathbb{Z}_8$-II	$(1/8,\ 3/8,\ \text{-}1/2)$
$\mathbb{Z}_{12}$-I	$(1/12,\ 1/3,\ \text{-}5/12)$
$\mathbb{Z}_{12}$-II	$(1/12,\ 5/12,\ \text{-}1/2)$

Point Group	Twist v_1	Twist v_2
$\mathbb{Z}_2 \times \mathbb{Z}_2$	$(1/2,\ 0,\ \text{-}1/2)$	$(0,\ 1/2,\ \text{-}1/2)$
$\mathbb{Z}_3 \times \mathbb{Z}_3$	$(1/3,\ 0,\ \text{-}1/3)$	$(0,\ 1/3,\ \text{-}1/3)$
$\mathbb{Z}_2 \times \mathbb{Z}_4$	$(1/2,\ 0,\ \text{-}1/2)$	$(0,\ 1/4,\ \text{-}1/4)$
$\mathbb{Z}_4 \times \mathbb{Z}_4$	$(1/4,\ 0,\ \text{-}1/4)$	$(0,\ 1/4,\ \text{-}1/4)$
$\mathbb{Z}_2 \times \mathbb{Z}_6$-I	$(1/2,\ 0,\ \text{-}1/2)$	$(0,\ 1/6,\ \text{-}1/6)$
$\mathbb{Z}_2 \times \mathbb{Z}_6$-II	$(1/2,\ 0,\ \text{-}1/2)$	$(1/6,\ 1/6,\ \text{-}1/3)$
$\mathbb{Z}_3 \times \mathbb{Z}_6$	$(1/3,\ 0,\ \text{-}1/3)$	$(0,\ 1/6,\ \text{-}1/6)$
$\mathbb{Z}_6 \times \mathbb{Z}_6$	$(1/6,\ 0,\ \text{-}1/6)$	$(0,\ 1/6,\ \text{-}1/6)$

Abelian point groups with $\mathcal{N} = 1$ SUSY in 4 dimensions

B. Kyae / J. E. Kim et al. (2006)

How to choose the Compactification Lattice

Point Group	Twist v
$\mathbb{Z}_3$	$(1/3,\ 1/3,\ \text{-}2/3)$
$\mathbb{Z}_4$	$(1/4,\ 1/4,\ \text{-}1/2)$
$\mathbb{Z}_6$-I	$(1/6,\ 1/6,\ \text{-}1/3)$
$\mathbb{Z}_6$-II	$(1/6,\ 1/3,\ \text{-}1/2)$
$\mathbb{Z}_7$	$(1/7,\ 2/7,\ \text{-}3/7)$
$\mathbb{Z}_8$-I	$(1/8,\ 1/4,\ \text{-}3/8)$
$\mathbb{Z}_8$-II	$(1/8,\ 3/8,\ \text{-}1/2)$
$\mathbb{Z}_{12}$-I	$(1/12,\ 1/3,\ \text{-}5/12)$
$\mathbb{Z}_{12}$-II	$(1/12,\ 5/12,\ \text{-}1/2)$

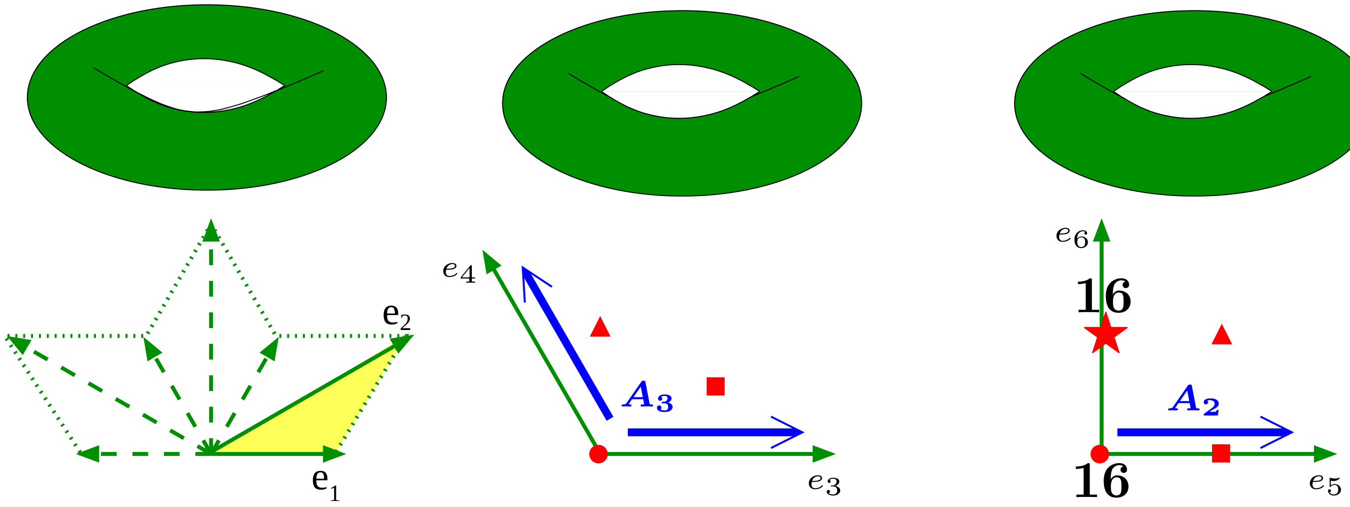
Point Group	Twist v_1	Twist v_2
$\mathbb{Z}_2 \times \mathbb{Z}_2$	$(1/2,\ 0,\ \text{-}1/2)$	$(0,\ 1/2,\ \text{-}1/2)$
$\mathbb{Z}_3 \times \mathbb{Z}_3$	$(1/3,\ 0,\ \text{-}1/3)$	$(0,\ 1/3,\ \text{-}1/3)$
$\mathbb{Z}_2 \times \mathbb{Z}_4$	$(1/2,\ 0,\ \text{-}1/2)$	$(0,\ 1/4,\ \text{-}1/4)$
$\mathbb{Z}_4 \times \mathbb{Z}_4$	$(1/4,\ 0,\ \text{-}1/4)$	$(0,\ 1/4,\ \text{-}1/4)$
$\mathbb{Z}_2 \times \mathbb{Z}_6$-I	$(1/2,\ 0,\ \text{-}1/2)$	$(0,\ 1/6,\ \text{-}1/6)$
$\mathbb{Z}_2 \times \mathbb{Z}_6$-II	$(1/2,\ 0,\ \text{-}1/2)$	$(1/6,\ 1/6,\ \text{-}1/3)$
$\mathbb{Z}_3 \times \mathbb{Z}_6$	$(1/3,\ 0,\ \text{-}1/3)$	$(0,\ 1/6,\ \text{-}1/6)$
$\mathbb{Z}_6 \times \mathbb{Z}_6$	$(1/6,\ 0,\ \text{-}1/6)$	$(0,\ 1/6,\ \text{-}1/6)$

