MET Cone and Mass Measurement @ LHC

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Base on the work with Jay Hubisz arxiv: 1009.1148

UC Davis LHC Lunch
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Plan

- Warm-up for Mass Measurement with Missing Energy
- Boosted Decay Chain and Collinearity
- MET-cone Method
- 1D projection of MET-cone: M_{test} Variable
  - Definition, analytic solution and endpoints
  - Numerical results
- Test consistency
- Conclusion

Missing Energy Event

- Missing energy event is not unusual
  -- neutrino in SM

e.g. $W \to e\nu$

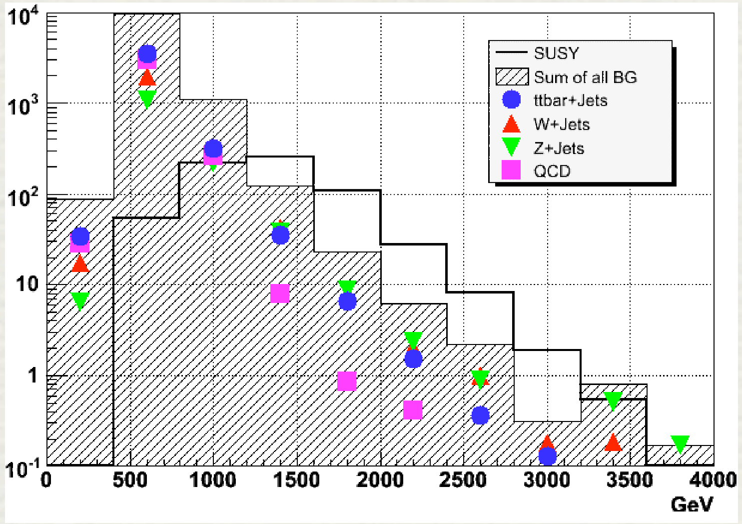
- We are interested in the missing energy from new physics

- Dark matter motivation : exist (meta)stable exotic particle

- New symmetry to protect it from decay

- Z_2 parity --> pair production of stable exotics at LHC

- SUSY, UED ...



Mass Reconstruction is Important

- Crucial for understanding the underlying physics
  - distinguish different physical models

- The dark matter connection:
  - the mass of the missing particle determines the relic density

$$\Omega_\chi h^2 \propto \frac{1}{\langle \sigma v \rangle} \sim \frac{m_\chi^2}{\alpha^2}.$$

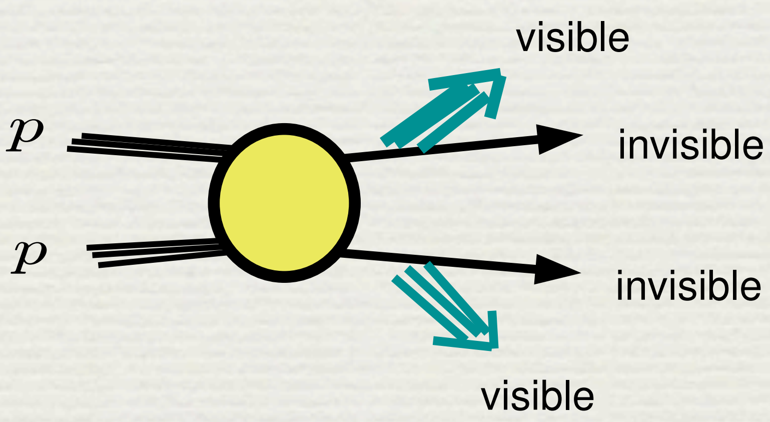
- Comparison with direct detection and indirect detection

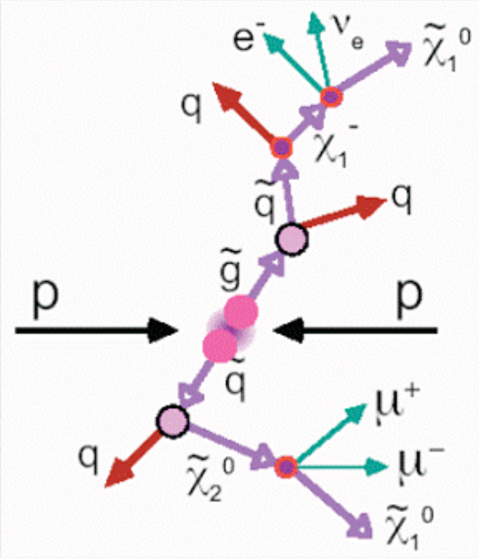
Baltz, Battaglia, Peskin and Wizansky, hep-ph/0602187.

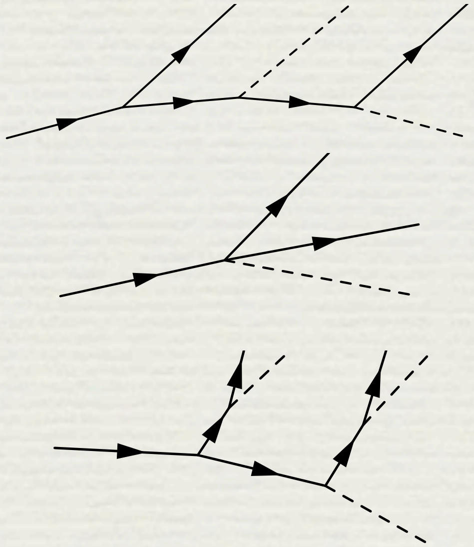
Determine the Dark Matter Mass

Challenging at the LHC

- Two missing particles in each event
- Unknown parton frame leads to less constrained kinematics
- Interpretation of the signal as a particular physics process maybe complicated -- different underlying topologies or a mixture of them







Kinematic Approaches

Demand that at least some particles are sufficiently close to their mass shells that their energy-momentum Lorentz invariant can be used to constrain their masses

Advantage: do not need to know many details of the underlying physical model (gauge group, spin etc)

see a recent review:
Barr and Lester, arXiv:1004.2732

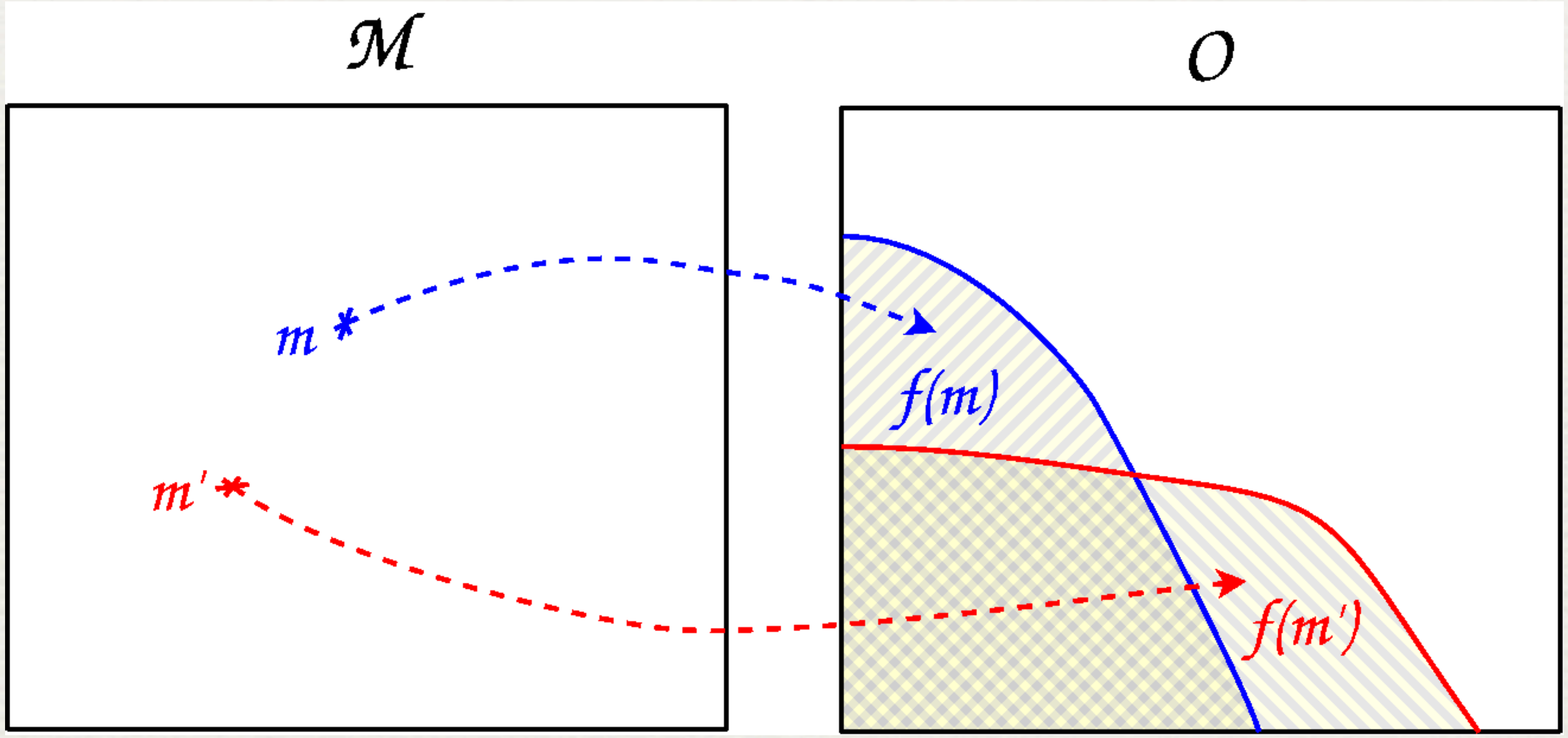
Three main categories:

- ☆ Invariant Mass endpoint — **Bachacou, Hinchliffe and Paige**; **Lester and Summers**
- ☆ MT2 variable and Kink; variations: subsystem MT2, Mct, — **W.S. Cho,K.Choi,Y.G.Kim,C.B.Park**; **K.Kong, K. Matchev, M.Park**
- ☆ Polynomial method/Mass relation method — **Kawagoe, Nojiri and Polesello**; **Cheng, Gunion, Han & McElrath**

......

General Picture

- Mapping from mass space to observable space



Han, cheng
JHEP 0812 (2008) 063

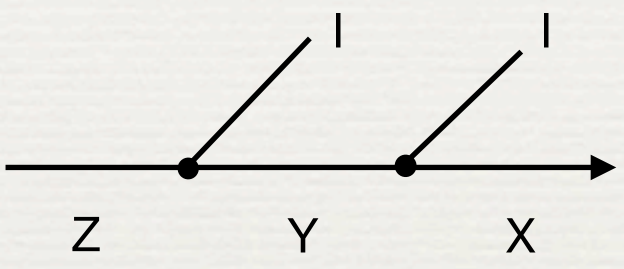
- Consistent regions f(m)

- Boundary of f(m) --> constrain masses

Choose the right observable is important !!

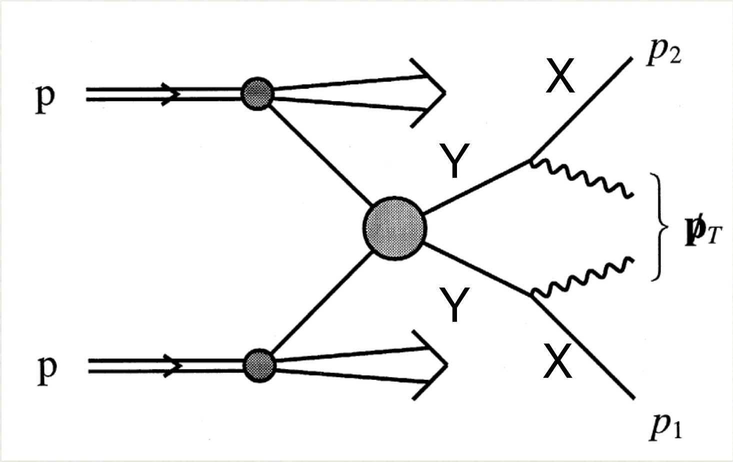
Examples:

- Invariant Mass Endpoint



$$m_{ll} \leq \sqrt{(M_Z^2 - M_Y^2)(M_Y^2 - M_X^2)}/M_Y$$

- MT2 Endpoint

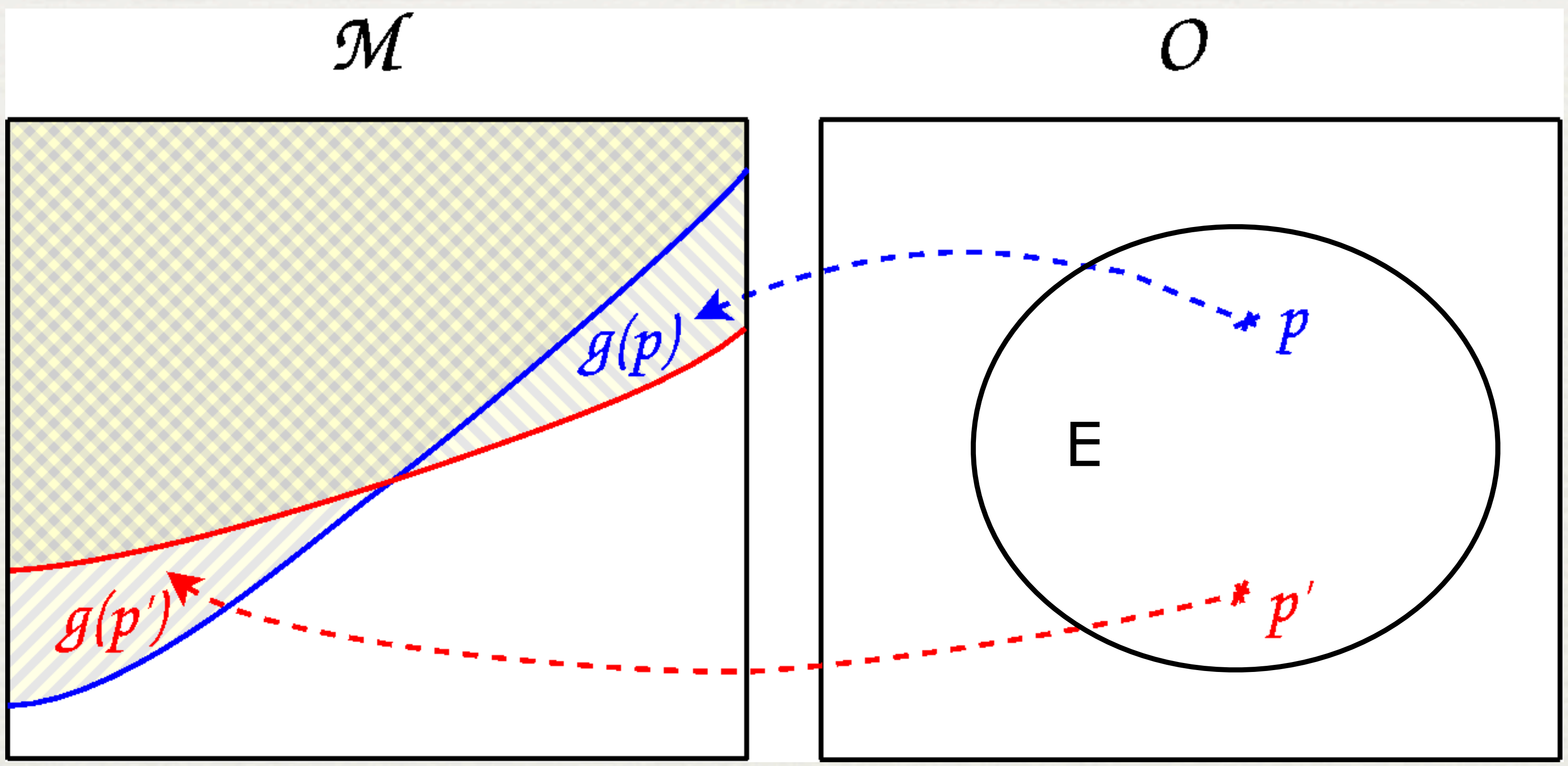


$$m_{T2}^{\max}(M_Y, M_X)$$

- ☆ They alone can’t determine the masses in events with missing energy
- ☆ Combine several such observables, or looking for extra structure such as “kinks”

General Picture

- Alternative view: Inverse map from observable space to mass space



- Consistent mass regions g(p)

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- For an event sample E, the intersection of g(pi) ideally shrink to a point, but not always.

