A Composite Little Higgs

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Introduction

What is the dynamics that drives electroweak symmetry breaking? Roughly speaking, there are two posibillities.

- **electroweak symmetry is broken by a light Higgs**
- **electroweak symmetry is broken by strong dynamics, in analogy to QCD. There is no light Higgs.**

Precision electroweak data favors a light Higgs. What then protects the Higgs mass parameter against quadratic divergences from higher scales?

One possibility is that the Higgs is a composite pseudo-Goldstone boson, similar to the pion in QCD.

However, any model of the Higgs as a composite pseudo-Goldstone faces an immediate obstacle.

The compositeness scale is constrained to be greater than about 5 TeV. This is because the scale Λ appearing in operators such as those below, which we expect to arise from integrating out new physics at the compositeness scale, is constrained to be greater than about 5 TeV.

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$$\frac{\bar{D}^2 H^\dagger \; D^2 H}{\Lambda^2} \qquad\qquad \frac{|H^\dagger D_\mu H|^2}{\Lambda^2}$$

The problem is that radiative corrections from scales of order 5 TeV naturally generate a Higgs mass much larger than 200 GeV , the precision electroweak upper bound.

In order to understand this let us consider the situation in QCD with just two light flavors, the up and down.

In the limit that the quark masses are zero, the theory has an approximate $SU(2)_L$ X $SU(2)_R$ global symmetry, which is broken down to the diagonal SU(2). The global symmetry is explicitly broken by U(1) electromagnetism, so that only a U(1) X U(1) subgroup of the SU(2) X SU(2) symmetry is exact.

Then, in this limit the neutral pion is a massless Nambu-Goldstone boson, while the charged pions acquire a mass at one loop.

$$m_\pi^2 \sim \frac{e^2}{16\pi^2}\Lambda_{\mathrm{QCD}}^2$$

If we were to construct a similar model for the Higgs, the analogous formula for the mass of the Higgs would be

$$m_{\mathsf{H}}^2 \sim 3\frac{\lambda_t^2}{8\pi^2}\Lambda^2$$

where Λ is the compositeness scale and λ_t is the top Yukawa coupling. For Λ of order 5 TeV, this results in a Higgs mass of order 1 TeV, well above the precision electroweak upper bound of 200 GeV. Such a theory is ruled out unless there are accidental cancellations at the 1% percent level.

We see from this analysis that theories with a composite pseudo-Goldstone Higgs are disfavored unless a mechanism can be found which suppresses these contributions to the Higgs mass.

This is where the little Higgs mechanism comes in. To illustrate the basic idea, consider again QCD with two light flavors, but with a somewhat different assignment of the electric charges of the quarks.

We assign the up-quarks a charge of +(½), and the down quarks a charge of –(½) . Furthermore we gauge U(1) separately on the left- and right-handed quarks, so that the gauge symmetry is now $U(1)_L$ x $U(1)_R$. This charge assignment is anomaly free.

Under $U(1)_L$, the charge of u_L is (½) , the charge of d_L is - (½), and the charges of u_R and d_R are both 0.

Under $U(1)_R$, the charge of u_R is (½) , the charge of d_R is - (½) , and the charges of u_L and d_L are both zero.

When the $SU(2)_L$ X $SU(2)_R$ global symmetry is broken down to the diagonal SU(2), the $U(1)_L$ X $U(1)_R$ gauge symmetry is broken down to the diagonal U(1) `electromagnetism'. One of the Goldstone bosons, corresponding to the neutral pion, is eaten. The two charged pions survive in the low-energy theory as pseudo-Goldstone bosons, just as in QCD.

What are the masses of the charged pions? A detailed calculation shows that the result is given by

$$m_\pi^2 \sim \frac{e_{\mathrm{L}}^2 e_{\mathrm{R}}^2}{(16\pi^2)^2} \Lambda_{\mathrm{QCD}}^2$$

This is a loop factor smaller than in real QCD.

What is the origin of the suppression? In the limit that $e_L = 0$, the $SU(2)_L$ symmetry is exact. After symmetry breaking there would be 3 massless Nambu-Goldstones, one of which, the neutral pion, is eaten. The charged pions are exactly massless in this limit.

