Deformations, bions, and (de)confinement

Erich Poppitz   U of T oronto

with Mithat Ünsal   SLAC/Stanford   (2008-present)

also, work in progress
with Mohamed Anber (Toronto)

This talk is about gauge dynamics.

There are many things one would like to understand about any gauge theory:

- does it confine?
- does it break its (super) symmetries?
- is it conformal?
- what are the spectrum, interactions...?

These are tough to address, in almost all theories.

“gauge theory space” conventional wisdom:

pure YM - formal but see www.claymath.org/millennium/

SUSY - very “friendly” to theorists
beautiful - exact results

QCD-like (vectorlike) - hard, leave it to lattice folks
(a,m,V, $)

non-SUSY chiral gauge theories - poorly understood strong dynamics
...(almost) nobody talks about them anymore

“gauge theory space” “applications”:

SUSY

superpartner masses;
supersymmetry breaking
in chiral SUSY theories;
flavor in SUSY

QCD-like (vectorlike)

W, Z-masses:
“walking” or
“conformal”
technicolor

non-SUSY chiral gauge theories

extended technicolor -
fermion mass generation;
quark and lepton compositeness;
& recent speculations of W, Z, t masses by monopole condensation

"gauge theory space"

SUSY → One of the most important "applications" of supersymmetry is to teach us about the many "weird" things gauge field theories could "do" - often very much unlike QCD:

QCD-like (vectorlike)

non-SUSY chiral gauge theories

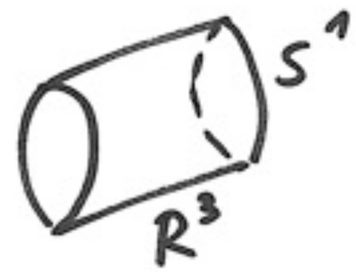
- massless monopole/dyon condensation causing confinement and chiral symmetry breaking
- "magnetic free phases" - dynamically generated gauge fields and fermions
- chiral-nonchiral dualities
- last but not least: gauge-gravity dualities made concrete...

This talk is another example of use of observations first made in SUSY and string theory to non-SUSY gauge dynamics.

What I'll talk about applies to the entire "theory space" above...

The theme of my talk is about inferring properties of infinite-volume theory by studying (arbitrarily) small-volume dynamics.

The small volume may be



most of this talk

or

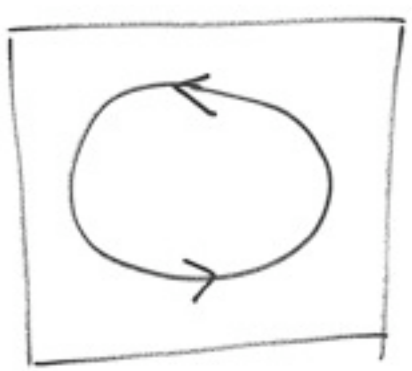
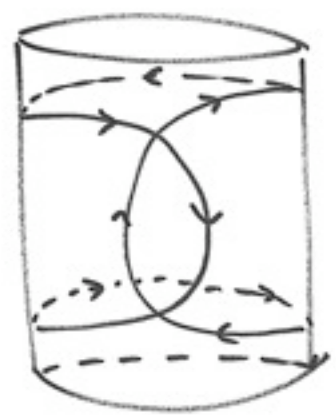
of characteristic size "L"

... isn't this crazy?

To put my talk in context, some relevant history:

Eguchi and Kawai (1982) showed that the infinite set of loop (Schwinger-Dyson) equations for Wilson loops in pure Yang-Mills theory is identical in small-V and infinite-V theory, to leading order in 1/N, **provided**:

- "center-symmetry" unbroken
- translational symmetry unbroken (see Yaffe, 1982)

 =  + O(1/N) **provided**

expectation value of any Wilson loop at infinite-L

expectation value of (folded) Wilson loop at small-L

topologically nontrivial (winding) Wilson loops have vanishing expectation value (= unbroken center)

"EK reduction" or "large-N reduction" or "large-N volume-independence"

Note: this is an exact result in QFT - so long as unbroken center.

It could be potentially exciting, since:

1) **simulations may be cheaper** (use single-site lattice?)
2) **raises theorist's hopes** (that small-L easier to solve?)