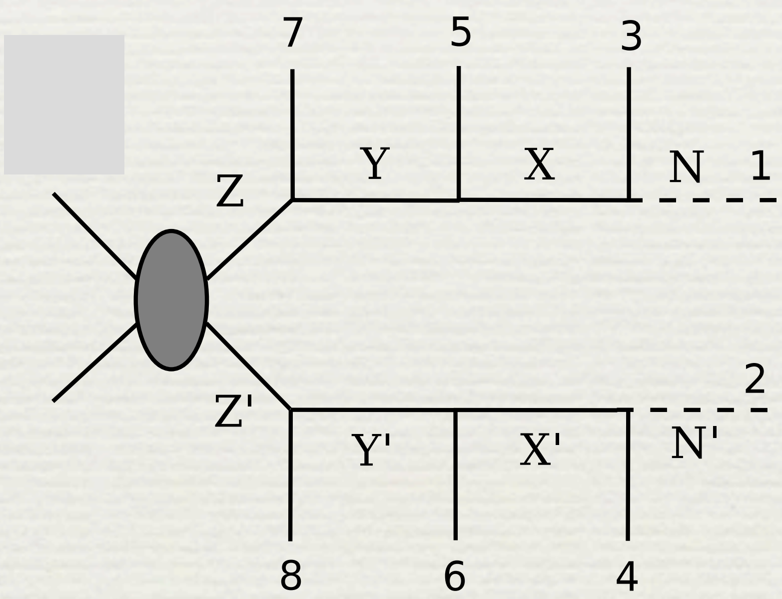
Polynomial method or Mass relation method

Kawagoe, Nojiri & Polesello;
Cheng, Gunion, Han & McElrath

- Using On-shell conditions event-by-event

$$\text{constraints} \geq \text{unknowns}$$

$$10n. \qquad 4+8n$$



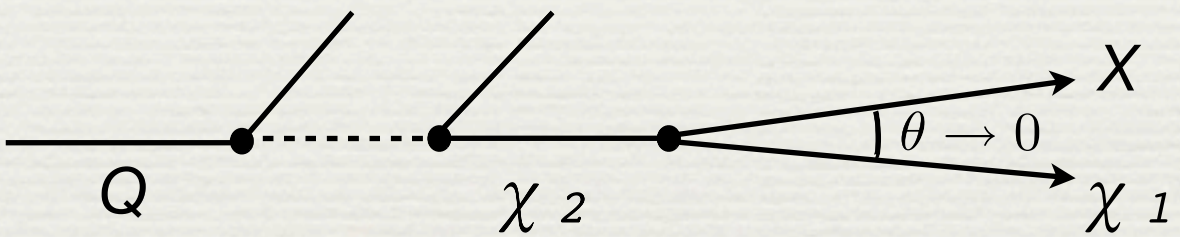
- For n>2, over-constrained system
- Require long decay chains -- at least four on-shell particles in each chain

Having multiple methods is crucial

Any new ideas?

Boosted decay is generic

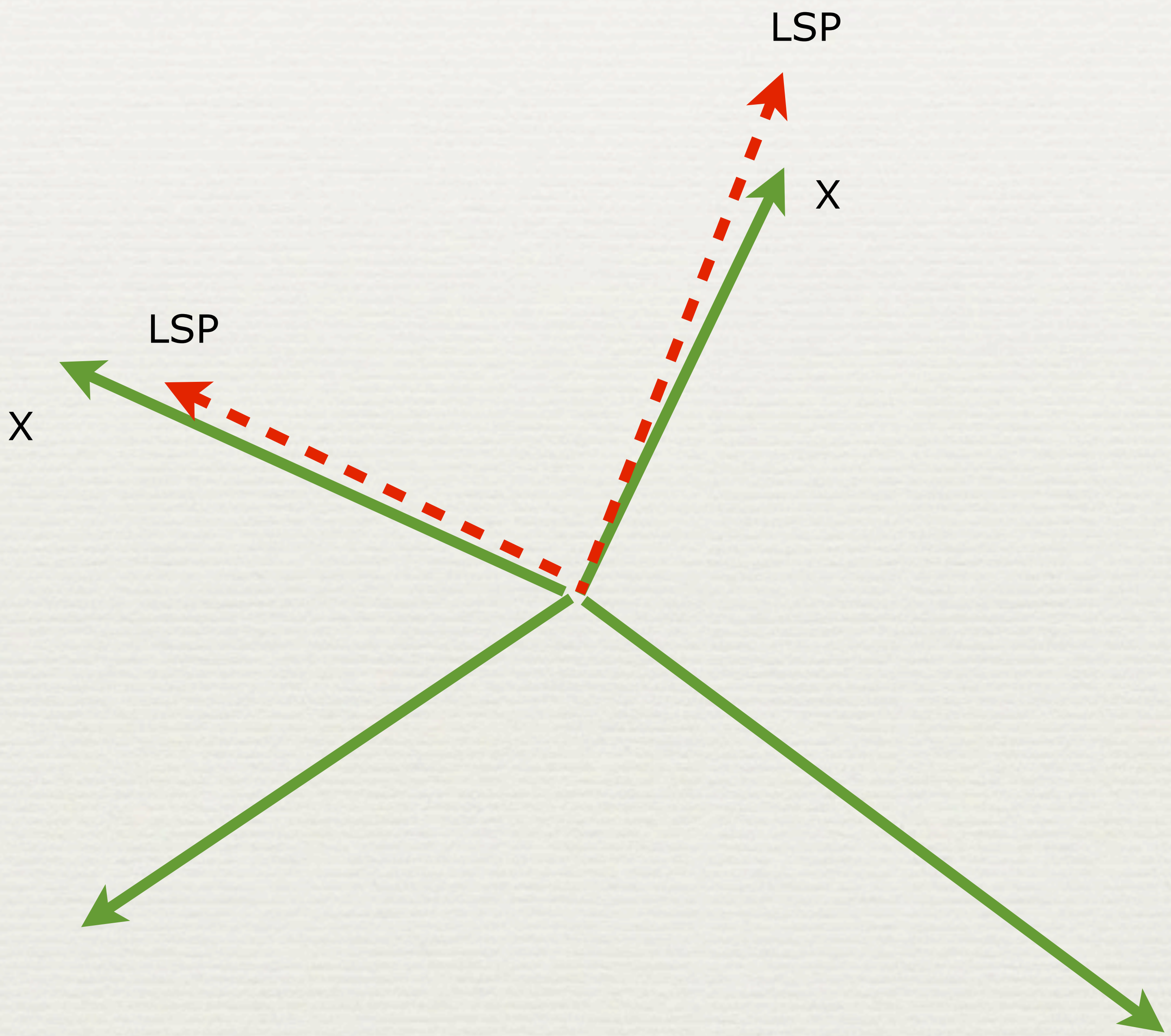
- In many new physics models: there are both heavy(~TeV) exotics as well as light(~100GeV) ones.



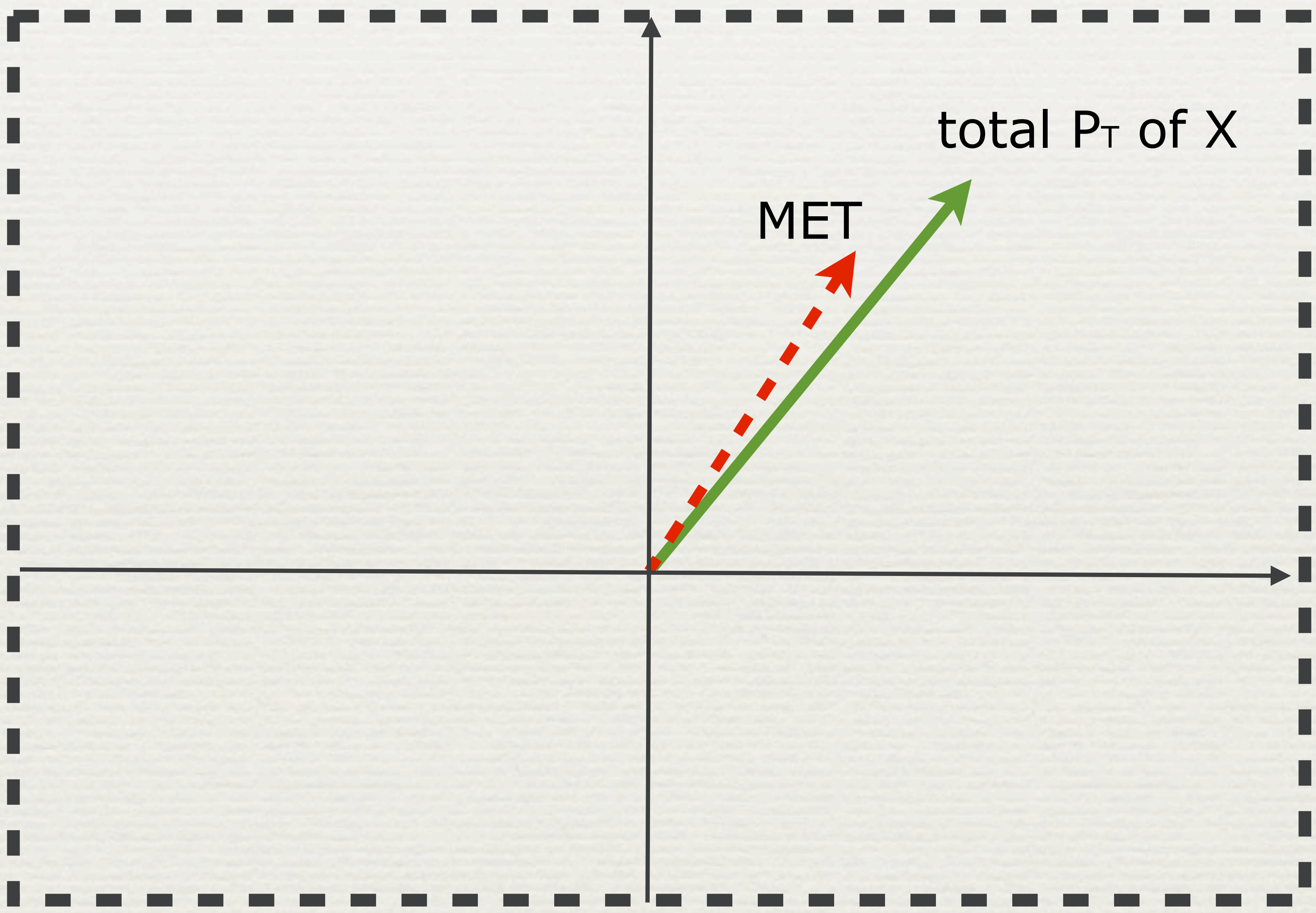
- *SUSY little hierarchy*

- SUSY example: squark --> q + NLSP -- > q + Z + LSP

- Can we get additional handle if missing particle is approximately collinear with visible particles ?