Abelian point groups with $\mathcal{N} = 1$ SUSY in 4 dimensions

$\rightsquigarrow$ We will work with this orbifold

The $\mathbb{Z}_6$-II Orbifold

The geometry of compact space



G_2 root lattice

SU(3) root lattice

SO(4) root lattice

Gauge embedding

- "Shift" V : 61 choices
- "Wilson lines" A_3 and A_2 : Thousands of choices

$\hookrightarrow$ Need guidelines

Adding Phenomenological Priors

- Grand Unified Group in intermediate step

$$\mathsf{E}_8 \xrightarrow{V} \mathsf{SO}(10) \times \ldots \xrightarrow{A_3} \ldots \xrightarrow{A_2} \mathsf{SU}(3) \times \mathsf{SU}(2) \times \mathsf{U}(1)$$

$\leadsto$ leaves us with 15 shifts

- One family in <u>complete</u> multiplet of $\mathsf{SO}(10)$

$$\mathbf{16} \xrightarrow{A_3} \ldots \xrightarrow{A_5} (\mathbf{3},\mathbf{2})_{1/3} + (\overline{\mathbf{3}},\mathbf{1})_{-4/3} + (\mathbf{1},\mathbf{1})_{-2} + (\overline{\mathbf{3}},\mathbf{1})_{2/3} + (\mathbf{1},\mathbf{2})_{-1} + (\mathbf{1},\mathbf{1})_0 \quad ✔$$

$\leadsto$ leaves us with 2 shifts : V_{22} and V_{56}

Recent Orbifold Publications

Shift V_{56}

T. Kobayashi, S. Raby, R.-J. Zhang, *Phys. Lett.* **B593** (2004)

T. Kobayashi, S. Raby, R.-J. Zhang, *Nucl. Phys.* **B704** (2005)

W. Buchmuller, K. Hamaguchi, O. Lebedev and M. Ratz, *Nucl. Phys.* **B712** (2005)

Shift V_{22}

W. Buchmuller, K. Hamaguchi, O. Lebedev and M. Ratz, *Phys. Rev. Lett.* **96** (2006)

W. Buchmuller, K. Hamaguchi, O. Lebedev and M. Ratz, **hep-th/0606187**

Lebedev, Nilles, Raby, Ratz, Ramos-Sanchez, Vaudrevange, A.W., *Phys. Lett.* **B645** (2007)

Lebedev, Nilles, Raby, Ratz, Ramos-Sanchez, Vaudrevange, A.W., *Phys. Rev. Lett.* **98**, 181602 (2007)

S. Raby, A.W., *Phys. Rev. Lett.* **99**, 051802 (2007)

S. Raby, A.W., *Phys. Rev.* **D76**, 086006 (2007)

Lebedev, Nilles, Raby, Ratz, Ramos-Sanchez, Vaudrevange, A.W., **arXiv:0708.2691**