To put my talk in context, some relevant history:

From a “modern” point of view EK reduction is a large-N orbifold with respect to the group of translations.

Kovtun, Unsal, Yaffe (2004)

Volume-independence viewed as an orbifold helps establish that VEVs and correlators of operators that are center-neutral and carry momenta quantized in units of 1/L (in compact direction) are the same on, say

as in infinite-L theory, to leading order in 1/N.

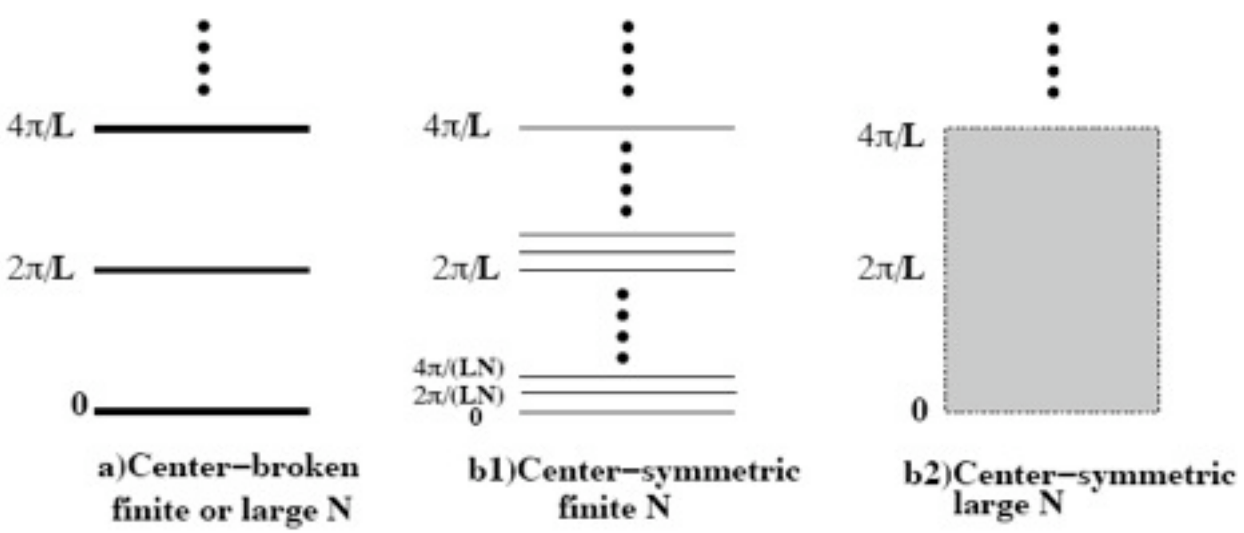
Thus, a working example of EK would be good for

- calculating vevs (symmetry breaking)
  - even if all dimensions small
- calculating spectra (for generic theories/reps)
  - need at least one large dimension

To put my talk in context, some relevant history:

Some intuition of how EK reduction works (note EK valid at any coupling).

in perturbation theory: or
from spectra (& Feynman graphs)
in appropriate background



at strong coupling:

- use lattice strong-coupling expansion

- use gauge-gravity duality:

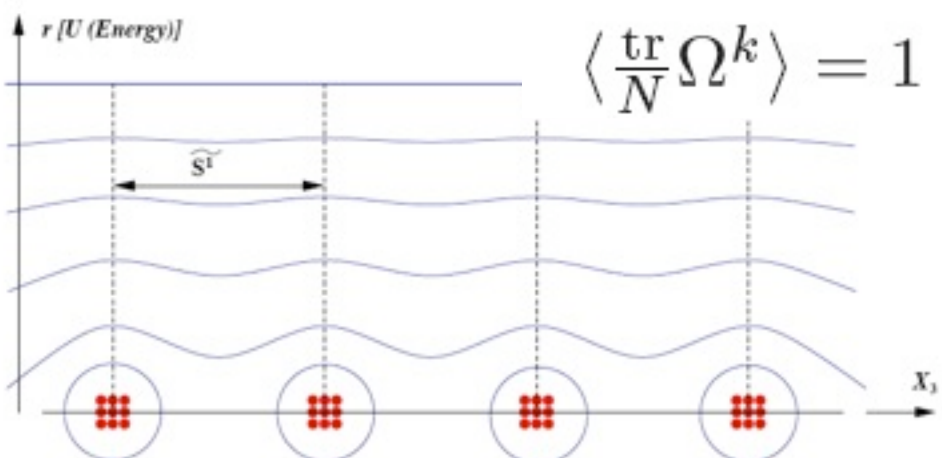
an exact correspondence for large-N
N=4 SYM - a conformal field theory;
since EK also exact, it must be that
non-winding Wilson loops & appropriate
correlators are insensitive to box
if center-symmetric vacuum