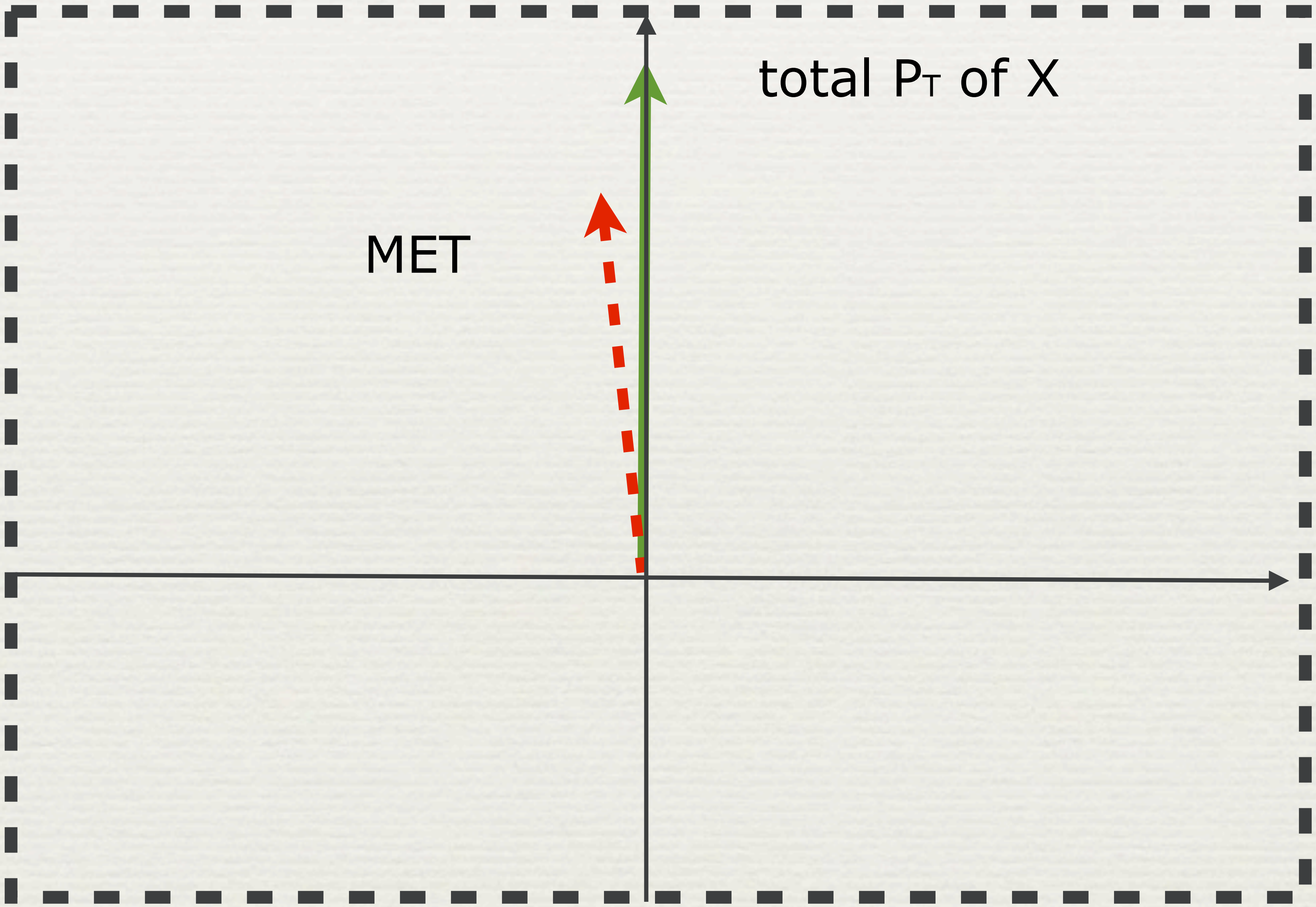

LSP
X
LSP
X

Transverse Plane



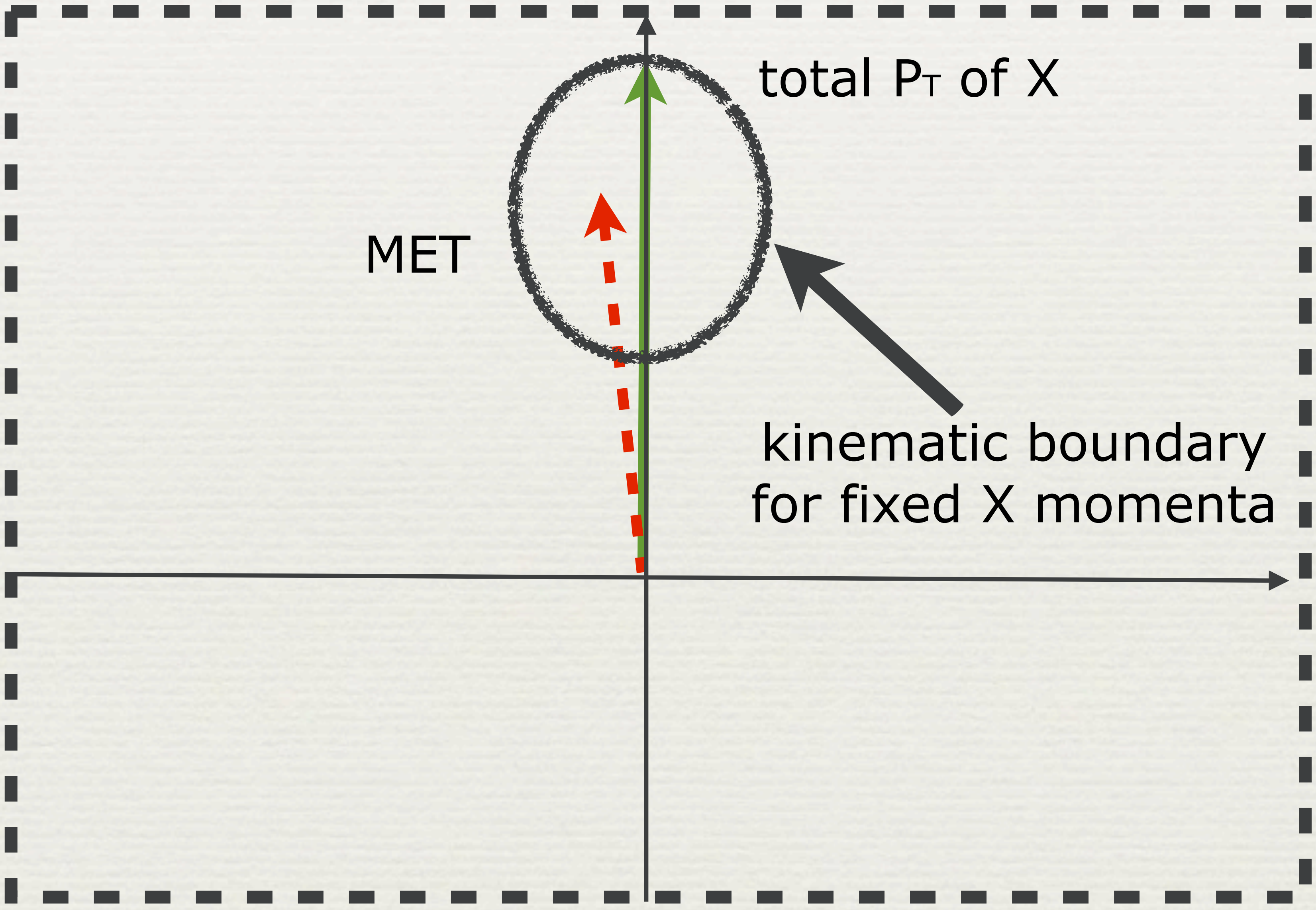
New coordinate

Transverse Plane

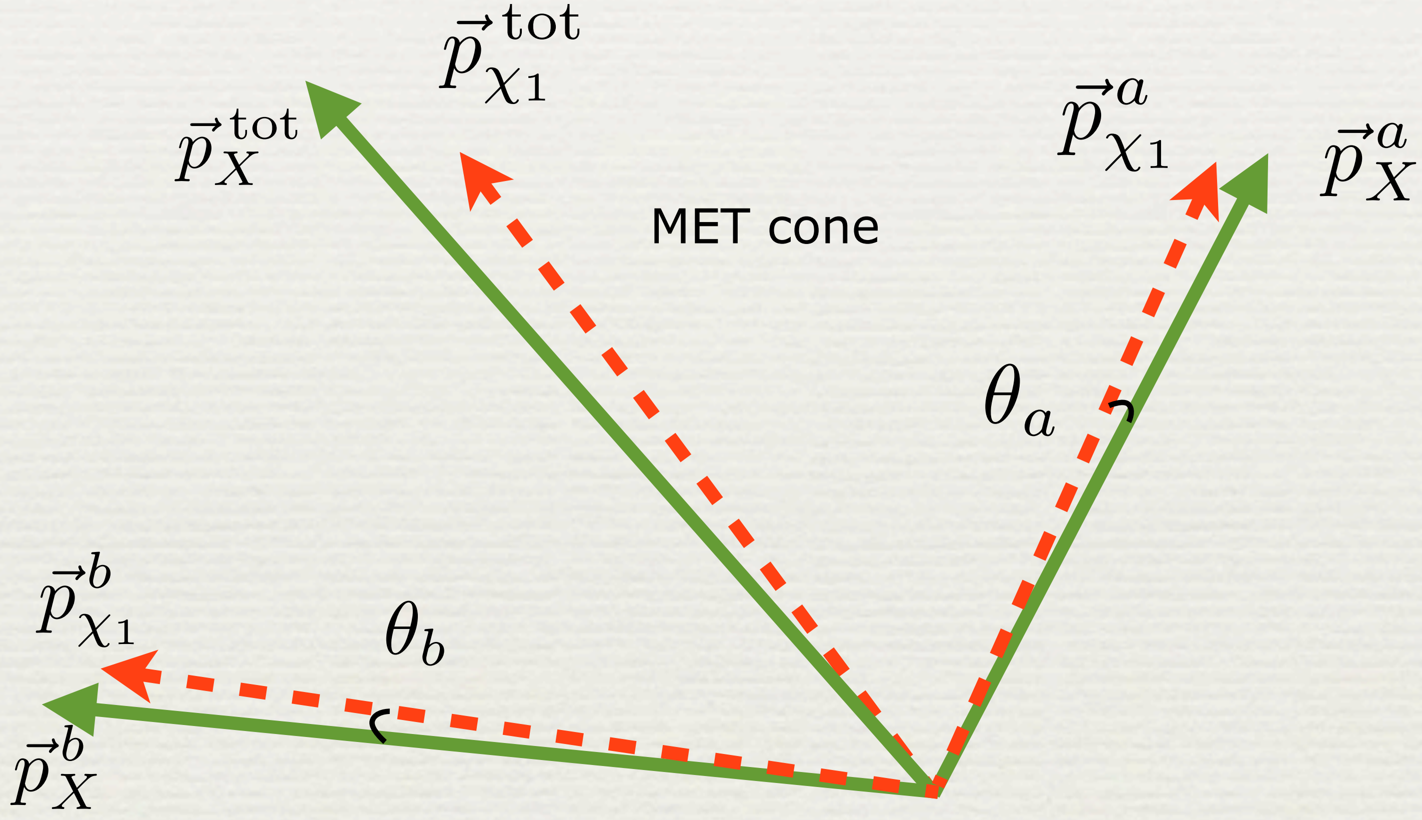


New coordinate

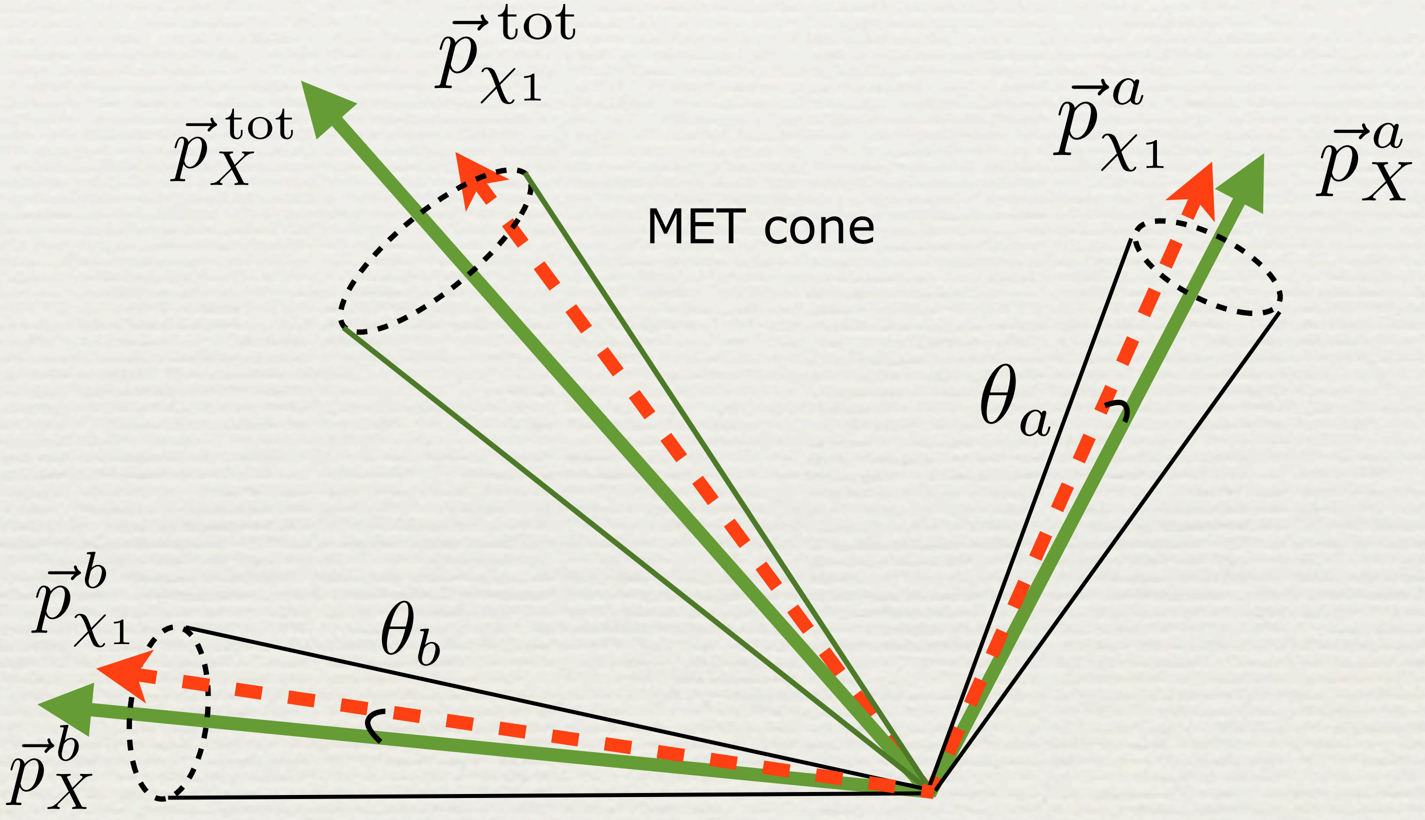
Transverse Plane



3D View


$\vec{p}_{\chi_1}^{\,\mathrm{tot}}$
$\vec{p}_X^{\,\mathrm{tot}}$
$\vec{p}_{\chi_1}^{\,a}$
$\vec{p}_X^{\,a}$
MET cone
θ_a
$\vec{p}_{\chi_1}^{\,b}$
θ_b
$\vec{p}_X^{\,b}$

3D View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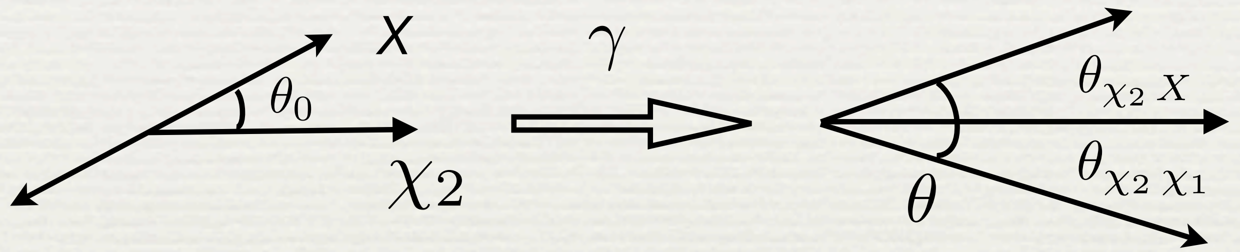
MET-cone method

- Based on simple observations:
  - Missing momentum only allowed to vary a narrow region around visible momentum -- “MET-cone”
  - MET-cone boundary is sensitive to the underlying masses
  - This kinematic boundary depends on the visible momenta, need event-by-event analysis
- Different from other methods, we consider the missing transverse momentum as our observables for mass measurement

Collinearity of the decay

- Parametrize the opening angle in the lab frame

$$\tan\theta_{\chi_2 X} = \frac{\beta_0^X}{\gamma}\left(\frac{\sin\theta_0}{\beta_0^X\cos\theta_0 + \beta}\right)$$



β, γ Velocity & boost factor of NLSP

β_0, γ_0 Velocity & boost factor in the rest frame of NLSP

- Narrow range of variation $\beta_0^X < \beta$

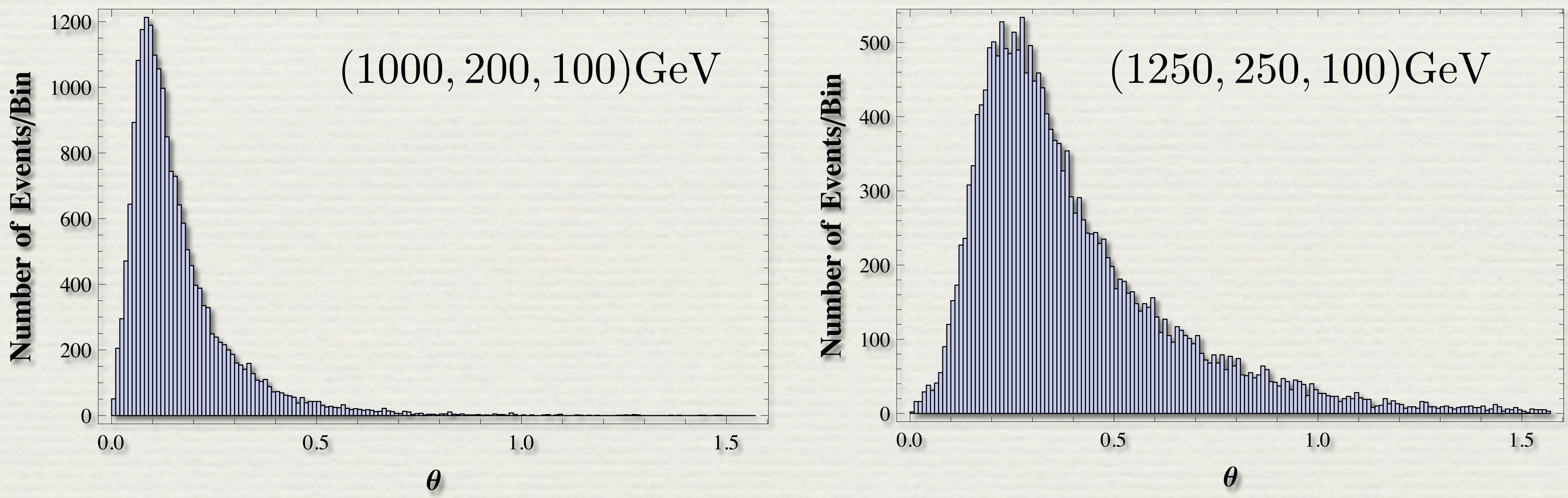
$$0 \le \tan\theta_{\chi_2 X} \le \frac{\beta_0^X}{\gamma\beta}\frac{1}{\sqrt{1-(\beta_0^X/\beta)^2}} \xrightarrow{\gamma \gg 1} \frac{\beta_0^X}{\gamma}\frac{1}{\sqrt{1-(\beta_0^X)^2}}.$$