A Landscape of Heterotic Orbifold Models

Criterion	$V^{\mathrm{SO}(\mathbf{10}),1}$	$V^{\mathrm{SO}(\mathbf{10}),2}$	$V^{\mathrm{E}_\mathbf{6},1}$	$V^{\mathrm{E}_\mathbf{6},2}$
① Inequivalent models with 2 Wilson lines	$22,000$	$7,800$	680	$1,700$
② SM gauge group $\subset$ SU(5) $\subset$ SO(10) (or $\mathrm{E}_\mathbf{6}$)	3563	1163	27	63
③ 3 net $(\mathbf{3},\mathbf{2})$	1170	492	3	32
④ Non–anomalous $\mathrm{U}(\mathbf{1})_Y \subset \mathrm{SU}(\mathbf{5})$	528	234	3	22
⑤ Spectrum $=$ 3 generations $+$ vector-like	127	90	3	2
⑥ Vector-like exotics decouple	106	85		
⑦ “Heavy” top	55	32		
⑧ Suitable B-L exists	34	5		
⑨ SM singlets w/ B-L $=$even/odd	85	8		
⑩ All $\mathrm{U}(\mathbf{1})$’s except $\mathrm{U}(\mathbf{1})_Y$ broken	42	0		
Decoupling along new set of singlets	15	0		

① Inequivalent Models

Criterion	$V^{\mathrm{SO}(\mathbf{10}),1}$	$V^{\mathrm{SO}(\mathbf{10}),2}$	$V^{\mathrm{E}_\mathbf{6},1}$	$V^{\mathrm{E}_\mathbf{6},2}$
① Inequivalent models with 2 Wilson lines	$22,000$	$7,800$	680	$1,700$
② SM gauge group $\subset$ SU(5) $\subset$ SO(10) (or $\mathrm{E}_\mathbf{6}$)	3563	1163	27	63
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④ Non–anomalous $\mathrm{U}(\mathbf{1})_Y \subset \mathrm{SU}(\mathbf{5})$	528	234	3	22
⑤ Spectrum $=$ 3 generations $+$ vector-like	127	90	3	2
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⑩ All $\mathrm{U}(\mathbf{1})$’s except $\mathrm{U}(\mathbf{1})_Y$ broken	42	0		
Decoupling along new set of singlets	15	0		

① Inequivalent Models

- Grand Unified Group in intermediate step

$$\mathsf{E}_8 \times \mathsf{E}_8{}' \xrightarrow{V_{22}} \mathsf{SO}(10) \times \mathsf{SU}(\mathbf{2})^2 \times \mathsf{U}(\mathbf{1}) \times \mathsf{SO}(\mathbf{14})' \times \mathsf{U}(\mathbf{1})'$$
$$\xrightarrow{A_3} \quad \ldots\ldots\ldots\ldots\ldots$$
$$\xrightarrow{A_2} \quad \ldots\ldots\ldots\ldots\ldots$$

$\rightsquigarrow$ 22,000 models

- Notion of inequivalence must be defined:

  Basically, we call 2 models equivalent, if their spectra coincide. There are some subtleties, e.g. spectrum may be complex conjugated columnwise, and gauge group factors may be permuted.

② Standard Model Gauge Group

Criterion	$V^{\mathsf{SO}(\mathbf{10}),1}$	$V^{\mathsf{SO}(\mathbf{10}),2}$	$V^{\mathsf{E}_{\mathbf{6}},1}$	$V^{\mathsf{E}_{\mathbf{6}},2}$
① Inequivalent models with 2 Wilson lines	$22,000$	$7,800$	680	$1,700$
② SM gauge group $\subset$ SU(5) $\subset$ SO(10) (or $\mathsf{E}_{\mathbf{6}}$)	3563	1163	27	63
③ 3 net $(\mathbf{3},\mathbf{2})$	1170	492	3	32
④ Non–anomalous $\mathsf{U}(\mathbf{1})_Y \subset \mathsf{SU}(\mathbf{5})$	528	234	3	22
⑤ Spectrum $=$ 3 generations $+$ vector-like	127	90	3	2
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⑩ All $\mathsf{U}(\mathbf{1})$’s except $\mathsf{U}(\mathbf{1})_Y$ broken	42	0		
Decoupling along new set of singlets	15	0		

② Standard Model Gauge Group

- 1 shift, all Wilson lines:

$$\mathsf{E}_8 \times \mathsf{E}_8{}' \overset{V_{22}}{\longrightarrow} \mathsf{SO}(10) \times \mathsf{SU}(\mathbf{2})^2 \times \mathsf{U}(\mathbf{1}) \times \mathsf{SO}(\mathbf{14})' \times \mathsf{U}(\mathbf{1})'$$
$$\overset{A_3}{\longrightarrow} \quad \ldots\ldots\ldots\ldots\ldots$$
$$\overset{A_2}{\longrightarrow} \quad \mathsf{SU}(\mathbf{3}) \times \mathsf{SU}(\mathbf{2}) \times \text{anything}$$

$\rightsquigarrow$ 3,563 models

- Note that we require

$$\mathsf{SU}(\mathbf{3}) \times \mathsf{SU}(\mathbf{2}) \subset \mathsf{SU}(\mathbf{5}) \subset \mathsf{SO}(\mathbf{10}) \subset \mathsf{E}_8$$