Similarly, in the limit that $e_R = 0$, the $SU(2)_R$ symmetry is exact, and the charged pions remain massless after symmetry breaking.

We see from this that the charged pions are massless if either $e_L = 0$ OR $e_R = 0$. This implies that the charged pions can only acquire masses from diagrams that involve BOTH e_L AND e_R. Any diagram that contributes to the Higgs mass must have loops involving both left and right gauge bosons and is suppressed by at least two loop factors. The underlying concept is `collective symmetry breaking' → `little Higgs mechanism'.

We can obtain a diagrammatic understanding of this result. The contribution to the mass of the charged pion from electromagnetism is cancelled by diagrams involving the heavy U(1)' gauge boson.

The next step is to apply these ideas to obtain a light Higgs as a composite pseudo-Goldstone. However, most little Higgs theories have been constructed only as non-linear sigma models, and it is not clear that the corresponding symmetry breaking patterns can be realized through strong dynamics. In this talk I show how the `Simplest Little Higgs' can be realized as a composite pseudo-Goldsone.

The Simplest Little Higgs

Consider a model where the SU(2) X U(1) gauge symmetry of the Standard Model is enlarged to a global SU(3) X U(1) symmetry, with its SU(2) X U(1) subgroup gauged. The Standard Model Higgs is the pseudo-Goldstone boson associated with the breaking of the global symmetry.

We gauge SU(2) X U(1) rather than the full SU(3) X U(1) because otherwise all the Goldstones will be eaten.

We parametrize the Goldstone bosons associated with the breaking of the global symmetry as

$$\phi = e^{i\pi/f} \begin{pmatrix} \\ f \end{pmatrix}$$

where

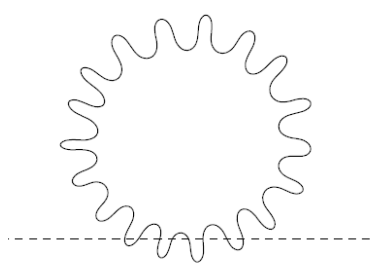
$$\pi = \left(\begin{array}{cc|c} -\eta/2 & 0 & h \\ 0 & -\eta/2 & \\ \hline \multicolumn{2}{c|}{h^\dagger} & \eta \end{array} \right)$$

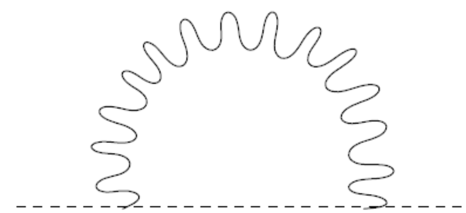
The field *h* is a doublet under the unbroken SU(2) and is to be identified with the Standard Model Higgs. The low energy effective theory for the Goldstone bosons will consist of all operators involving Φ consistent with the non-linearly realized SU(3) X U(1) symmetry.

The interactions of the pseudo-Goldstones arise from the gauge covariant kinetic terms.

$$|D_\mu \phi\ |^2 \longrightarrow |g \begin{pmatrix} W_\mu & \\ & 0 \end{pmatrix} \phi\,|^2$$

Note that the gauge interactions do not have an SU(3) X U(1) invariant form because we gauged only the SU(2) X U(1) subgroup. As a consequence diagrams such as those below give *h* a quadratically divergent mass.





This radiatively generated mass is then of order

$$\delta m^2 = \frac{9g^2}{64\pi^2}\Lambda^2$$ **(UNSATISFACTORY)**

We see that the Higgs mass in this model scales like the pion mass in QCD.

One way to get around this problem is to break SU(3) X U(1) to SU(2) X U(1) twice. The pattern of symmetry breaking is then $[SU(3) \times U(1)/SU(2) \times U(1)]^2$. The vector SU(3) X U(1) subgroup of the $[SU(3) \times U(1)]^2$ global symmetry is gauged. The Standard Model Higgs emerges as the pseudo-Goldstone associated with the breaking of the approximate global symmetry and is free of quadratic divergences.