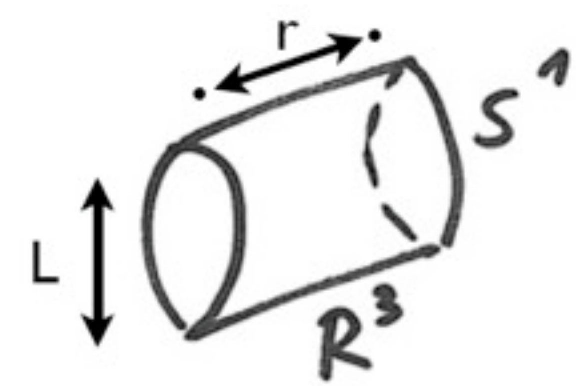
To put my talk in context, some relevant history:

How does volume independence show up in the gravity duals?

V(r) ~ 1/r : CFT result obtains
in center-symmetric vacuum
for any r (<L or >L)
insensitive to box size

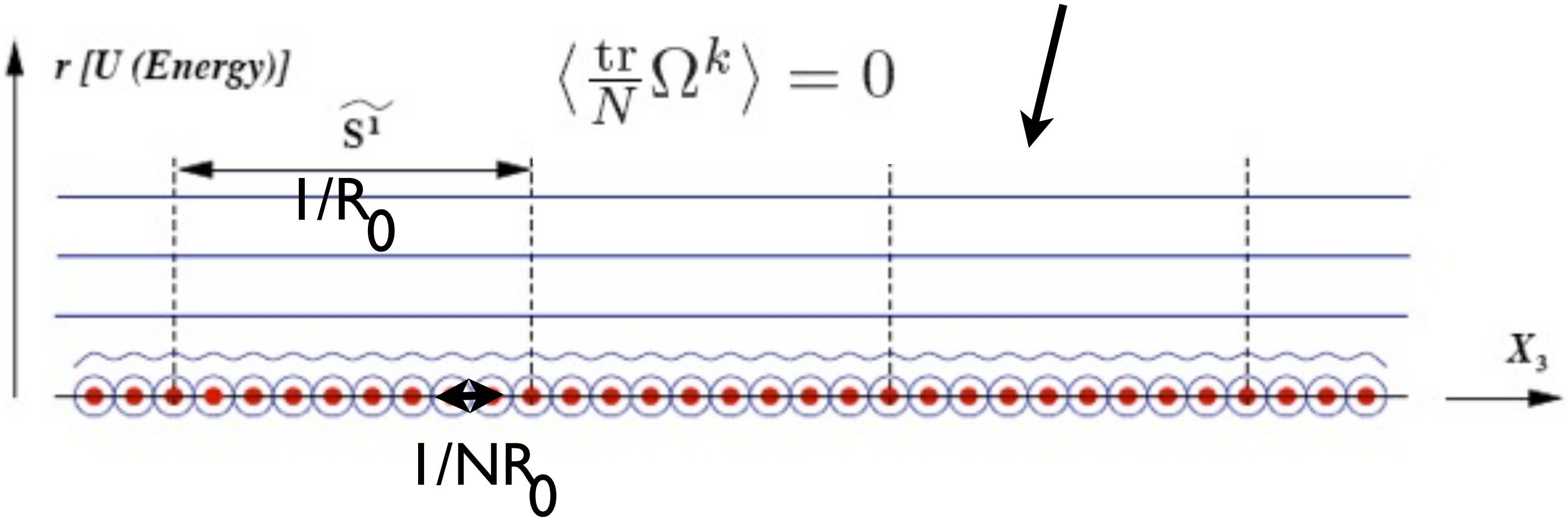
gravity dual of center-broken vacuum





Unsal, EP 2010

gravity dual of center-symmetric vacuum



To put my talk in context, some relevant history:

However, Bhanot, Heller, Neuberger (1982) noticed immediate problem with EK in pure YM, QCD...:

center symmetry breaks for $L < L_c$ (e.g. deconfinement transition)

and thus invalidates EK reduction

Older proposed remedies: e.g., Gonzalez-Arroyo, Okawa (1982) - TEK... + others later argued to have problems (Bringoltz/Sharpe 2009) (some recent "twists" on TEK ?)