③ Spectrum is 3-2 Standard Model

Criterion	$V^{\mathsf{SO}(\mathbf{10}),1}$	$V^{\mathsf{SO}(\mathbf{10}),2}$	$V^{\mathsf{E}_{\mathbf{6}},1}$	$V^{\mathsf{E}_{\mathbf{6}},2}$
① Inequivalent models with 2 Wilson lines	$22,000$	$7,800$	680	$1,700$
② SM gauge group ⊂ SU(5) ⊂ SO(10) (or $\mathsf{E}_{\mathbf{6}}$)	3563	1163	27	63
③ 3 net $(\mathbf{3},\mathbf{2})$	1170	492	3	32
④ Non–anomalous $\mathsf{U}(\mathbf{1})_Y \subset \mathsf{SU}(\mathbf{5})$	528	234	3	22
⑤ Spectrum $=$ 3 generations $+$ vector-like	127	90	3	2
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⑩ All $\mathsf{U}(\mathbf{1})$'s except $\mathsf{U}(\mathbf{1})_Y$ broken	42	0		
Decoupling along new set of singlets	15	0		

③ Spectrum is 3-2 Standard Model

$$\text{SU}(\mathbf{3})\times\text{SU}(\mathbf{2})\times\text{U}(\mathbf{1})^5\times\text{SU}(\mathbf{5})'\times\text{U}(\mathbf{1})'^4$$

$3\times(\mathbf{3},\mathbf{2},\mathbf{1})$	$12\times(\overline{\mathbf{3}},\mathbf{1},\mathbf{1})$	$29\times(\mathbf{1},\mathbf{2},\mathbf{1})$	$8\times(\mathbf{1},\mathbf{1},\mathbf{5})$
	$6\times(\mathbf{3},\mathbf{1},\mathbf{1})$	$136\times(\mathbf{1},\mathbf{1},\mathbf{1})$	$8\times(\mathbf{1},\mathbf{1},\overline{\mathbf{5}})$

- Spectrum "factorizes" into SM and hidden sector:
  Particles that transform under SM gauge group do not transform under hidden gauge group.

- Pair up exotic particles:

$3\times(\mathbf{3},\mathbf{2},\mathbf{1})$	$6\times(\overline{\mathbf{3}},\mathbf{1},\mathbf{1})$	$3\times(\mathbf{1},\mathbf{2},\mathbf{1})$	$8\times(\mathbf{1},\mathbf{1},\mathbf{5})$
	$6\times(\overline{\mathbf{3}},\mathbf{1},\mathbf{1})$	$1\times(\mathbf{1},\mathbf{2},\mathbf{1})$	$8\times(\mathbf{1},\mathbf{1},\overline{\mathbf{5}})$
	$6\times(\mathbf{3},\mathbf{1},\mathbf{1})$	$1\times(\mathbf{1},\mathbf{2},\mathbf{1})$	
		$24\times(\mathbf{1},\mathbf{2},\mathbf{1})$	
		$136\times(\mathbf{1},\mathbf{1},\mathbf{1})$	

④ Construct Hypercharge from SU(5)

Criterion	$V^{\mathsf{SO}(\mathbf{10}),1}$	$V^{\mathsf{SO}(\mathbf{10}),2}$	$V^{\mathsf{E}_\mathbf{6},1}$	$V^{\mathsf{E}_\mathbf{6},2}$
① Inequivalent models with 2 Wilson lines	$22,000$	$7,800$	680	$1,700$
② SM gauge group $\subset$ SU(5) $\subset$ SO(10) (or $\mathsf{E}_\mathbf{6}$)	3563	1163	27	63
③ 3 net $(\mathbf{3},\mathbf{2})$	1170	492	3	32
④ Non–anomalous $\mathsf{U}(\mathbf{1})_Y \subset \mathsf{SU}(\mathbf{5})$	528	234	3	22
⑤ Spectrum $=$ 3 generations $+$ vector-like	127	90	3	2
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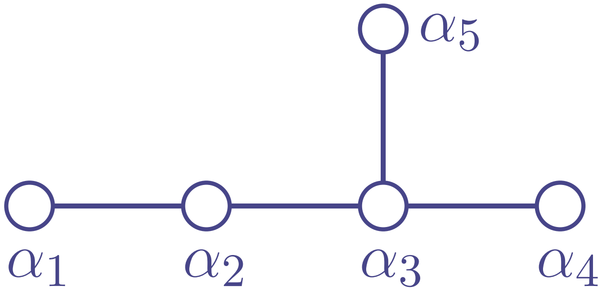
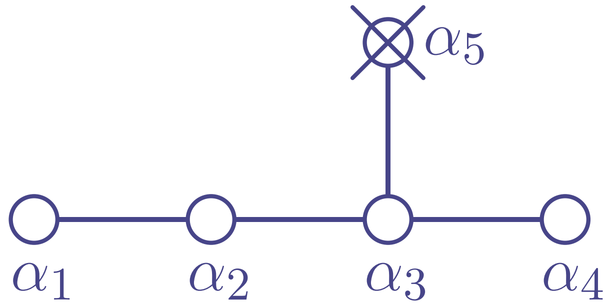
⑩ All $\mathsf{U}(\mathbf{1})$'s except $\mathsf{U}(\mathbf{1})_Y$ broken	42	0		
Decoupling along new set of singlets	15	0		

④ Construct Hypercharge from SU(5)



SO(10)