(Schmaltz)

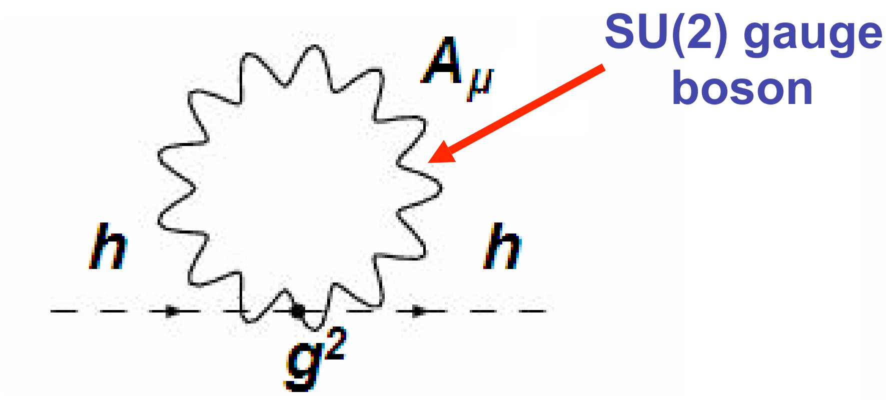
There are now two sets of pseudo-Goldstone bosons

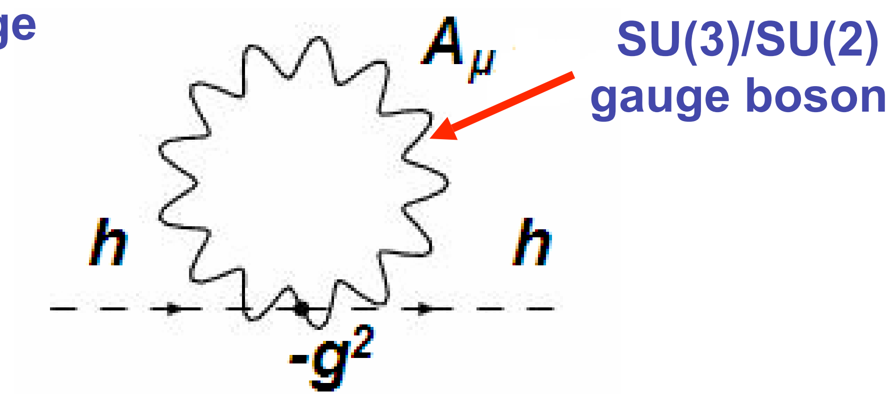
$$\phi_1 = e^{i\pi_1/f} \begin{pmatrix} \\ f \end{pmatrix} \qquad \phi_2 = e^{i\pi_2/f} \begin{pmatrix} \\ f \end{pmatrix}$$

Interactions of these fields again arise from the covariant kinetic terms

$$|D_\mu \phi_1|^2 + |D_\mu \phi_2|^2$$

where the full SU(3) X U(1) symmetry has now been gauged.





Explicit calculation shows that quadratic divergences arising from loops involving the SU(2) X U(1) gauge bosons are cancelled by loops involving the heavy [SU(3) x U(1)]/[SU(2) X U(1)] gauge bosons.

The first non-vanishing contribution to the mass of *h* from gauge interactions is of order

$$\delta m^2 \sim \frac{g^4}{16\pi^2} f^2 \sim \frac{g^4}{(16\pi^2)^2} \Lambda^2$$

This is a loop factor smaller than before! For Λ of order 10 TeV the Higgs mass parameter is weak scale size.

What is the origin of this cancellation? Consider the relevant interactions

$$|gA_\mu\phi_1|^2 + |gA_\mu\phi_2|^2$$

Let us denote the gauge coupling constant of Φ_1 by g_1 and Φ_2 by g_2. In the limit that g_1 is zero the model has an exact SU(3) X U(1) global symmetry which is broken to SU(2) X U(1), and there are 5 exactly massless Goldstone bosons. Similarly, in the limit that g_2 is zero the model has a different exact SU(3) X U(1) global symmetry, and therefore there are again 5 exactly massless Goldstone bosons.