A more recent cure is argued to allow reduction valid to arbitrarily small L (e.g., single-site) if one adds either

- periodic adjoint fermions **aka "twisted partition function"** $\mathrm{tr} e^{-\beta H}(-1)^F$

(in SUSY = Witten index)

or

- appropriate double-trace deformations Unsal, Yaffe 2008

To put my talk in context, some relevant history:

Unsal, Yaffe 2008

Remedies proposed: reduction valid to arbitrarily small L (single-site) if:

periodic adjoint fermions (more than one Weyl) - no center breaking, so reduction holds at all L

used for current **lattice studies** of "minimal walking technicolor"

is 4 ...3,5... Weyl adjoint theory conformal or not?

small-L(=1) large-N (~20 or more...) simulations (2009-)
Hietanen-Narayanan; Bringoltz-Sharpe; Catterall et al

small-N large-L simulations (2007-)
Catterall et al; del Debbio et al; Hietanen et al...

... who "wins"?

To put my talk in context, some relevant history:

Unsal, Yaffe 2008

Remedies proposed: reduction valid to arbitrarily small L (single-site) if:

periodic adjoint fermions (more than one Weyl) - no center breaking, so reduction holds at all L

double-trace deformations:
deform measure to prevent center breaking
at infinite-N, deformation does not affect (connected correlators of "untwisted") ***observables***

used for current **lattice studies** of "minimal walking technicolor"

is 4 ...3,5... Weyl adjoint theory conformal or not?

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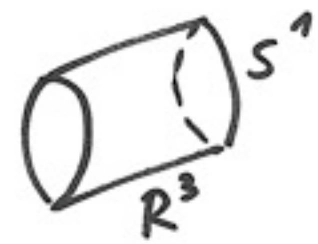
small-N large-L simulations (2007-)
Catterall et al; del Debbio et al; Hietanen et al...

... who "wins"?

THIS TALK:

theoretical studies

Unsal; Unsal-Yaffe; Unsal-Shifman; Unsal-EP 2007-



fix-N, take L-small: semiclassical studies of **confinement** due to novel strange "oddball" (nonselfdual) topological excitations, whose nature depends on fermion content

- for vectorlike or chiral theories, with or without supersymmetry

- **a complementary regime to that of volume independence, which requires infinite N - a (calculable!) shadow of the 4d "real thing".**

First, the key players:

3d Polyakov model & "monopole-instanton"-induced confinement

Polyakov, 1977

"monopole-instantons" on $R^3 \times S^1$

K. Lee, P. Yi, 1997
P. van Baal, 1998

the relevant index theorem

Nye, Singer, 2000
Unsal, EP, 2008

center-symmetry on $R^3 \times S^1$ - adjoint fermions or double-trace deformations

Shifman, Unsal, 2008
Unsal, Yaffe, 2008

"bions", "triplets", "quintets"... - new non-self-dual topological excitations and confinement

Unsal, 2007
Unsal, EP, 2009

First, the key players:

3d Polyakov model & "monopole-instanton"-induced confinement

Polyakov, 1977

continuum picture: 3d Georgi-Glashow [on the lattice - compact U(1)]

$$L \sim \frac{1}{g_3^2}\left(F^a_{\mu\nu} F^a_{\mu\nu} + D_\mu \phi^a D^\mu \phi^a\right) \qquad \mu, \nu = 1, 2, 3$$

$$[A_\mu] = [\phi] = 1 \qquad [g_3^2] = 1$$

due to some Higgs potential $\langle \phi \rangle = (0, 0, v)$

$SU(2) \xrightarrow{v} U(1)$ at low energies, $E \ll m_W \sim v$

free U(1) theory $A^3_\mu \equiv A_\mu$

$$\mathcal{L}_{eff} = \frac{1}{g_3^2} F^2_{\mu\nu} + \ldots$$

"..." are perturbatively calculable & not very interesting

$$B_\mu = \epsilon_{\mu\nu\lambda} F_{\nu\lambda}$$

"magnetic field" - by Bianchi identity, it is also a topologically conserved current of
"emergent topological U(1) symmetry"
responsible for conservation of magnetic charge

$$B_\mu = g_3^2 \, \partial_\mu \sigma$$

3d photon dual to scalar (as one polarization only)

$$\partial_\mu B_\mu = 0 \qquad \text{Abelian duality} \qquad \partial_\mu^2 \sigma = 0$$