④ Construct Hypercharge from SU(5)



$$\mathsf{SO}(\mathbf{10}) \quad \rightarrow \quad \mathsf{SU}(\mathbf{5}) \times \mathsf{U}(\mathbf{1})_X$$

$$\mathsf{U}(\mathbf{1})_X \;=\; 4\,\alpha_5^* \;=\; 4\sum_{j=1}^{5}(A^{-1}_{\mathsf{SO}(10)})_{5j}\,\alpha_j \;=\; (2\alpha_1+4\alpha_2+6\alpha_3+3\alpha_4+5\alpha_5)$$

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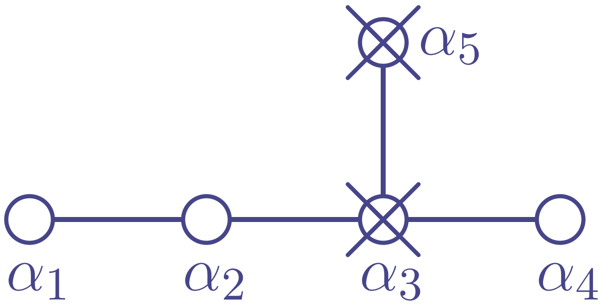
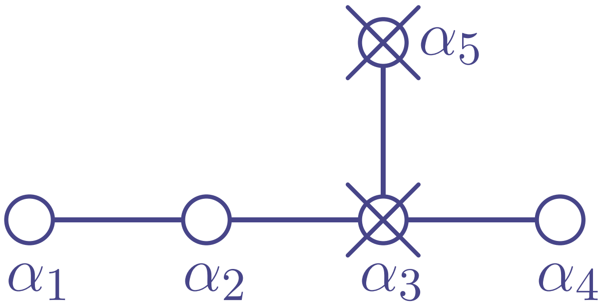


$$\mathsf{SO}(\mathbf{10}) \quad \rightarrow \quad \mathsf{SU}(\mathbf{5}) \times \mathsf{U}(\mathbf{1})_X \quad \rightarrow \quad \mathsf{SU}(\mathbf{3})_c \times \mathsf{SU}(\mathbf{2})_L \times \mathsf{U}(\mathbf{1})_Y \times \mathsf{U}(\mathbf{1})_X$$

$$\mathsf{U}(\mathbf{1})_X = 4\,\alpha_5^* = 4\sum_{j=1}^{5} (A^{-1}_{\mathsf{SO}(10)})_{5j}\,\alpha_j = (2\alpha_1 + 4\alpha_2 + 6\alpha_3 + 3\alpha_4 + 5\alpha_5)$$

$$\mathsf{U}(\mathbf{1})_Y = \tfrac{5}{3}\,\alpha_3^* = \tfrac{5}{3}\sum_{j=1}^{4} (A^{-1}_{\mathsf{SU}(5)})_{3j}\,\alpha_j = \tfrac{1}{3}(2\alpha_1 + 4\alpha_2 + 6\alpha_3 + 3\alpha_4)$$

④ Construct Hypercharge from SU(5)



$$\mathsf{SO}(\mathbf{10}) \quad \to \quad \mathsf{SU}(\mathbf{5})\times\mathsf{U}(\mathbf{1})_X \quad \to \quad \mathsf{SU}(\mathbf{3})_c\times\mathsf{SU}(\mathbf{2})_L\times\mathsf{U}(\mathbf{1})_Y\times\mathsf{U}(\mathbf{1})_X$$

$$\mathsf{U}(\mathbf{1})_X = 4\,\alpha_5^* = 4\sum_{j=1}^{5}(A^{-1}_{\mathsf{SO}(10)})_{5j}\,\alpha_j = (2\alpha_1+4\alpha_2+6\alpha_3+3\alpha_4+5\alpha_5)$$

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$$\mathsf{U}(\mathbf{1})_Y \perp \mathsf{U}(\mathbf{1})_A \;?$$

⑤ Spectrum is 3-2-1 Standard Model

Criterion	$V^{\mathsf{SO}(\mathbf{10}),1}$	$V^{\mathsf{SO}(\mathbf{10}),2}$	$V^{\mathsf{E}_\mathbf{6},1}$	$V^{\mathsf{E}_\mathbf{6},2}$
① Inequivalent models with 2 Wilson lines	$22,000$	$7,800$	680	$1,700$
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Decoupling along new set of singlets	15	0		

⑤ Spectrum is 3-2-1 Standard Model

$$\mathsf{SU}(\mathbf{3}) \times \mathsf{SU}(\mathbf{2}) \times \mathsf{U}(1)^5 \times \mathsf{SU}(\mathbf{5})' \times \mathsf{U}(1)'^4$$

$3 \times (\mathbf{3}, \mathbf{2}, \mathbf{1})$	$12 \times (\overline{\mathbf{3}}, \mathbf{1}, \mathbf{1})$	$29 \times (\mathbf{1}, \mathbf{2}, \mathbf{1})$	$8 \times (\mathbf{1}, \mathbf{1}, \mathbf{5})$
	$6 \times (\mathbf{3}, \mathbf{1}, \mathbf{1})$	$136 \times (\mathbf{1}, \mathbf{1}, \mathbf{1})$	$8 \times (\mathbf{1}, \mathbf{1}, \overline{\mathbf{5}})$