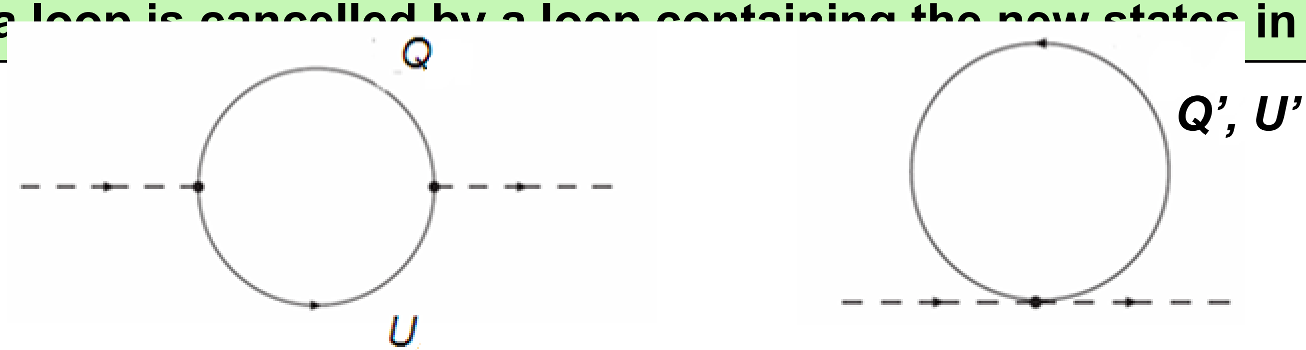
This implies that for the Goldstone bosons to acquire a mass, both g_1 and g_2 must be non-zero. Therefore only diagrams involving both sets of couplings g_1 and g_2 contribute to the masses of the Goldstone bosons. The absence of quadratically divergent contributions to the pseudo-Goldstone mass at one loop is a consequence of the fact that none of the potentially dangerous diagrams involve both g_1 and g_2, but only one of them.

We see again that the symmetry of which the Higgs is a pseudo-Goldstone is explicitly broken, but only COLLECTIVELY. The symmetry is only broken when two or more couplings in the theory are non-zero. The absence of quadratically divergent diagrams that involve both sets of couplings implies that the Higgs mass is protected.

This mechanism can be extended to Yukawa couplings as well. We promote the SM SU(2) doublets to SU(3) triplets or anti-triplets as required. Some additional new fermions are also necessary. The up-type Yukawa couplings take the form

$$y_1 Q \phi_1 U_1 + y_2 Q \phi_2 U_2$$

Here Q, which contains the SU(2) doublet quarks, is now a triplet under SU(3), while the SM SU(2) singlet quarks emerge from a linear combination of U_1 and U_2. The theory has an exact SU(3) X U(1) global symmetry in limit that either y_1 or y_2 is zero. The Higgs mass is again protected by collective breaking. The SM Yukawa loop is cancelled by a loop containing the new states in Q and U.



For a compositeness scale of 5 TeV, the overall fine-tuning in this model is of order 10%. This is an order of magnitude better than our earlier naïve estimate for a composite Higgs. The next step is to construct a theory where this pattern of breaking is realized through strong dynamics.

A Composite Little Higgs

In trying to realize a composite Simplest Little Higgs we are immediately faced with a serious obstacle.

Among the operators allowed by the symmetry breaking pattern are

$$c\ \frac{|\phi^\dagger D_\mu \phi|^2}{\Lambda^2}$$

The coefficient *c* is of order $16\pi^2$. This operator violates the custodial SU(2) symmetry of the Standard Model. Its effect is to alter the ratio of the W and Z masses from the Standard Model prediction → requires $\Lambda > 50$ TeV.

Strongly coupled UV completions of the Simplest Little Higgs seem to be strongly disfavored!