Bianchi identity $\longleftrightarrow$ equation of motion

$$\mathcal{L}_{eff} = \frac{1}{g_3^2} F_{\mu\nu}^2 + \ldots \qquad \mathcal{L}_{eff} = g_3^2 (\partial_\mu \sigma)^2 + \ldots$$

topological U(1) symmetry = shift of "dual photon"

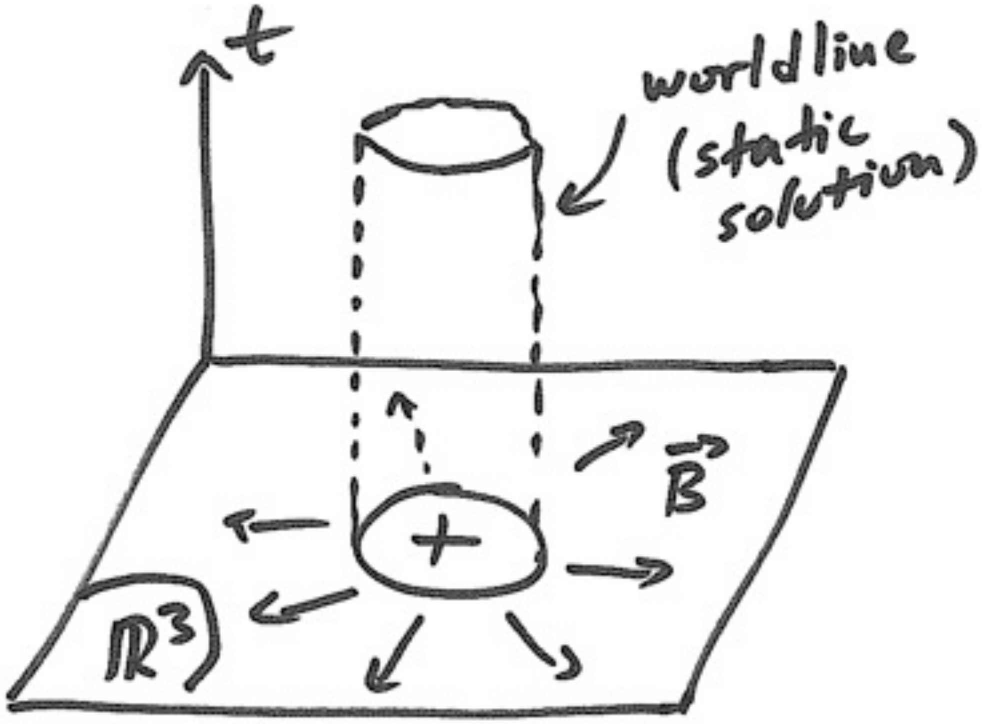
a rather ***"boring-boring" duality*** **-** if not for the existence of monopoles:

monopoles $\partial_\mu B_\mu$ = quantized magnetic charge - shift symmetry broken

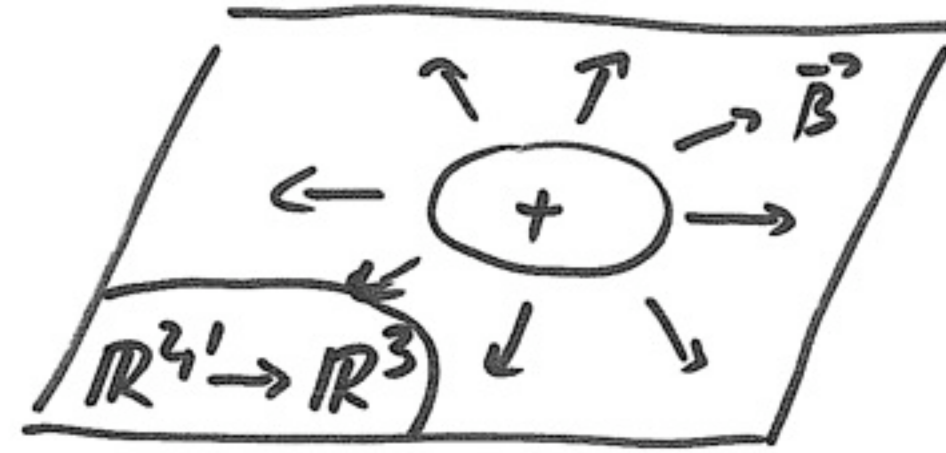
- dual photon gains mass & **electric charges confined**