- Two ways to have collinear decay
  - Large boost factor $\gamma \gg 1$
  - Moderate boost factor; decay products are non-relativistic in the rest frame of the decay $\beta_0 \ll 1$

Collinearity of the decay

- For a given underlying physics, both boost factor and vary θ_0 according to the matrix element

$$\tilde{q}_L \rightarrow \chi_2 q \rightarrow \chi_1 \, Z \, q$$



- Boost factor decrease with increased number of steps in the cascade

Correlation in the magnitude

- Boost factors are correlated

$$\begin{aligned} p_{\chi_1} &= \gamma_{\chi_1}\beta_{\chi_1}m_{\chi_1} \\ p_X &= \gamma_X\beta_X m_X. \end{aligned} \qquad \begin{aligned} \gamma_{\chi_1} &= \gamma\,\gamma_0^{\chi_1}(1+\beta\,\beta_0^{\chi_1}\cos\theta) \\ \gamma_X &= \gamma\,\gamma_0^{X}(1-\beta\,\beta_0^{X}\cos\theta) \end{aligned}$$

$$\gamma_0^{\chi_1} = \frac{m_{\chi_2}^2 + m_{\chi_1}^2 - m_X^2}{2m_{\chi_2}m_{\chi_1}}$$

$$\beta_0^{\chi_1} = \frac{\sqrt{\left(m_{\chi_2}^2 - (m_{\chi_1}+m_X)^2\right)\left(m_{\chi_2}^2 - (m_{\chi_1}-m_X)^2\right)}}{m_{\chi_2}^2 + (m_{\chi_1}^2 - m_X^2)}$$

- In the limit $\beta_0 \ll 1$, two boost factors equal
- the ratio mainly depend on θ_0 , mildly dependence on the boost factor γ

Finding MET-Cone boundary

- For a given visible particle configuration, what is the allowed region of MET ?

Given $\gamma_a^X, \gamma_b^X, \theta_{ab}^X \quad \theta_{\text{beam}}, \phi_{\text{beam}} \quad m_{\chi_1}, m_{\chi_2}, m_X$

No need to know the boost factor of NLSP!

Parameterize MET by the rest frame angles

$\theta_{a,0}, \theta_{b,0}, \phi_{a,0}, \phi_{b,0}$

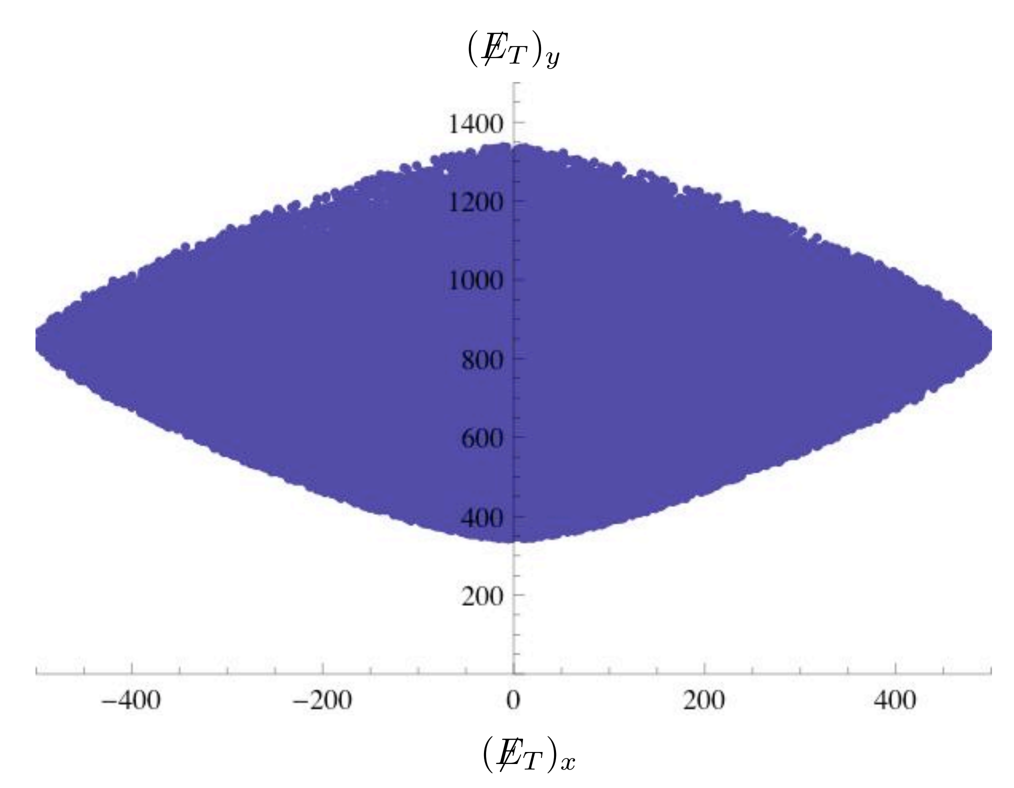
- No analytic formula for the boundary. Need sampling the phase space

- A simple example:

$\chi_2 \to \chi_1 Z$

$m_{\chi_2} = 200$ GeV, $m_{\chi_1} = 100$ GeV.

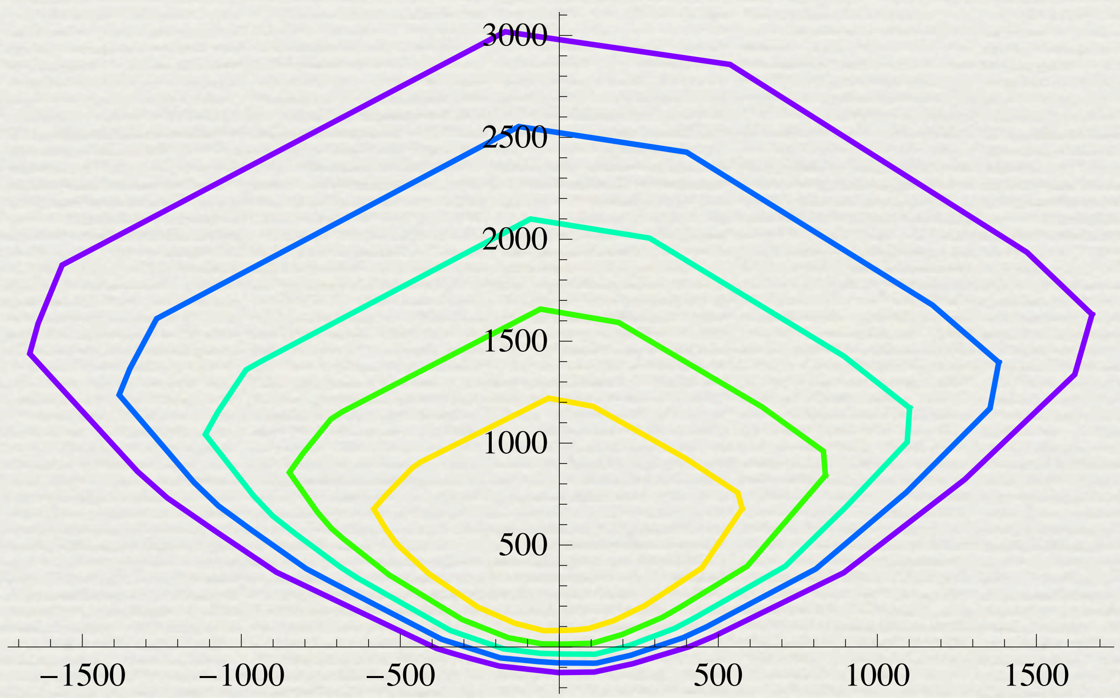
$\gamma_{a,b}^X = 5 \quad \theta_{ab}^X = \pi/2 \quad \theta_{\text{beam}} = 0$



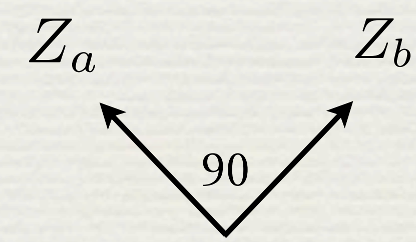
MET-Cone : mass dependence

- MET cone boundary sensitive to the exotic masses

Shift m_{χ_2} uniformly from 220 to 300 GeV



$$\gamma_a^Z = \gamma_b^Z = 3.0$$



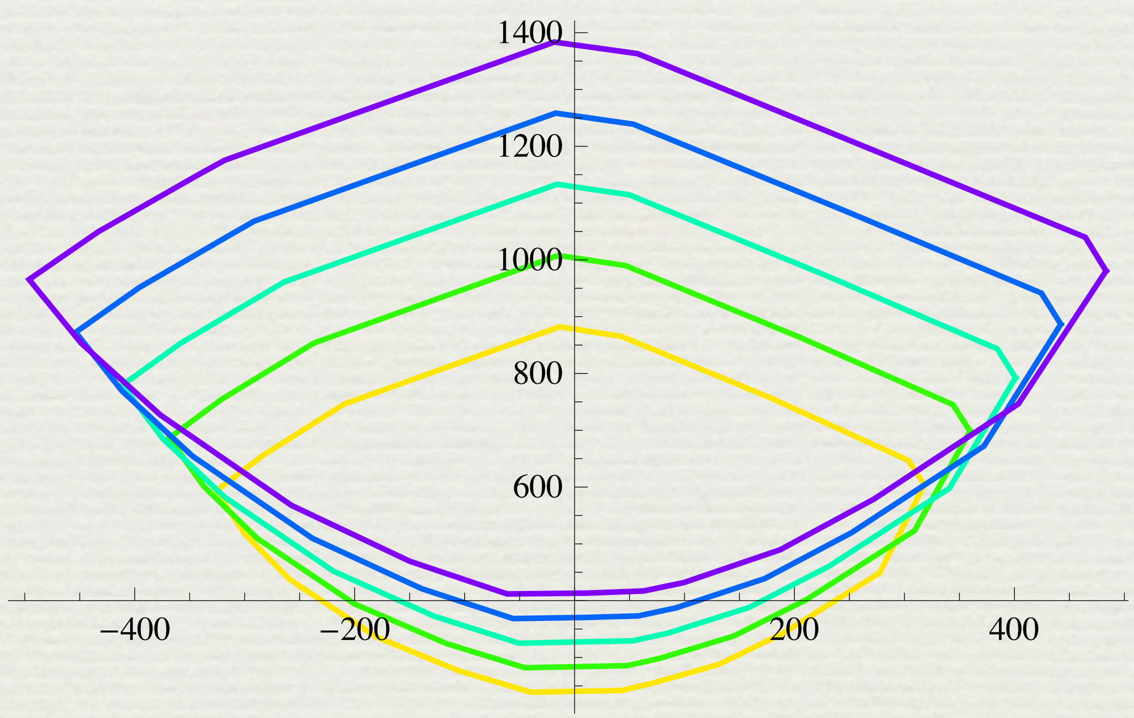
$$(m_{\chi_2}, m_{\chi_1}, m_Z)$$

= (220-300, 100, 91)

MET-Cone : mass dependence

- MET cone boundary sensitive to the exotic masses

Shift m_{χ_2} uniformly from 220 to 300 GeV, but keep $m_{\chi_2} - m_{\chi_1}$ fixed



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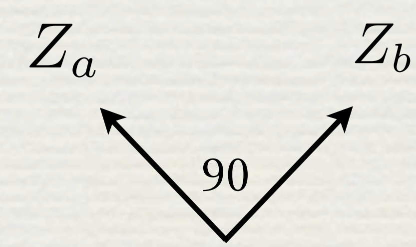
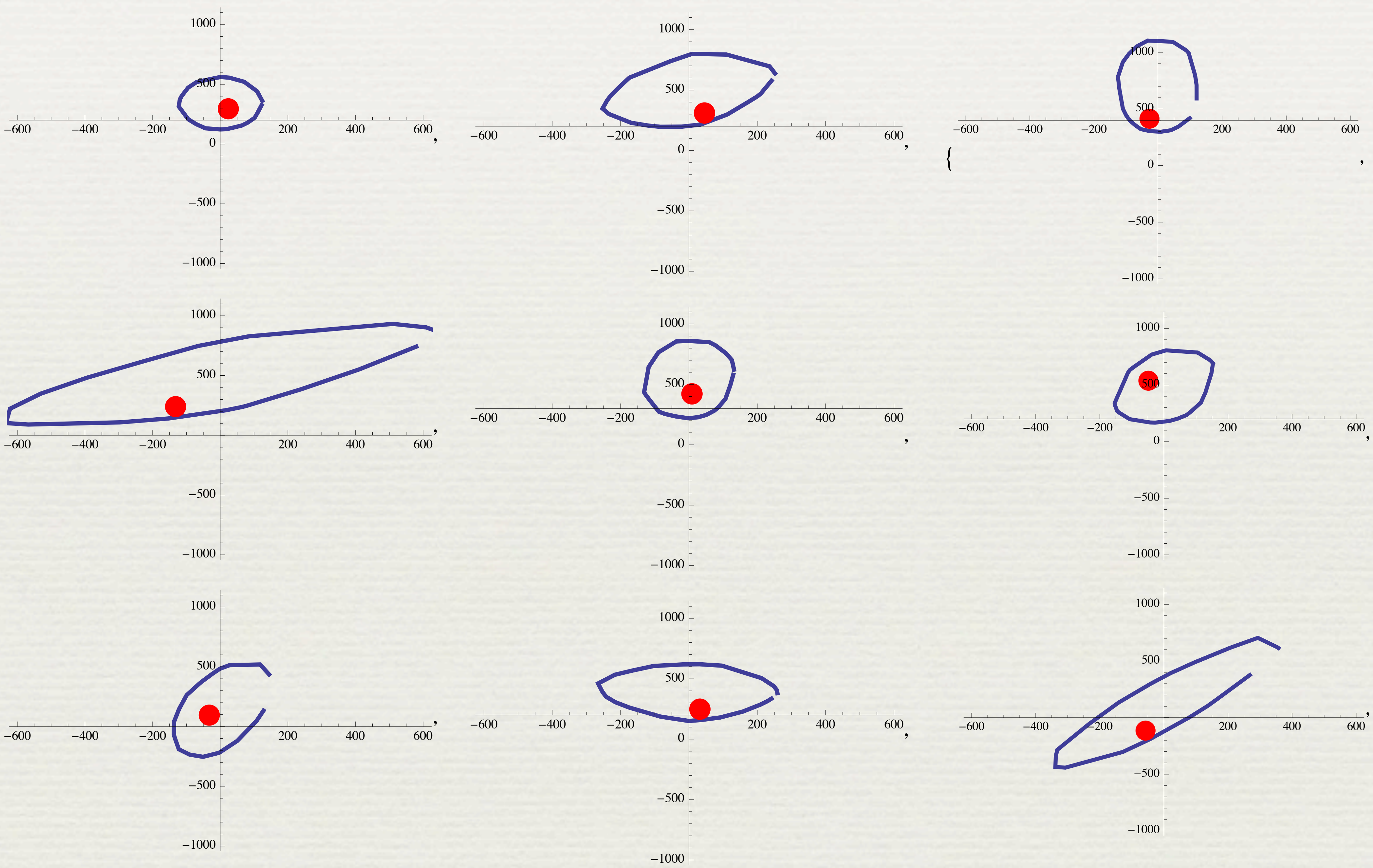