⑤ Spectrum is 3-2-1 Standard Model

SU($\mathbf{3}$) $\times$ SU($\mathbf{2}$) $\times$ something

$3 \times (\mathbf{3}, \mathbf{2})$	$12 \times (\overline{\mathbf{3}}, \mathbf{1})$	$29 \times (\mathbf{1}, \mathbf{2})$
	$6 \times (\mathbf{3}, \mathbf{1})$	$216 \times (\mathbf{1}, \mathbf{1})$

⑤ Spectrum is 3-2-1 Standard Model

$$\mathrm{SU}(\mathbf{3}) \times \mathrm{SU}(\mathbf{2}) \times \text{something}$$

$3 \times (\mathbf{3}, \mathbf{2})$	$12 \times (\overline{\mathbf{3}}, \mathbf{1})$	$29 \times (\mathbf{1}, \mathbf{2})$
	$6 \times (\mathbf{3}, \mathbf{1})$	$216 \times (\mathbf{1}, \mathbf{1})$

- Particles w/hypercharge

$3 \times (\mathbf{3}, \mathbf{2})_{1/3}$	$5 \times (\overline{\mathbf{3}}, \mathbf{1})_{-4/3}$	$7 \times (\overline{\mathbf{3}}, \mathbf{1})_{2/3}$	$16 \times (\mathbf{1}, \mathbf{2})_{-1}$	$45 \times (\mathbf{1}, \mathbf{1})_{2}$	$129 \times (\mathbf{1}, \mathbf{1})_{0}$
	$2 \times (\mathbf{3}, \mathbf{1})_{4/3}$	$4 \times (\mathbf{3}, \mathbf{1})_{-2/3}$	$13 \times (\mathbf{1}, \mathbf{2})_{1}$	$42 \times (\mathbf{1}, \mathbf{1})_{-2}$	

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- Vectorlike?

$3 \times (\mathbf{3},\mathbf{2})_{1/3}$	$3 \times (\overline{\mathbf{3}},\mathbf{1})_{-4/3}$	$3 \times (\overline{\mathbf{3}},\mathbf{1})_{2/3}$	$3 \times (\mathbf{1},\mathbf{2})_{-1}$	$3 \times (\mathbf{1},\mathbf{1})_{2}$	$3 \times (\mathbf{1},\mathbf{1})_{0}$
	$2 \times (\overline{\mathbf{3}},\mathbf{1})_{-4/3}$	$4 \times (\overline{\mathbf{3}},\mathbf{1})_{2/3}$	$1 \times (\mathbf{1},\mathbf{2})_{-1}$	$42 \times (\mathbf{1},\mathbf{1})_{2}$	$126 \times (\mathbf{1},\mathbf{1})_{0}$
	$2 \times (\mathbf{3},\mathbf{1})_{4/3}$	$4 \times (\mathbf{3},\mathbf{1})_{-2/3}$	$1 \times (\mathbf{1},\mathbf{2})_{1}$	$42 \times (\mathbf{1},\mathbf{1})_{-2}$	
			$12 \times (\mathbf{1},\mathbf{2})_{1}$		
			$12 \times (\mathbf{1},\mathbf{2})_{-1}$		

⑥ Exotics Decouple

Criterion	$V^{\mathrm{SO}(\mathbf{10}),1}$	$V^{\mathrm{SO}(\mathbf{10}),2}$	$V^{\mathrm{E}_\mathbf{6},1}$	$V^{\mathrm{E}_\mathbf{6},2}$
① Inequivalent models with 2 Wilson lines	$22,000$	$7,800$	680	$1,700$
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Decoupling along new set of singlets	15	0		

⑥ Exotics Decouple

- Decoupling: Mass term?

$$M\phi\bar{\phi}$$

- Generic situation: Interaction term w/ singlets

$$\phi\,\bar{\phi}\,\tilde{s}_1\tilde{s}_2\ldots\tilde{s}_n$$

  Singlet fields acquire a vev $\rightsquigarrow$ Effective mass term

$$\phi\,\bar{\phi}\,\langle\tilde{s}_1\tilde{s}_2\ldots\tilde{s}_n\rangle$$

- Couplings must be allowed by the string selection rules !
- Allowing for all singlets may break Standard Model symmetries:

  $s_i^+ = (\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(1/2,*)}\,,\quad \langle s_i^+\rangle \neq 0$ breaks $\mathrm{U}(1)_Y$ ✗

- Find subset of singlet fields which decouple exotics and preserve Standard Model symmetries $\rightsquigarrow$ Highly non-trivial, superpotential has 286,781 terms

⑦ Heavy Top

Criterion	$V^{\mathsf{SO}(\mathbf{10}),1}$	$V^{\mathsf{SO}(\mathbf{10}),2}$	$V^{\mathsf{E}_\mathbf{6},1}$	$V^{\mathsf{E}_\mathbf{6},2}$
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⑦ Heavy Top

- Demand existence of Yukawa coupling

$$\bar{u} H_u Q \quad \leftrightarrow \quad (\overline{\mathbf{3}}, \mathbf{1})_{\text{-}4/3} (\mathbf{1}, \mathbf{2})_1 (\mathbf{3}, \mathbf{2})_{1/3}$$

at trilinear level, i.e. w/o singlets

- Singlets $\rightarrow$ interactions suppressed

$$\bar{u} H_u Q \tilde{s}_1 \ldots \tilde{s}_n \quad \leftrightarrow \quad \bar{u} H_u Q \frac{\langle \tilde{s}_1 \ldots \tilde{s}_n \rangle}{M^n}$$