how?
...in pictures:

“ ‘t Hooft-Polyakov monopole” - *static finite energy solution of Georgi-Glashow model in 4d*



get Euclidean 3d by “forgetting time”



solution of Euclidean eqns. of motion of finite action: a “monopole-instanton”

$$E_M = \frac{4\pi v}{g_4^2} \longrightarrow S_0 = \frac{4\pi v}{g_3^2}$$

$$e^{-S_0} \xrightarrow[g_3^2/v \to 0]{} 0$$

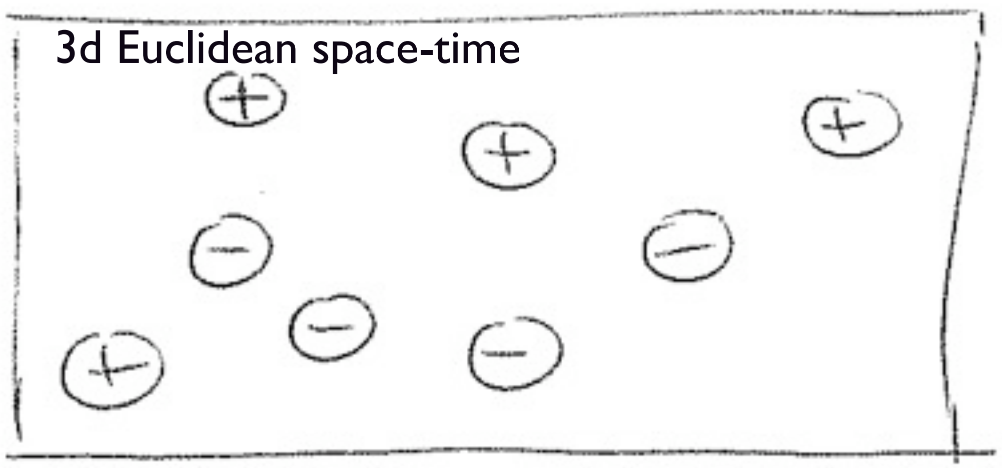
M-M* pairs give exponentially suppressed (at weak coupling) “semiclassical” contributions to the vacuum functional

vacuum “is” a dilute monopole-antimonopole plasma

number of M’s per unit volume ~ $v^3 e^{-S_0}$

(analogous to B+L violation in electroweak model; at T=0 exponentially small)

vacuum is a dilute M-M* plasma - interacting, unlike instanton gas in 4d (in say, electroweak theory)



physics is that of Debye screening

analogy:

electric fields are screened in a charged plasma ("Debye mass for photon"), so in the monopole-antimonopole plasma, the dual photon obtains mass from screening of magnetic field:

$$\mathcal{L}_{eff} = g_3^2 (\partial\sigma)^2 + (\#)\, v^3 e^{-S_0}\left(e^{i\sigma} + e^{-i\sigma}\right) + \ldots$$

"(anti-)monopole operators"

aka **"disorder operators"** - not locally expressed in terms of original gauge fields (Kadanoff-Ceva; 't Hooft - 1970s)