$(m_{\chi_2}, m_{\chi_1}, m_Z)$

= (220-300, 120-200, 91)

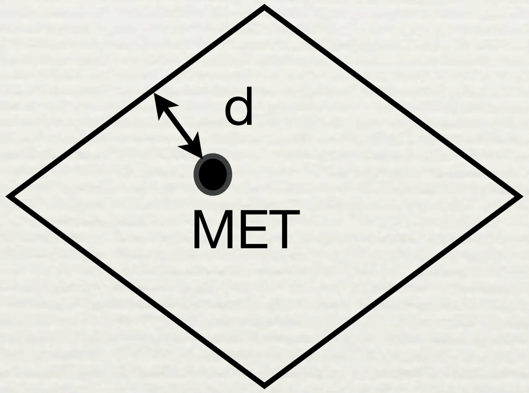
Reconstructed MET-cone boundary from random events -- Assume correct mass



MET must be inside if the correct masses were used

MET-cone: application for mass measurement

- For a set of events and trial masses, the MET-cone boundary can be determined by the Z momenta event-by-event.



- The correct masses are those that lead to the smallest MET-cone that enclose all the MET points

$$d_{\min} \to 0$$

- More systematically, compare the statistical likelihood of a MET data under different mass hypotheses.

Detailed numerical evaluation of this method is under investigation.

Quick Summary

- MET-cone method is different from other methods; only need information of the visible particles in the final-step decay and MET
- Although motivated from boosted decay chain, the general idea of the method doesn’t require boost.
- It should work best in the boosted case

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- MET-cone method is different from other methods; only need information of the visible particles in the final-step decay and MET
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Is there a simple way to access the power of MET-cone?

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Is there a simple way to access the power of MET-cone?

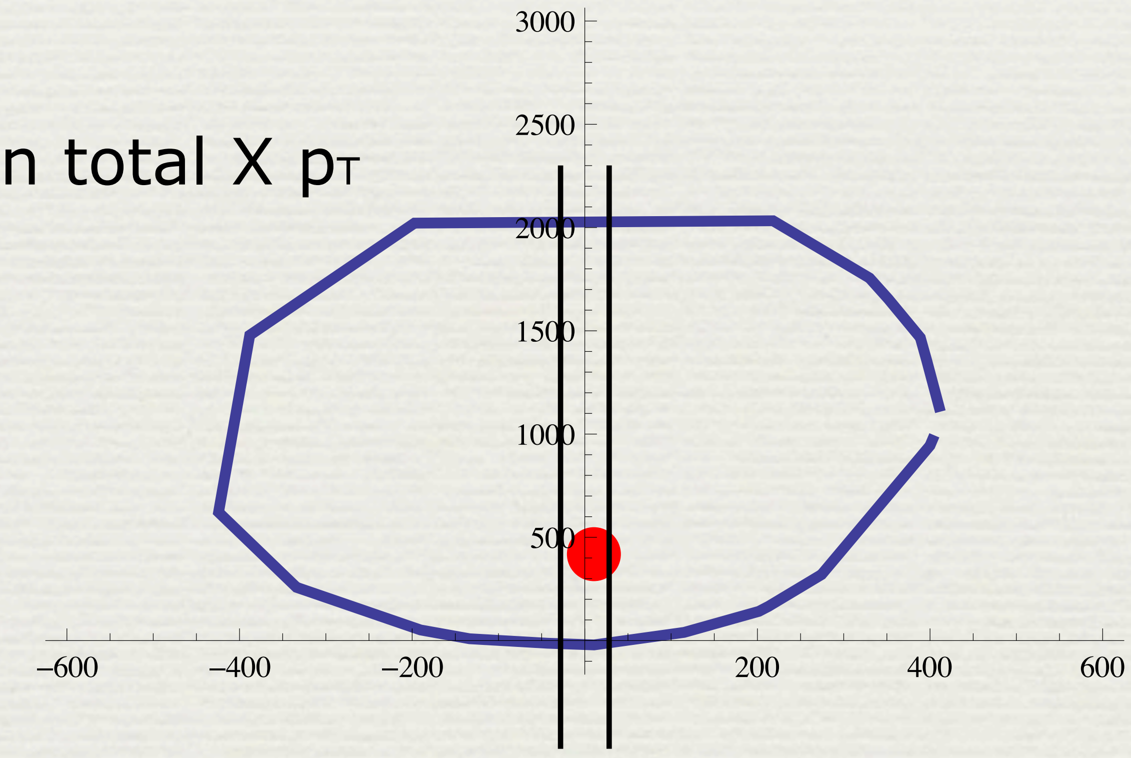
Yes, We can construct a variable independent of the X momenta, and has lower and upper endpoints.

A 1D projection of the MET-cone

- Focus on events where MET is in narrow window around y-axis (i.e. the direction of the total X p_T)
- Expect two boundaries, but vary event by event
- Finite variation in the ratio between total X p_T and total missing p_T
- Rescale x & y coordinates:

$$\not{p}_{T,y} \rightarrow m_X \not{p}_{T,y} / p^{\text{tot}}_{X,T}$$

$$\not{p}_{T,x} \rightarrow \not{p}_{T,x} / \not{E}_T$$



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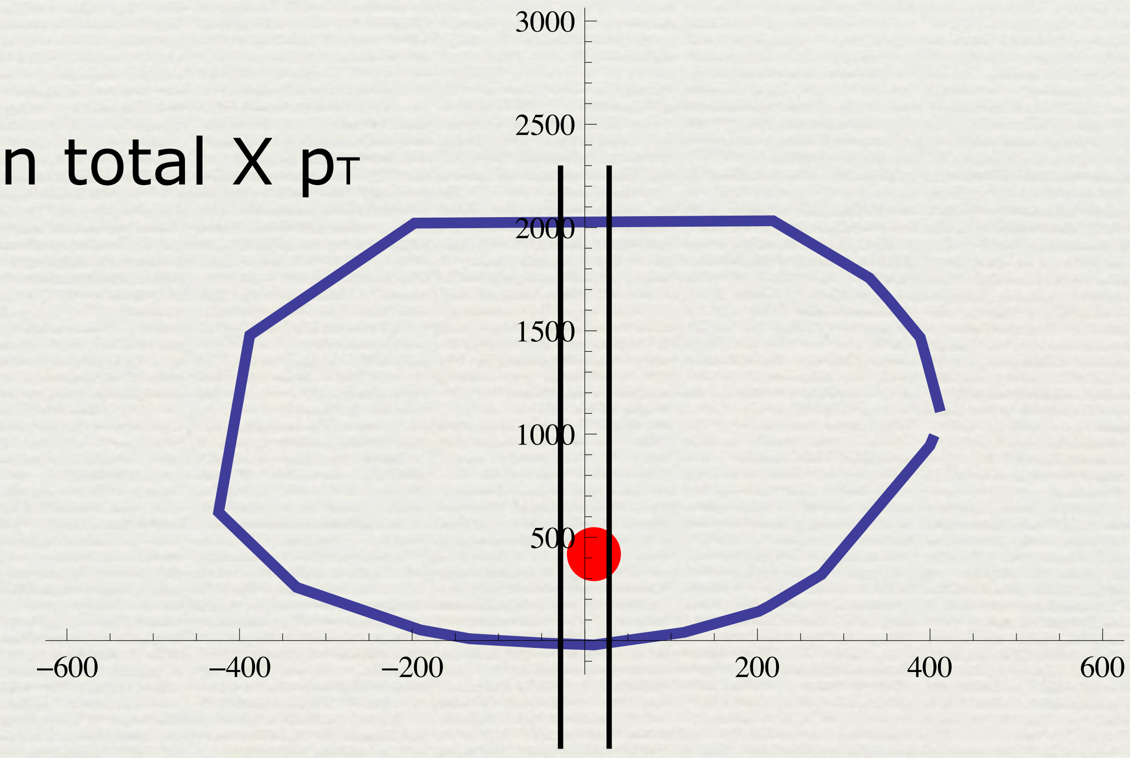
$$\not{p}_{T,x} \rightarrow \not{p}_{T,x} / \not{E}_T$$

$m_{\chi_1}^{\text{test}}$



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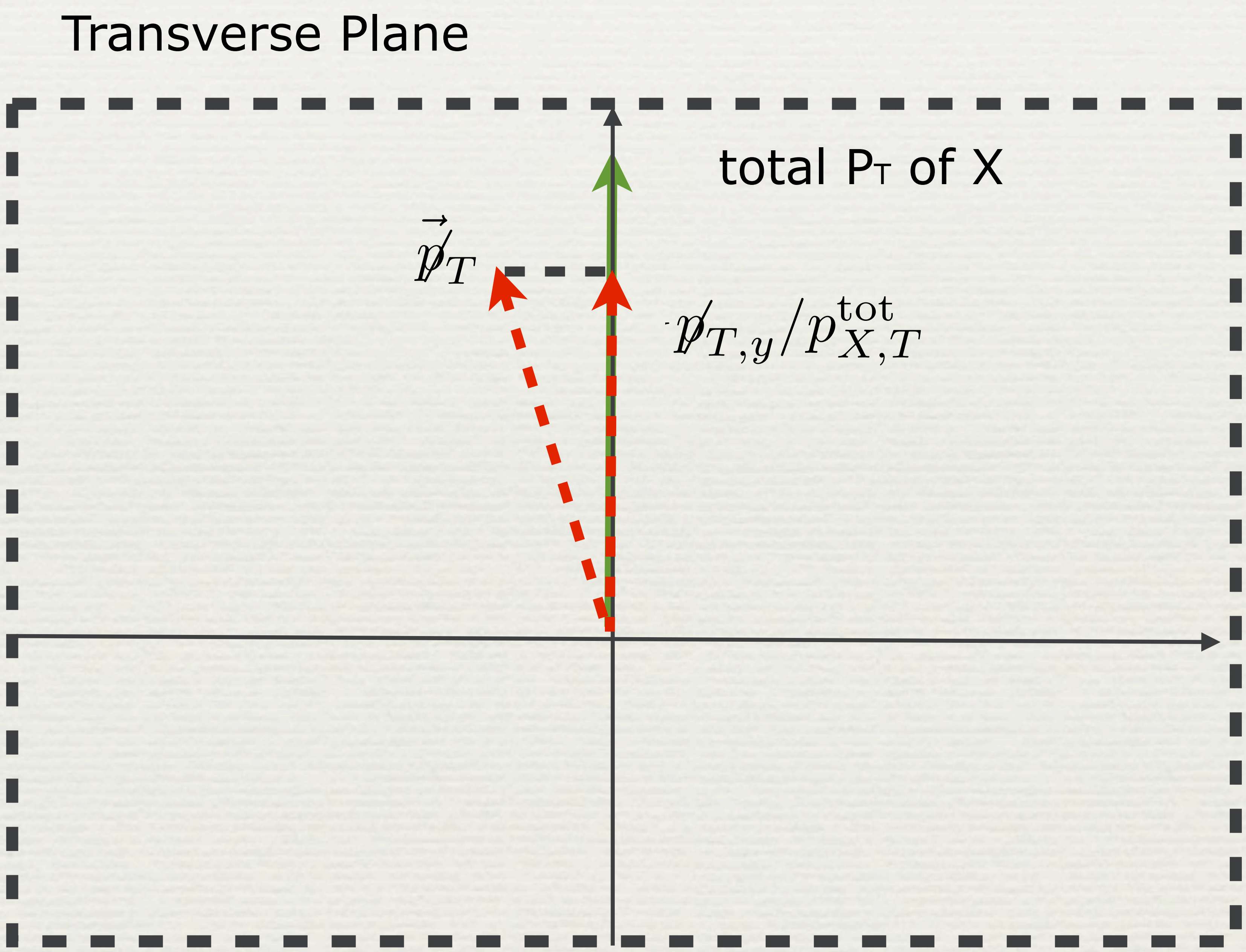
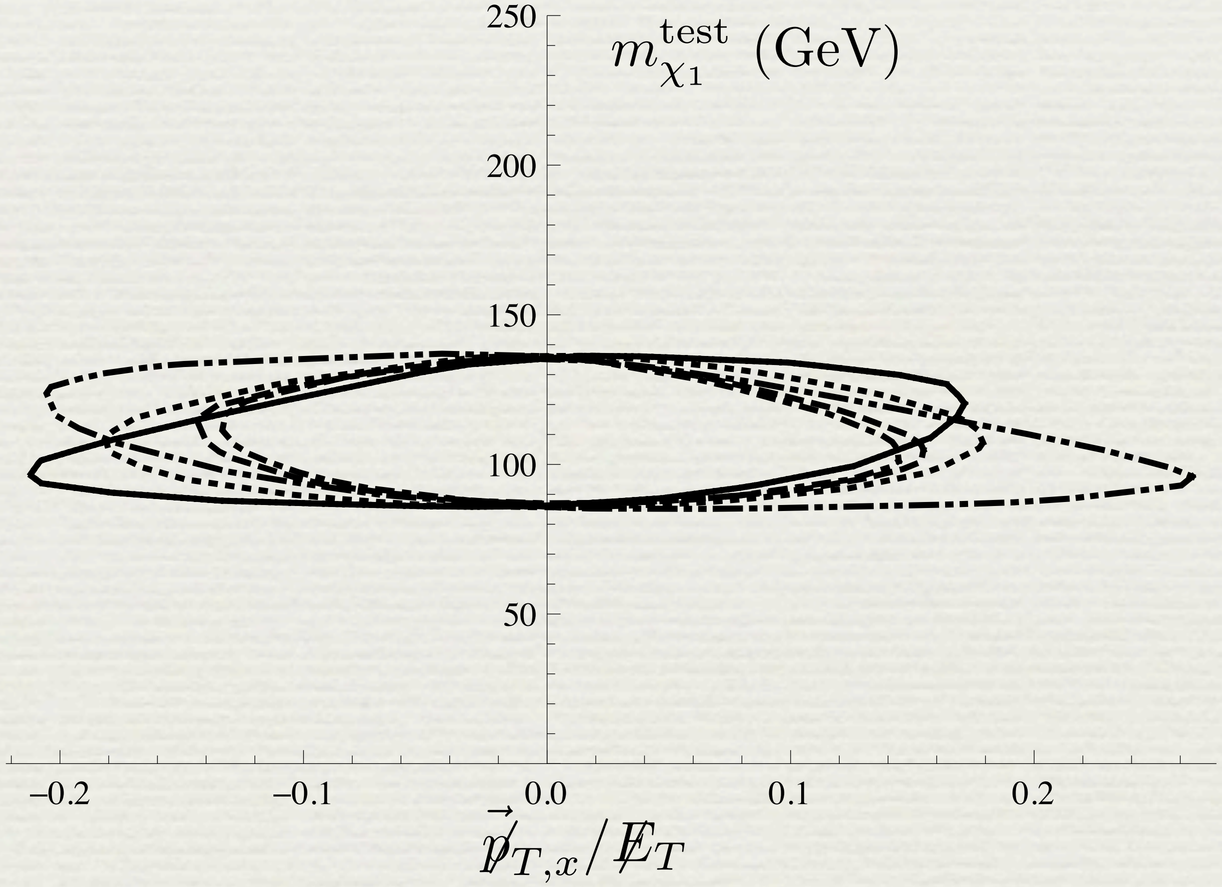