However, there is a way out. Consider a linear realization of the symmetry breaking pattern.

$$\left(|D_\mu\phi_1|^2 + m^2|\phi_1|^2 - \lambda|\phi_1|^4\right) + (\phi_1 \to \phi_2)$$

Here Φ_1 and Φ_2 are scalar fields which transform as fundamentals under the SU(3) X U(1) gauge symmetry.

The crucial observation is that in the limit that the gauge interactions are turned off, the Lagrangian for Φ_1 is invariant under an accidental O(6) global symmetry which is broken to O(5). The 5 Goldstones that arise when Φ_1 gets a VEV can just as well be thought of as arising from this pattern as from SU(3) x U(1) → SU(2) x U(1).

The same is true for Φ_2.

Once we gauge SU(3) x U(1), 5 of the 10 Goldstone bosons are eaten, just as before.

Consider now a model where the breaking of O(6) to O(5) is realized through strong dynamics. Since this pattern preserves a custodial SU(2) symmetry, the unwanted operators are forbidden in the low energy effective theory even though the number of Goldstone bosons and their gauge quantum numbers are exactly the same as in the original Simplest Little Higgs model!

In this scenario, the bad operators can only be generated through loops involving interactions that break the custodial symmetry, specifically the gauge and Yukawa interactions. Then the coefficient *c* is of order one, rather than 4π, and so the compositeness scale can be as low as 5 TeV.

The first step in UV completing the Simplest Little Higgs is to find a way to break O(6) to O(5) by strong dynamics.

Note that O(6) and SU(4) have the same Lie algebra, as do O(5) and Sp(4). The problem is then to break SU(4) to Sp(4) dynamically.

Consider an SU(2) gauge theory with 4 fields $\psi_{\alpha i}$ in the fundamental representation. Here α is an SU(2) color index while i is a flavor index and takes values 1 through 4.

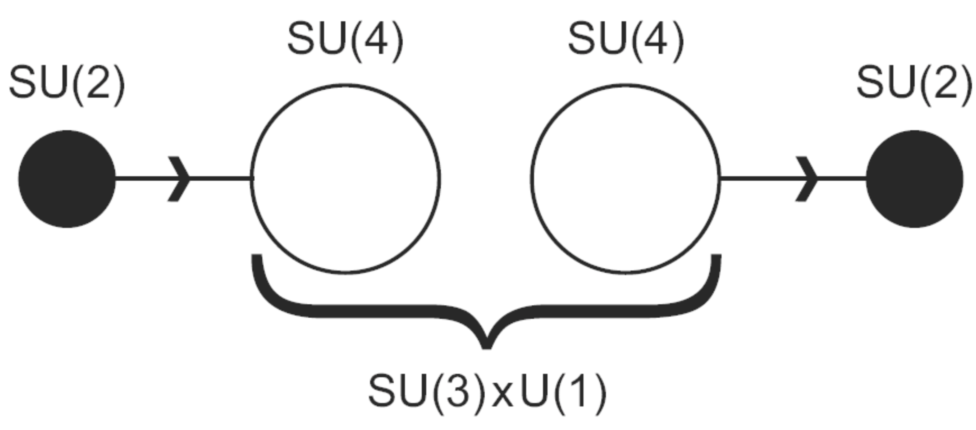
When the SU(2) gauge theory gets strong a condensate $\langle \epsilon_{\alpha\beta}\, \psi_{\alpha i}\psi_{\beta j} \rangle$ α J_{ij} forms along the SU(2) singlet direction. Here J is the matrix

$$J = \begin{pmatrix} i\sigma^2 & 0 \\ 0 & i\sigma^2 \end{pmatrix}$$

The condensate J_{ij} is anti-symmetric in the indices i and j, thereby breaking the SU(4) global symmetry down to Sp(4).

In order to break SU(4) → Sp(4) twice we just begin with two different copies ψ_1 and ψ_2 of the SU(2) gauge theory, each with 4 fields in the fundamental representation. We end up with two separate anti-symmetric condensates, resulting in the desired pattern.

We are free to gauge a vector SU(3) X U(1) subgroup of the SU(4) X SU(4) global symmetry, as shown in figure.