⑧ B-L exists

Criterion	$V^{\mathsf{SO}(\mathbf{10}),1}$	$V^{\mathsf{SO}(\mathbf{10}),2}$	$V^{\mathsf{E}_{\mathbf{6}},1}$	$V^{\mathsf{E}_{\mathbf{6}},2}$
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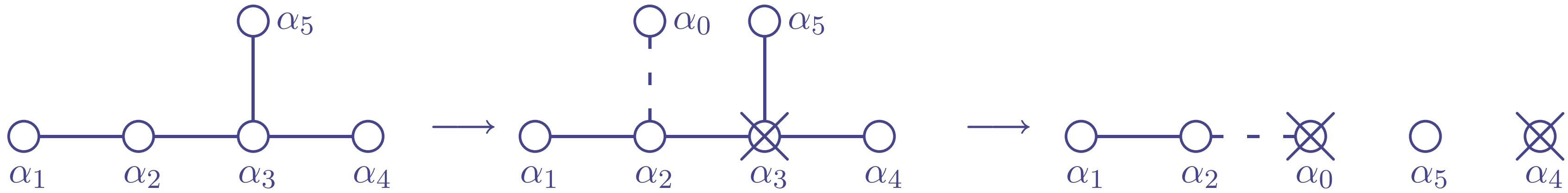

SO(10) — SU$(4)_c \times$ SU$(2)_L \times$ SU$(2)_R$ — SU$(3)_c \times$ SU$(2)_L \times$ U$(1)_{\text{B-L}} \times$ U$(1)_R$

- Default candidate for B-L in SO(10):

$$\mathsf{SO}(\mathbf{10}) \supset \mathsf{SU}(\mathbf{4})_c \times \mathsf{SU}(\mathbf{2})_L \times \mathsf{SU}(\mathbf{2})_R \rightarrow \mathsf{SU}(\mathbf{3})_c \times \mathsf{U}(\mathbf{1})_{\text{B-L}} \times \mathsf{SU}(\mathbf{2})_L \times \mathsf{U}(\mathbf{1})_R$$

- Too restrictive, need to construct more general $B - L$ generator:
  $\rightsquigarrow$ S. Raby, A.W., **arXiv:0706.0217**

One Model in Detail

$SU(\mathbf{3})\times SU(\mathbf{2})\times SU(\mathbf{4})'\times SU(\mathbf{2})'\times U(\mathbf{1})^9$

#	irrep	label		#	irrep	label
3	$(\mathbf{3},\mathbf{2};\mathbf{1},\mathbf{1})_{(1/6,1/3)}$	Q_i		3	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})_{(-2/3,-1/3)}$	$\bar{u}_i$
3	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(1,1)}$	$\bar{e}_i$		8	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})_{(0,*)}$	m_i
4	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})_{(1/3,-1/3)}$	$\bar{d}_i$		1	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1})_{(-1/3,1/3)}$	d_i
4	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})_{(-1/2,-1)}$	L_i		1	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})_{(1/2,1)}$	$\bar{L}_i$
1	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})_{(-1/2,0)}$	H_i		1	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})_{(1/2,0)}$	$\bar{H}_i$
6	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})_{(1/3,2/3)}$	$\bar{\delta}_i$		6	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1})_{(-1/3,-2/3)}$	δ_i
14	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(1/2,*)}$	s_i^+		14	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(-1/2,*)}$	s_i^-
16	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(0,1)}$	$\bar{\nu}_i$		13	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(0,-1)}$	ν_i
5	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})_{(0,1)}$	$\bar{\eta}_i$		5	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})_{(0,-1)}$	η_i
10	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})_{(0,0)}$	h_i		2	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{2})_{(0,0)}$	y_i
6	$(\mathbf{1},\mathbf{1};\mathbf{4},\mathbf{1})_{(0,*)}$	f_i		6	$(\mathbf{1},\mathbf{1};\overline{\mathbf{4}},\mathbf{1})_{(0,*)}$	$\bar{f}_i$
2	$(\mathbf{1},\mathbf{1};\mathbf{4},\mathbf{1})_{(-1/2,-1)}$	f_i^-		2	$(\mathbf{1},\mathbf{1};\overline{\mathbf{4}},\mathbf{1})_{(1/2,1)}$	$\bar{f}_i^+$
4	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(0,\pm 2)}$	χ_i		32	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(0,0)}$	s_i^0
2	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})_{(-1/6,2/3)}$	$\bar{v}_i$		2	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1})_{(1/6,-2/3)}$	v_i

Attention: Hypercharge is half the conventional value!!!