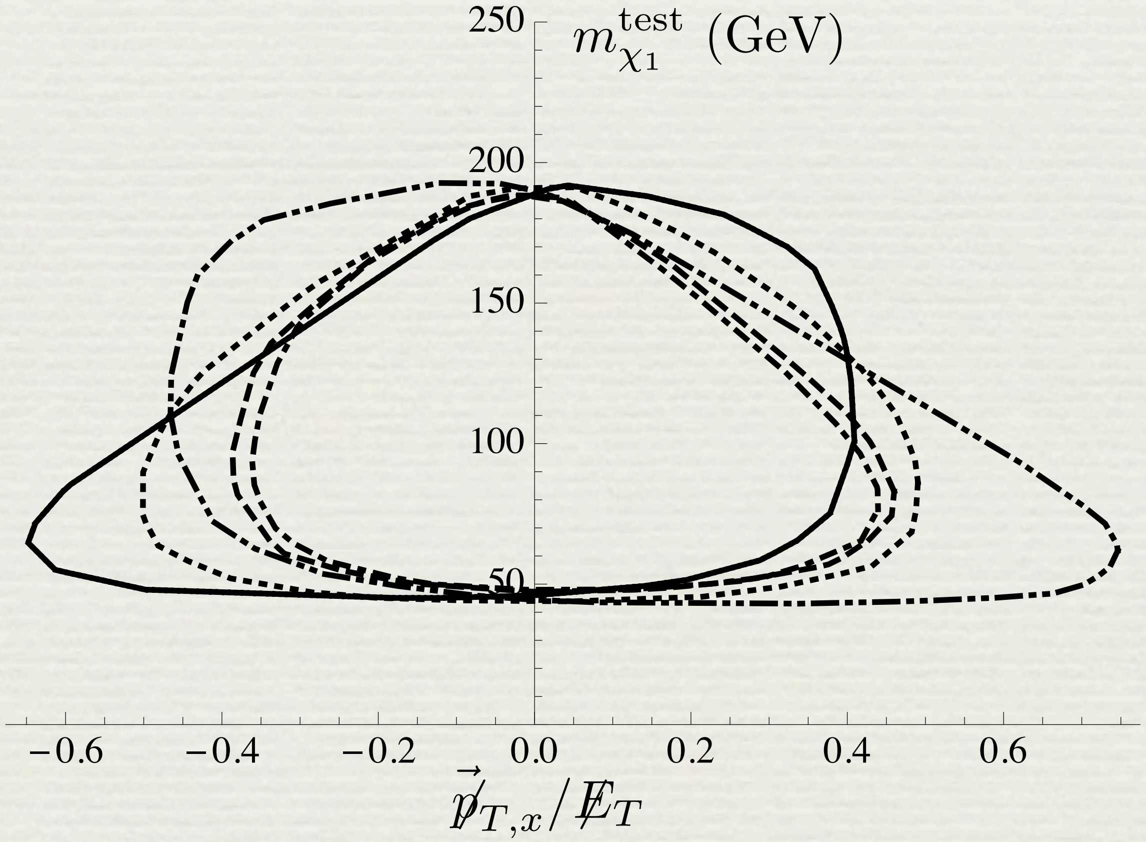
Rescaled MET-cone

- After rescaling, endpoints are fixed for all events !

$(m_{\chi_2}, m_{\chi_1}, m_Z)$ (200, 109.9, 90)



(200 100 90)



Analytic solution of M_{test}

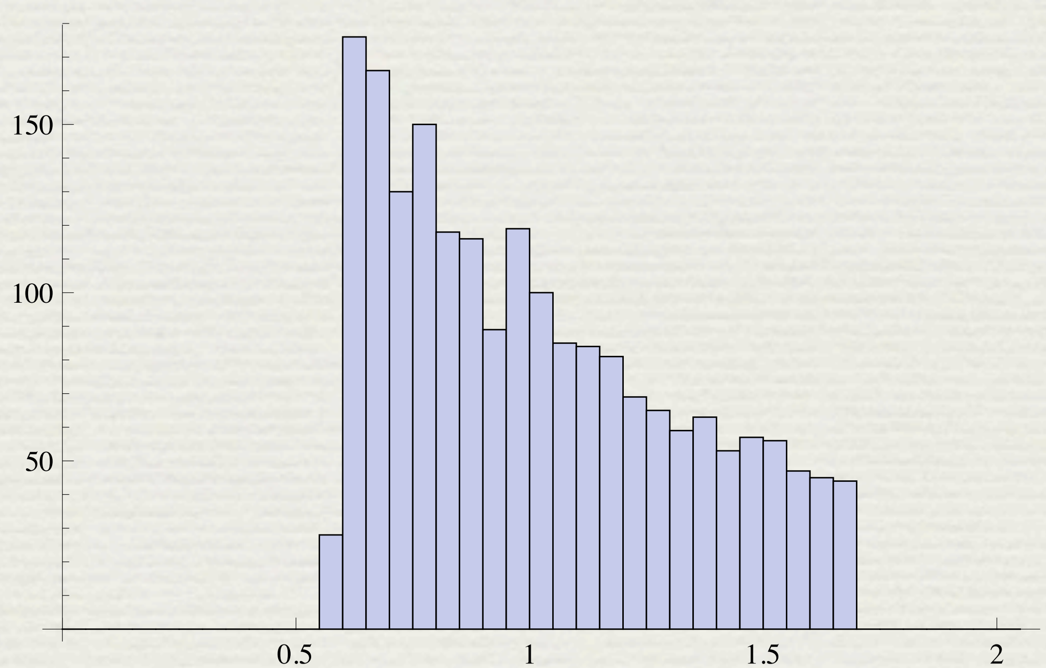
- Take the limit $|\not{p}_{T,x}/\not{E}_T| \to 0$, and use the collinear approx.

- Consider a simple case: X's in the transverse plane.

- Leading order result:

$$m^{\text{test}} \approx m_{\chi_1} \frac{\gamma_0^{\chi_1}}{\gamma_0^X} \frac{1 + \beta\, \beta_0^{\chi_1} \cos\theta_{a,0}}{1 - \beta\, \beta_0^X \cos\theta_{a,0}}$$

- Ideal shape (assume flat prior for theta)



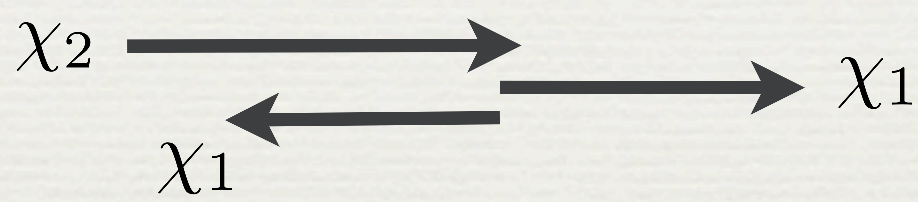
$$\begin{aligned} p_{\chi_1} &= \gamma_{\chi_1}\beta_{\chi_1} m_{\chi_1} \\ p_X &= \gamma_X \beta_X m_X . \end{aligned}$$

$$\begin{aligned} \gamma_{\chi_1} &= \gamma\, \gamma_0^{\chi_1} (1 + \beta\, \beta_0^{\chi_1} \cos\theta) \\ \gamma_X &= \gamma\, \gamma_0^X (1 - \beta\, \beta_0^X \cos\theta) \end{aligned}$$

$$\frac{\gamma_{\chi_1}^b \beta_{\chi_1}^b}{\gamma_X^b \beta_X^b} = \frac{\gamma_{\chi_1}^a \beta_{\chi_1}^a}{\gamma_X^a \beta_X^a} \Big(1 + \mathcal{O}(\theta_{a,b}) \Big)$$

Mtest endpoints

- There are two endpoints, corresponding to θ_0 --> 0, Pi



$$m_{\min}^{\text{test}} = m_{\chi_1} \frac{\gamma_0^{\chi_1}}{\gamma_0^{X}} \frac{1-\beta\,\beta_0^{\chi_1}}{1+\beta\,\beta_0^{X}} \qquad \gamma_0^{\chi_1} = \frac{m_{\chi_2}^2 + m_{\chi_1}^2 - m_X^2}{2 m_{\chi_2} m_{\chi_1}}$$

$$m_{\max}^{\text{test}} = m_{\chi_1} \frac{\gamma_0^{\chi_1}}{\gamma_0^{X}} \frac{1+\beta\,\beta_0^{\chi_1}}{1-\beta\,\beta_0^{X}} \qquad \beta_0^{\chi_1} = \frac{\sqrt{\left(m_{\chi_2}^2 - (m_{\chi_1}+m_X)^2\right)\left(m_{\chi_2}^2 - (m_{\chi_1}-m_X)^2\right)}}{m_{\chi_2}^2 + (m_{\chi_1}^2 - m_X^2)}$$

- Punchline: endpoints only depend on the masses
  --> measure these endpoints experimentally can determine these masses

non-collinear effects

- M_{test} not invariant under boost -- subjet to non-collinear correction

$$m_{\chi_1}^{\text{test}} \approx m_{\chi_1} \frac{\gamma_0^{\chi_1}}{\gamma_0^X} \frac{1 + \beta\, \beta_0^{\chi_1} \cos\theta_0^a}{1 - \beta\, \beta_0^X \cos\theta_0^a} \left(1 - \cot\theta_{ab} \cos\phi_a \theta_a + \frac{\cos\phi_b}{\sin\theta_{ab}} \theta_b\right)$$

  - endpoints get smeared;
  - prefer small θ , not too small θ_{ab}^X

- If X's not in the transverse plane, extra projection needed -- more complicated in the above θ expansion

Quick Summary

- MET-cone method
- A simple 1D variable m_{test} for mass measurement
- How well this works in practice?

Monte Carol simulation

- Generate 20k events for SUSY squark production, using MadGraph 2-->6 matrix element

$$pp \to \tilde{q}_L \tilde{q}_L \to q\,\tilde{\chi}_1\,Z\,q\,\tilde{\chi}_1\,Z$$

- Assume Z's are reconstructed using the leptonic decay. The SM backgrounds is negligible for 4 leptons, 2 jets plus MET

- No detector smearing included-- to be included later.

- Parton-level cuts

$p_T^Z > 50$ GeV $\quad |\eta^Z| < 3$

$|\eta^{Z,\mathrm{tot}}| < 1 \quad \pi/3 < \theta^Z_{a,b} < 2/3\pi$ two Z's opening angle

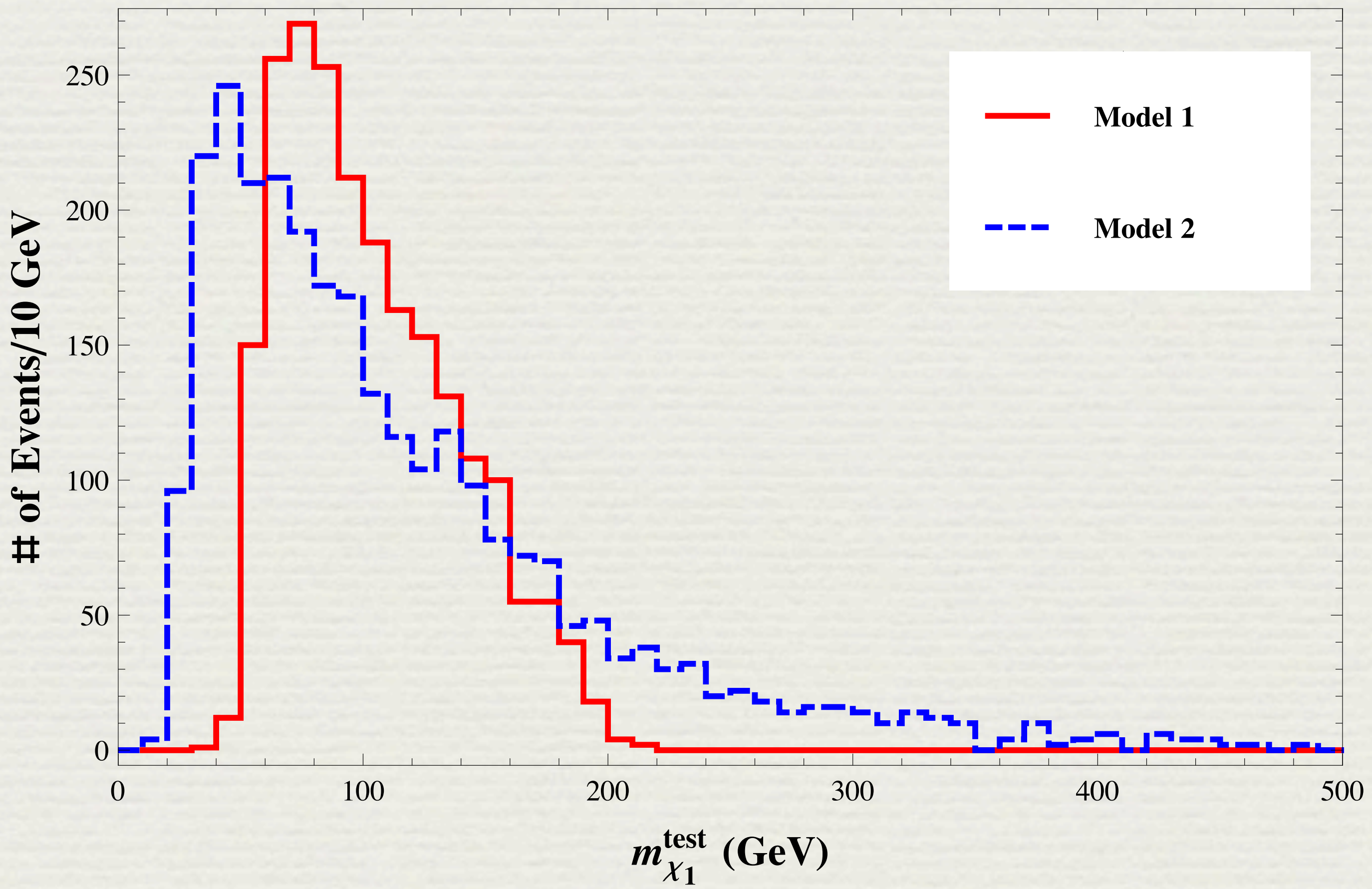
$\not{E}_T > 200$ GeV $\quad |\not{p}_{T,x}/\not{E}_T| < 0.15$