Without loss of generality we take the indices i = 1,2 and 3 to be SU(3) indices, while i =4 is an SU(3) singlet. The SU(3) gauge symmetry will be anomaly free provided the fields in ψ_1 transform in the fundamental representation of SU(3), while the fields in ψ_2 transform instead in the anti-fundamental representation of SU(3).

Further, the U(1) charge assignments are such that the fields in the fundamental of SU(3) in ψ_1 have charge (1/6), while the SU(3) singlet has charge - (½). Similarly, the singlet in ψ_2 has charge (½), while the anti-fundamental has charge - (1/6).

Then after condensation the SU(3) x U(1) gauge symmetry is broken to the SU(2) x U(1) of the Standard Model.

The low-energy effective theory can once again be described by a non-linear sigma model. We parametrize the (pseudo)Goldstone bosons arising from the ψ's as π^i, and define

$$A = f \exp(\frac{i}{f}\pi^a T^a)\ J\ \exp(\frac{i}{f}\pi^a T^a)^T$$

Here the 5 matrices T^a are the generators of SU(4)/Sp(4). There are two sets of A's, A_1 and A_2, corresponding to ψ_1 and ψ_2 respectively. The low energy effective theory consists of all operators involving A_1 and A_2 consistent with the non-linearly realized SU(4) x SU(4) symmetry

$$\{T^a\} = \left\{ \begin{pmatrix} 0 & i\sigma^a \\ -i\sigma^a & 0 \end{pmatrix}, \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \right\}$$

By choosing to keep only the SU(3) x U(1) subgroup of each SU(4) manifest, we can immediately carry over most of the results of the original Simplest Little Higgs to this new construction. To see this explicitly consider the decomposition of *A* under the SU(3) X U(1) subgroup.

$$A = \begin{pmatrix} 0 & \phi_3^\star & -\phi_2^\star & \phi_1 \\ -\phi_3^\star & 0 & \phi_1^\star & \phi_2 \\ \phi_2^\star & -\phi_1^\star & 0 & \phi_3 \\ -\phi_1 & -\phi_2 & -\phi_3 & 0 \end{pmatrix}$$

Here, (in an abuse of notation), Φ_1, Φ_2 and Φ_3 transform as components of an SU(3) triplet Φ. This allows the identification of the π's in *A* with the π's in the SU(3) triplets Φ that arise in the non-linear sigma model of the original Simplest Little Higgs model.

In other words, substituting the formula below from the original Simplest Little Higgs model

$$\phi \;\; = e^{i\pi \;\; /f} \begin{pmatrix} \\ f \end{pmatrix}$$

for Φ in the formula on the previous page will reproduce

$$A = f \exp(\frac{i}{f}\pi^a T^a) \; J \; \exp(\frac{i}{f}\pi^a T^a)^T$$

This means that the two sets of Goldstone bosons transform equivalently under SU(3) X U(1), as required.

Then the low-energy effective theory for the π's consists of all possible operators involving Φ_1 and Φ_2 consistent with the non-linearly realized $[SU(3) \times U(1)]^2$ symmetry, but with additional restrictions and relations among the coefficients of the various terms enforced by the larger $[SU(4)]^2$ symmetry.

In particular, the dangerous operators that violate the custodial SU(2) symmetry at leading order are forbidden.

Any potential for the pseudo-Goldstones can only arise those interactions that violate the global symmetry, in particular the gauge and Yukawa interactions. In the low energy effective theory these can be written down in terms of the fields Φ_1 and Φ_2, exactly as in the original Simplest Little Higgs.

How do the operators that generate Yukawa couplings arise in the low energy theory arise?