⑨ Break B-L to R-Parity

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① Inequivalent models with 2 Wilson lines	$22,000$	$7,800$	680	$1,700$
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Decoupling along new set of singlets	15	0		

⑨ Break B-L to R-Parity

- As before, giving vevs to arbitrary fields may break desirable symmetries. . .

$$\langle \bar{v}_i \rangle = \langle (\overline{\mathbf{3}}, \mathbf{1}; \mathbf{1}, \mathbf{1})_{\text{-}1/6,\ 2/3} \rangle \neq 0 \quad \rightsquigarrow \quad \not{\text{SU}(3)}_c \quad \not{\text{U}(1)}_Y \quad \not{\text{U}(1)}_{\text{B-L}}$$

- Take "special" fields:

$$\langle \chi_i \rangle = \langle (\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{1})_{0,\ \pm 2} \rangle \neq 0 \quad \rightsquigarrow \quad \text{U}(1)_{\text{B-L}} \to \mathbb{Z}_2$$

- R-parity $\rightsquigarrow$ Proton stability, etc.

- Note: This restricts the choice of singlet fields we may use to decouple exotics !!!

⑩ Break all spurious U(1)'s

Criterion	$V^{\mathsf{SO}(\mathbf{10}),1}$	$V^{\mathsf{SO}(\mathbf{10}),2}$	$V^{\mathsf{E}_\mathbf{6},1}$	$V^{\mathsf{E}_\mathbf{6},2}$
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⑩ Break all spurious U(1)'s

- At low energies, we only observe $\mathsf{U}(1)_Y$
- More than one $\mathsf{U}(1)$ $\rightsquigarrow$ Many Z' bosons
- Break these symmetries as before

$$\langle(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{0,\ \pm 2,\ *,\ *,\dots *}\rangle \neq 0 \quad \rightsquigarrow \quad \mathsf{U}(1)_Y \quad \cancel{\mathsf{U}(1)}_{\text{B-L}} \xrightarrow{} \mathbb{Z}_2 \quad \cancel{\mathsf{U}(1)} \dots$$

- Note: Again, this restricts the choice of singlet fields we may use to decouple exotics !!!

Decoupling Revisited

Criterion	$V^{\mathsf{SO}(\mathbf{10}),1}$	$V^{\mathsf{SO}(\mathbf{10}),2}$	$V^{\mathsf{E}_{\mathbf{6}},1}$	$V^{\mathsf{E}_{\mathbf{6}},2}$
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Decoupling Revisited

- Choice of vacuum:

$$\{\widetilde{s}_i\} = \{\chi_1, \chi_2, \chi_3, \chi_4, h_1, h_2, h_3, h_4, h_5, h_6, h_9, h_{10}, s^0_1, s^0_4, s^0_5, s^0_6, s^0_9, s^0_{11}, s^0_{13}, s^0_{15}, s^0_{16}, s^0_{17}, s^0_{18}, s^0_{20}, s^0_{21}, s^0_{22}, s^0_{23}, s^0_{25}, s^0_{26}, s^0_{27}, s^0_{30}, s^0_{31}\}$$

- Terms of arbitrary order appear in superpotential, one example is:

$$\phi\,\bar{\phi}\,\tilde{s}_1\tilde{s}_2\ldots\tilde{s}_n$$

Consider terms up to order 6 in singlets, i.e. $n = 6$: 286,781 terms

- Check that exotics decouple ✔

- Mass matrices: Collect terms quadratic in field $\times$ singlets

$$Y_u = \begin{pmatrix} \widetilde{s}^5 & \widetilde{s}^5 & \widetilde{s}^5 \\ \widetilde{s}^5 & \widetilde{s}^5 & \widetilde{s}^6 \\ \widetilde{s}^6 & \widetilde{s}^6 & 1 \end{pmatrix}, \quad Y_d = \begin{pmatrix} 0 & \widetilde{s}^5 & 0 \\ \widetilde{s}^5 & 0 & 0 \\ 0 & \widetilde{s}^6 & 0 \end{pmatrix}, \quad Y_e = \begin{pmatrix} 0 & \widetilde{s}^5 & \widetilde{s}^6 \\ \widetilde{s}^5 & 0 & 0 \\ \widetilde{s}^6 & \widetilde{s}^6 & 0 \end{pmatrix}.$$

F- and D-Flatness

- Theory supersymmetric $\leftrightarrow$ Minimum of potential is 0

$$V = \sum F_i^* F_i + \sum D^a D^a$$

$\leadsto$ Unbroken supersymmetry is tantamount to $F_i = D^a = 0$

- D-terms for U(1)'s

$$D^a = \sum \phi_i^* T^a \phi_i = \sum q_i |\phi_i|^2$$

- F-terms

$$F_{\phi_i} = \frac{\partial W}{\partial \phi_i}\Big|_{\phi_i = \langle \phi_i \rangle}$$

Under certain conditions, $F = 0$ implies $D = 0$

- Superpotential in singlets has 124 terms, there are $32 \times \tilde{s}$ fields

$\leadsto$ 32 polynomial equations up to order 5

Works ✔

Conclusions

- Standard Model leaves many questions unanswered
- SUSY GUTs promising candidates for physics beyond the Standard Model
- Extra dimensions may solve many problems from which ordinary GUTs suffer
- String theory is natural candidate for UV completion of 5- and 6 dimensional models, and includes quantized version of gravity
- Models presented come closer to MSSM than any other string construction so far (compare to D-brane models, Calabi-Yaus, Gepner-type models, . . .)
- Still a far way to go to reproduce other key features of MSSM