Mtest distributions

Model 1 : M_1=100, M_2=200, M_Q= 1TeV. Moderate boost with small β_0

Model 2 : M_1=100, M_2=250, M_Q= 1.25Tev. Moderate boost with larger β_0

True endpoints: Model 1 (54.6, 183.2) GeV; Model 2 (21.6, 463) GeV

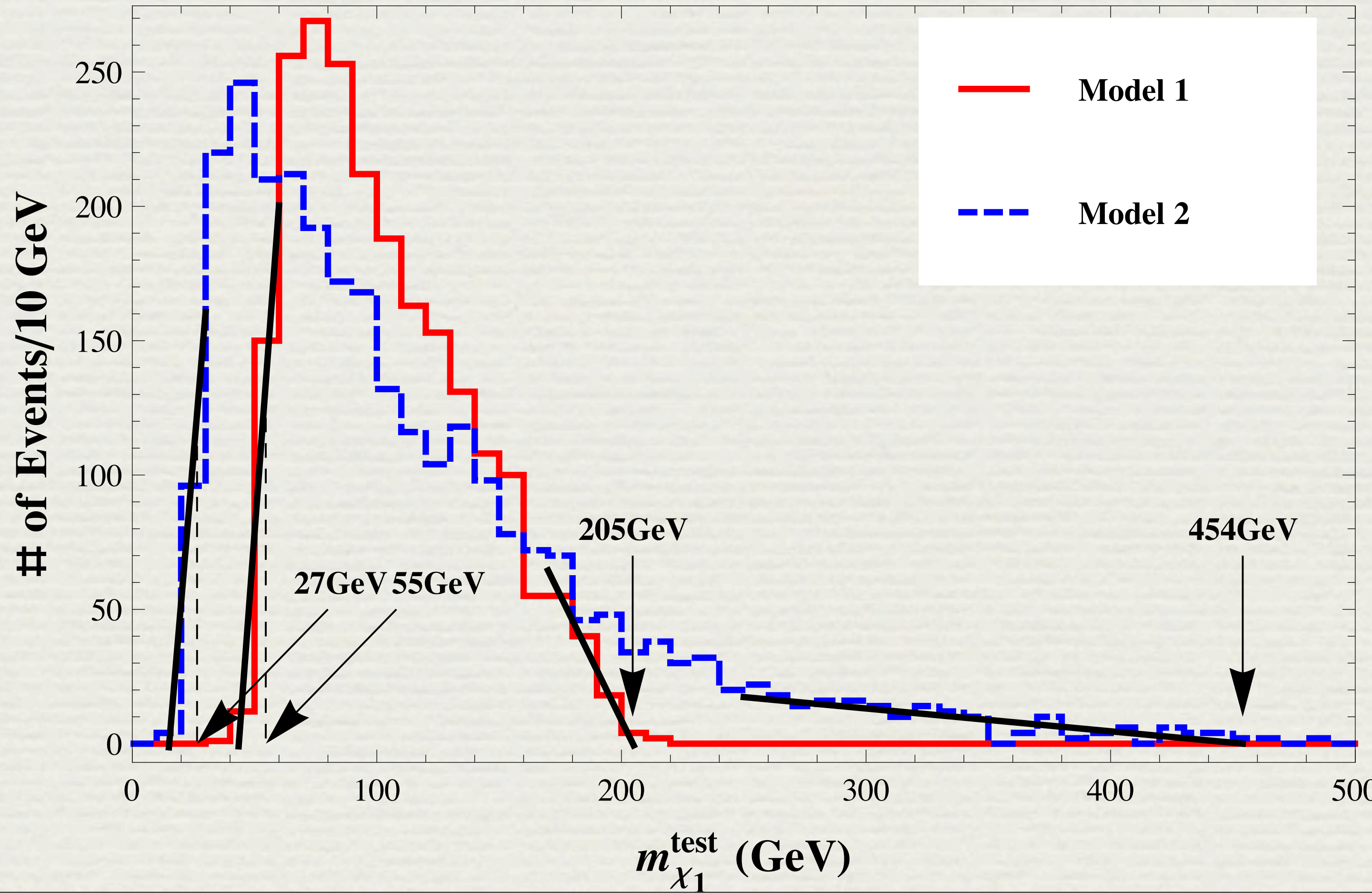


Fit of endpoints

- Use linear fits
  - Lower endpoint -- expected to be sharp edge, we take half-max point to reduce smearing effects
  - Upper endpoint -- less populated, and we take intercept position
- Larger sys. err. for Model 2

Model	M1	M2
1	106	208
2	110	253

Masses are in GeV

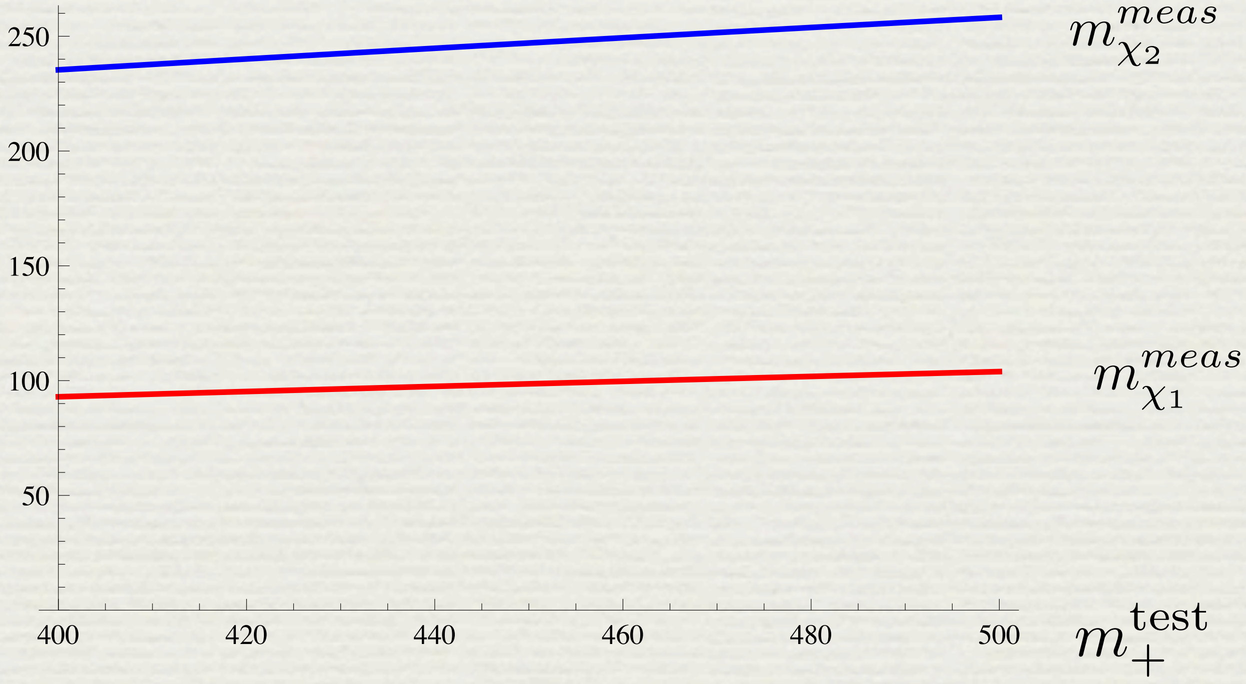


Fit of endpoints

The measured mass is not sensitive to the upper end point,

e.g. for Model 2:

vary upper end point 400 - 500 GeV



$$(m_{\chi_1}^{meas}, m_{\chi_2}^{meas}) = (103 \text{ GeV}, 241 \text{ GeV}) — (116 \text{ GeV}, 264 \text{ GeV})$$

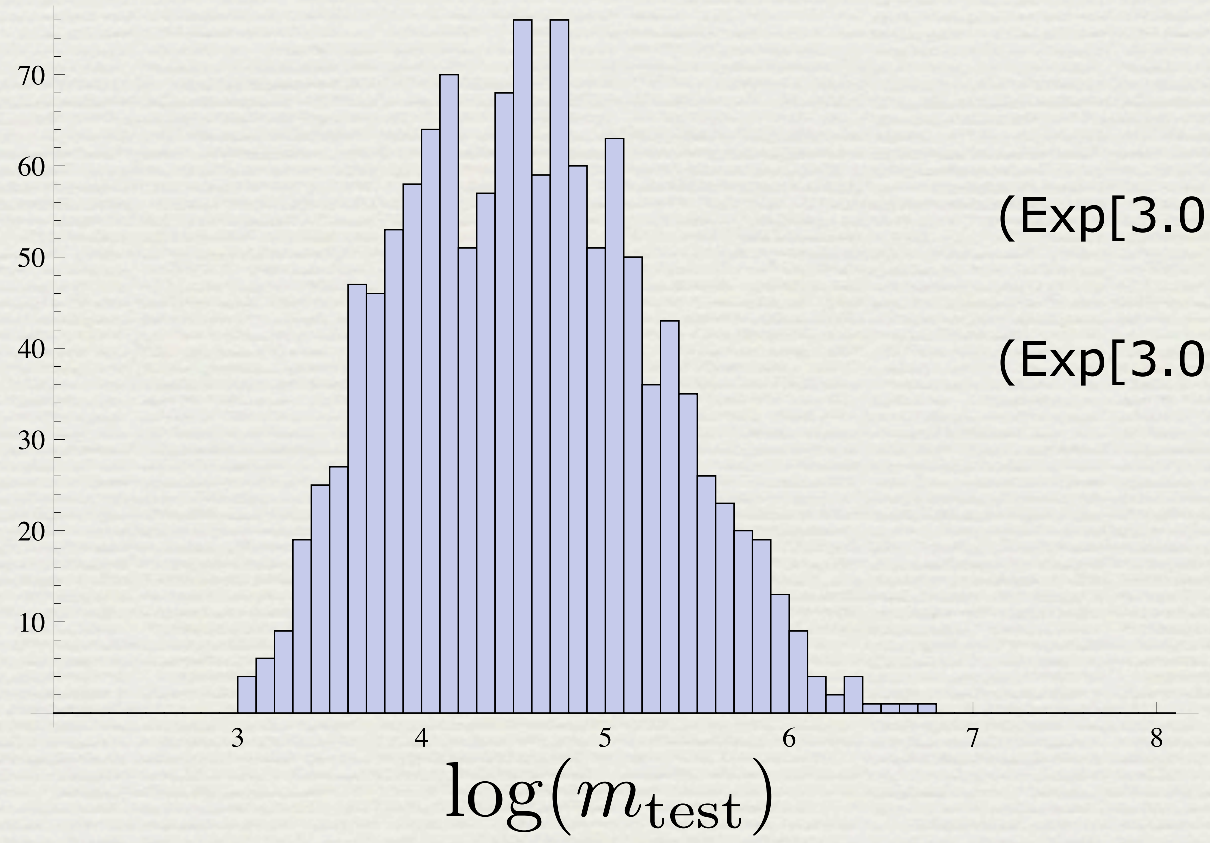
Fit of endpoints

- Better variable by taking logarithm

$$\log(m_{\rm test}) \sim \log(m_{\chi_1}) + (\beta_0^X + \beta_0^{\chi_1})\cos\theta_0$$

- more symmetric distribution --> easier to determined the endpoint

Model 2



(Exp[3.05], Exp[6.2]) =(21.1153, 492.749)

(Exp[3.0], Exp[6.3])=(20.0855, 544.572)

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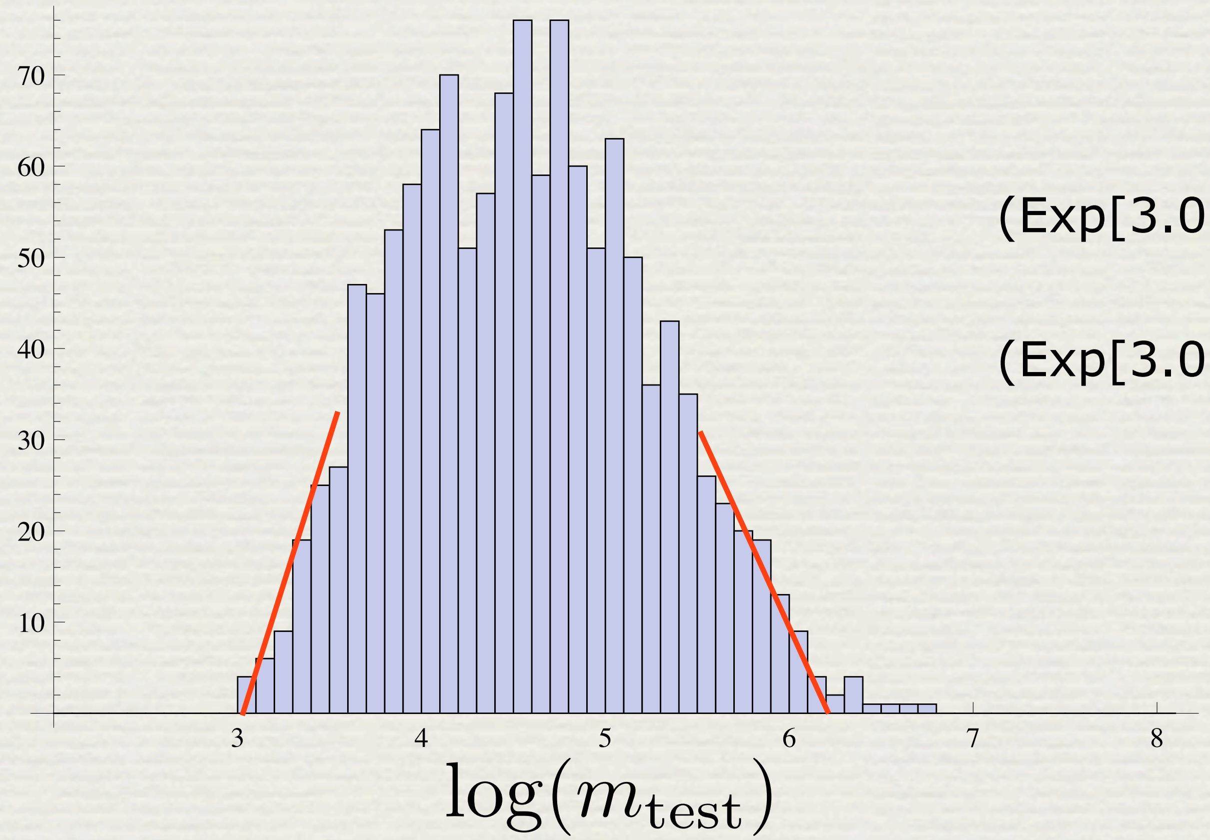
Model 2