Consider the operator

$$\frac{\lambda_1}{4\pi f^2} Q\left(\psi_i \psi_4\right)^\dagger U_1 + \frac{\lambda_2}{4\pi f^2} Q\left(\widehat{\psi}_i \widehat{\psi}_4\right) U_2$$

where, in an abuse of notation, in the formula above

$$\psi \longrightarrow \psi_1 \qquad \widehat{\psi} \longrightarrow \psi_2$$

In the low energy effective theory this operator leads to Yukawa couplings of the required form

$$y_1 Q \phi_1 U_1 + y_2 Q \phi_2 U_2$$

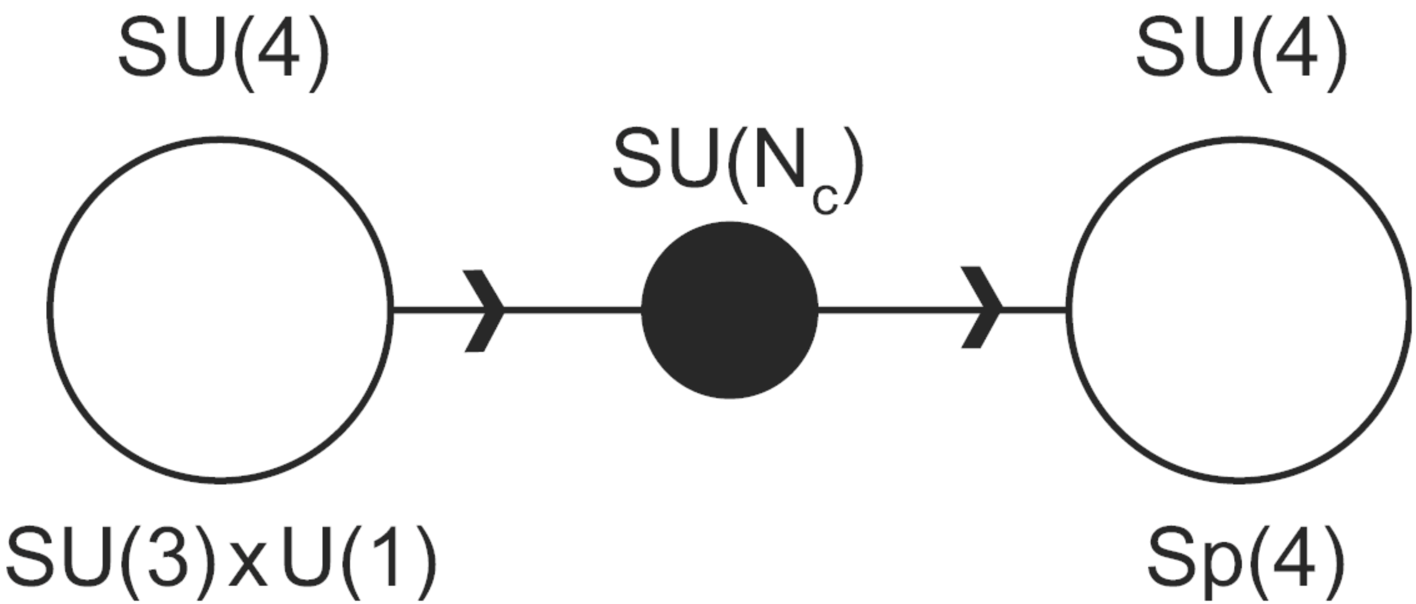
There is another construction which also generates the same symmetry breaking pattern!

The global symmetry breaking pattern $G \rightarrow H$, with a subgroup F of G gauged has exactly the same low-energy dynamics as a two-site non-linear sigma model with global symmetry breaking pattern $G \times G \rightarrow G$ and gauged subgroup $H \times F$, in limit that the gauge coupling constant of H is large.

Thaler

This two-site model can, in principle, be UV completed provided the $G \times G \rightarrow G$ breaking pattern is a simple generalization of QCD-like dynamics.

Since the pattern O(6)/O(5) is equivalent to SU(4)/Sp(4), we can use this technique to realize the symmetry breaking pattern we require. The appropriate construction is below.



To replicate exactly the desired pattern [SU(4)/Sp(4)]^2, we simply repeat the above pattern twice and gauge the same SU(3) x U(1) in each case.

Conclusions

We have shown two different ways of obtaining the symmetry breaking pattern of the Simplest Little Higgs, complete with a custodial SU(2), from strong dynamics.

This is an important first step in the construction of completely realistic composite little Higgs models.