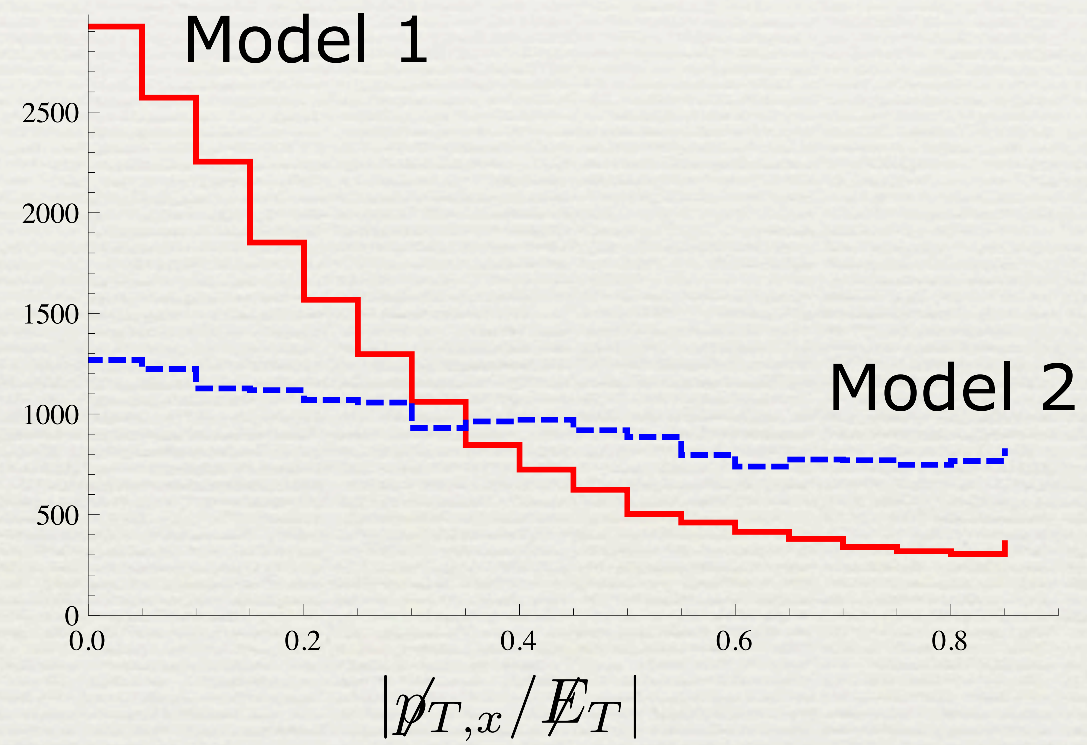


(Exp[3.05], Exp[6.2]) =(21.1153, 492.749)

(Exp[3.0], Exp[6.3])=(20.0855, 544.572)

Is it a boosted decay chain?

- $|\not{p}_{T,x}/\not{E}_T|$ distribution peak towards zero



- Sharp endpoints in M_{test} distribution

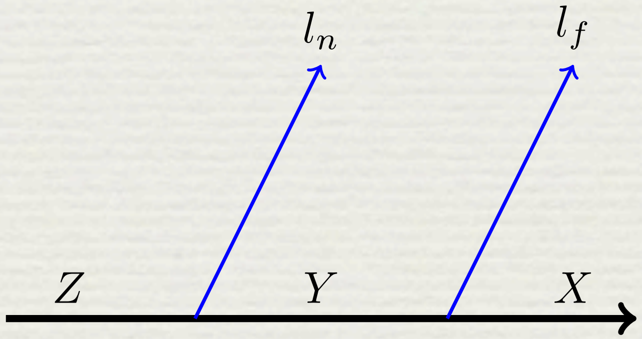
- Measure upstream exotica masses, check whether it is consistent

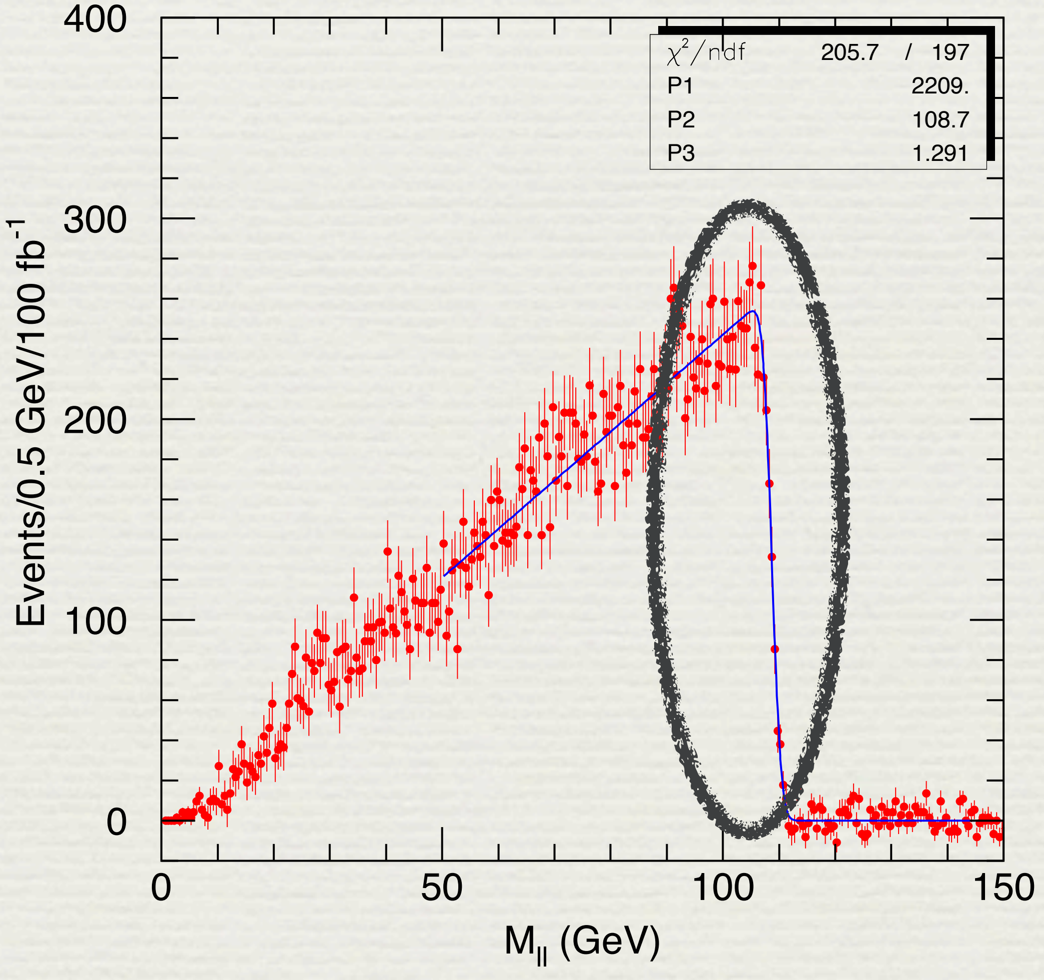
Other Channels ?

- χ_2 can also decay through slepton --> di-lepton
- Invariant mass is not fixed, but can select events near the upper endpoint.

$$M_{ll}^{\max} \sim m_{\chi_2} - m_{\chi_1}$$

- work in progress

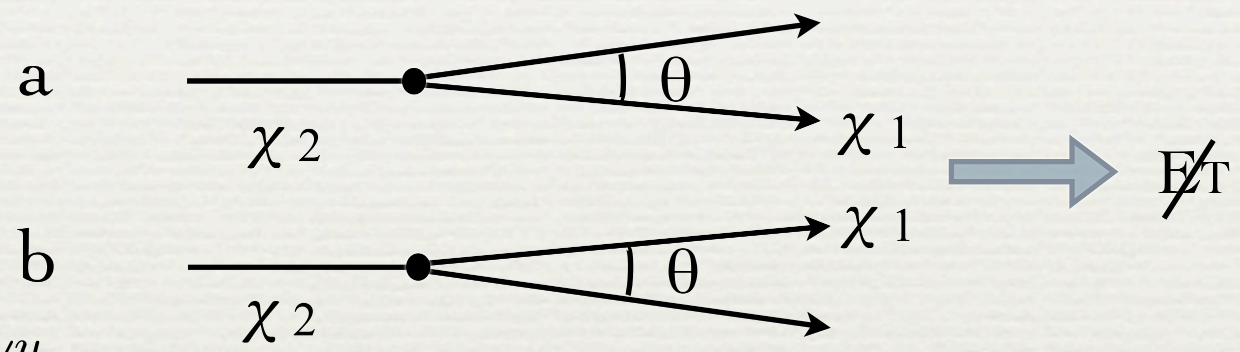




Use CM energy Variable

- Reconstruct missing particle momenta using collinear approx.

$$\vec{p}_{\chi_1,a} = k_a\,\vec{p}_{X,a}$$
$$\vec{p}_{\chi_1,b} = k_b\,\vec{p}_{X,b}$$



$$\longrightarrow \begin{cases} k_a = \dfrac{p^y_{X,b}\not{p}^x - p^x_{X,b}\not{p}^y}{-p^y_{X,a}p^x_{X,b}+p^x_{X,a}p^y_{X,b}} \\ k_b = \dfrac{-p^y_{X,a}\not{p}^x + p^x_{X,a}\not{p}^y}{-p^y_{X,a}p^x_{X,b}+p^x_{X,a}p^y_{X,b}} \end{cases}$$

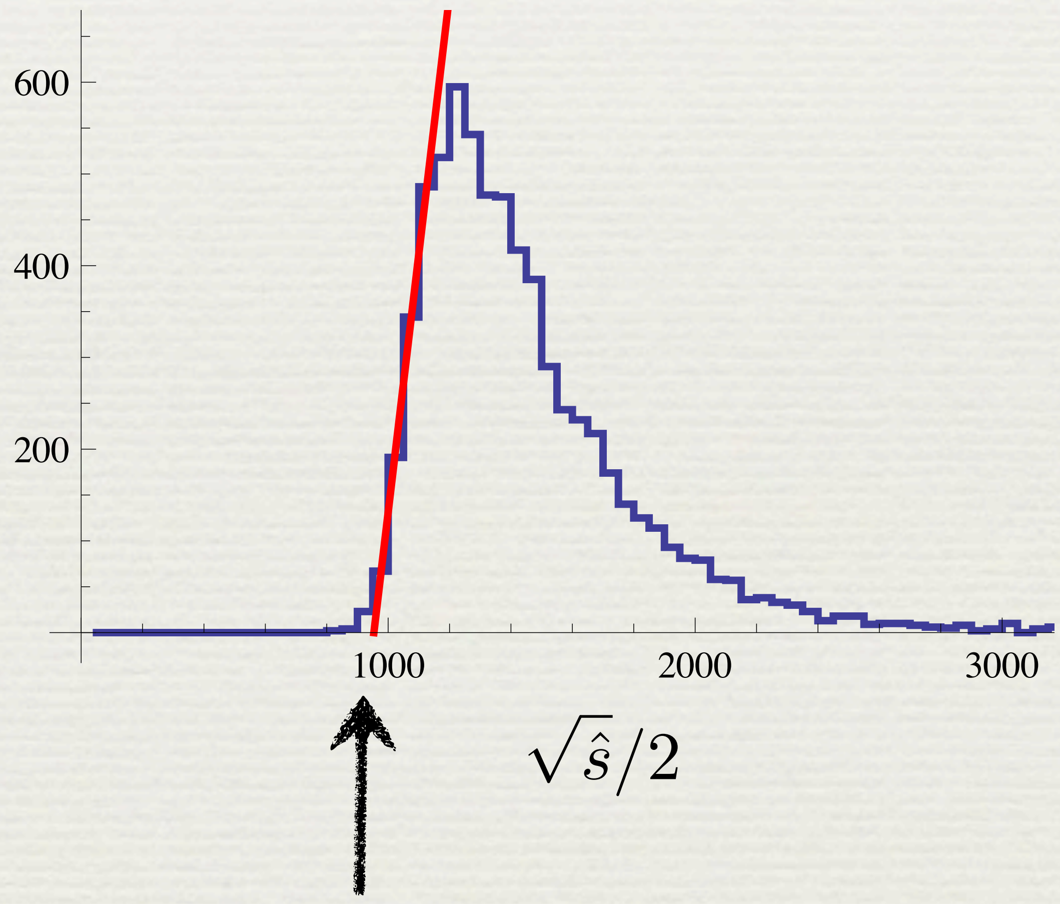
- Reconstruct CM energy of the collision $s = \left(\sum_i p_i\right)^2$

- lower endpoint provide an estimate of the mass of mother particle

$$\hat{s} \geq 4m_Q^2$$

Use CM energy Variable

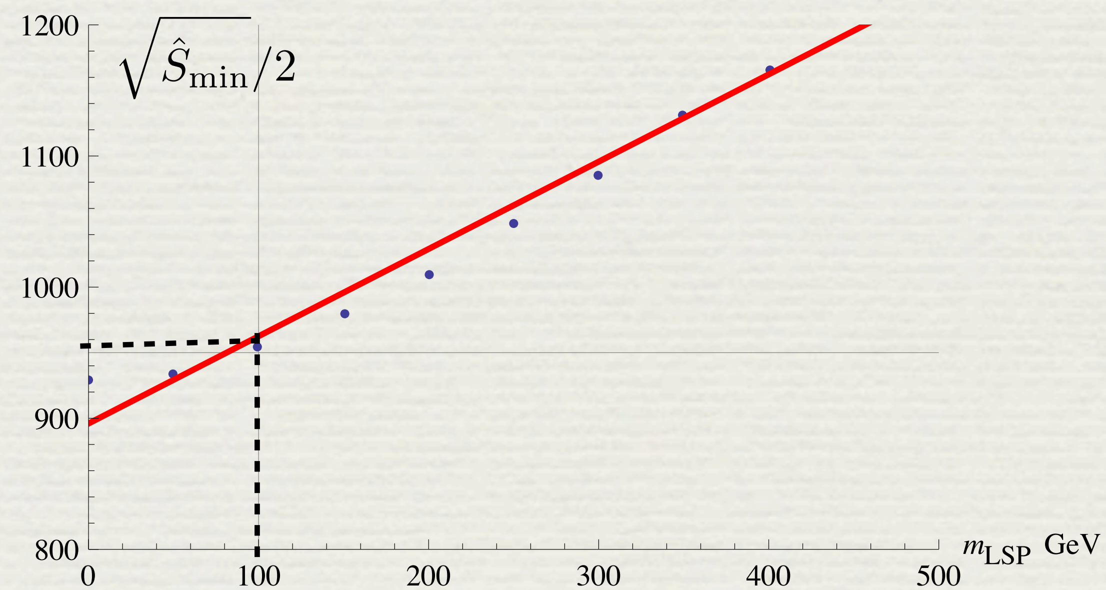
☆ Use the measured LSP mass and cuts



Lower endpoint ~ 960 GeV

- $p_T > 50$ GeV for jet
- $|\eta| < 3$ for jet
- missing E_T cut $E_T^{miss} > 100$ GeV

$p_Z > 300\,\text{GeV}$



Summary and Outlook

- LHC may discovery new physics via large $\not{E}_T$, difficult for mass measurement - key information for studying cosmic relic dark matter
- MET-cone and mtest variable are useful tools for mass measurement in boosted events with $\not{E}_T$.
- Further explore the idea of MET-cone and develop a more general method that can apply for less-collinear events.
- More realistic collider study: include detector effects on MET, initial/final-state radiation etal