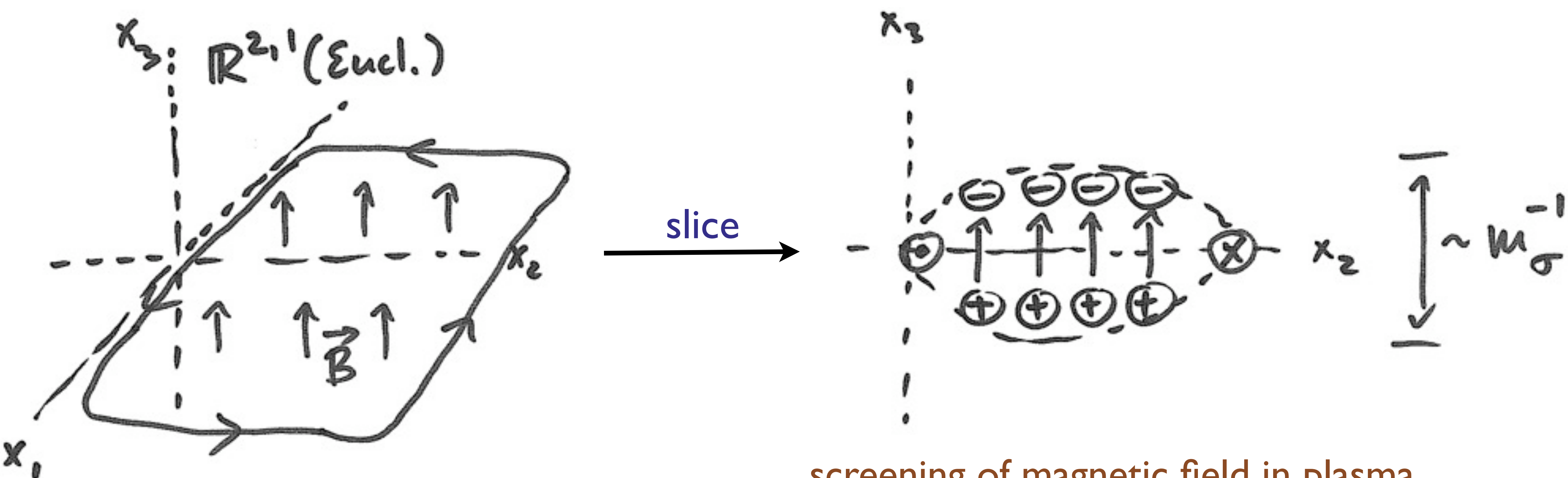
also by analogy with Debye mass:

dual photon mass^2 ~ M-M* plasma density

$$m_\sigma \sim v\, e^{-S_0/2} = v\, e^{-\frac{4\pi v}{2 g_3^2}}$$

now, dual photon mass ~ confining string tension:

in pictures:



screening of magnetic field in plasma

= Wilson loop area law: $\left\langle e^{i\oint A\,dx} \right\rangle \sim e^{-(\text{Area})\, m_\sigma g_3^2}$

Minkowski space interpretation of Wilson loop

confining flux tube: **tension^{-1} ~ thickness ~ inverse dual photon mass**

First, the key players:

3d Polyakov model & "monopole-instanton"-induced confinement — Polyakov, 1977

"monopole-instantons" on $R^3 \times S^1$ — K. Lee, P. Yi, 1997; P. van Baal, 1998

the relevant index theorem — Nye, Singer, 2000; Unsal, EP, 2008

center-symmetry on $R^3 \times S^1$ - adjoint fermions or double-trace deformations — Shifman, Unsal, 2008; Unsal, Yaffe, 2008

"bions", "triplets", "quintets"... - new non-self-dual topological excitations and confinement — Unsal, 2007; Unsal, EP, 2009

First, the key players:

we want to go to 4d - by "growing" a compact dimension: $S^1: \quad x^4 \sim x^4 + L$

"monopole-instantons" on $R^3 \times S^1$

K. Lee, P. Yi, 1997
P. van Baal, 1998

A_4 is now an adjoint 3d scalar Higgs field $\quad \partial_4 + A_4 \longrightarrow \frac{2\pi n}{L} + A_4$

but it is a bit unusual -
a compact Higgs field: $\quad \langle A_4 \rangle \sim \langle A_4 \rangle + \frac{2\pi}{L}$

such shifts of A_4 vev absorbed into shift of KK number "n" $\quad A_4 \to A_4 + \partial_4\left(\frac{2\pi x_4}{L}\right)$

"large" gauge transform

thus, natural
scale of "Higgs vev" is $\langle A_4 \rangle \sim \frac{\pi}{L}$ leading to $SU(2) \xrightarrow{\frac{1}{L}} U(1)$

(clearly, semiclassical and weakly coupled if L << inverse strong scale)

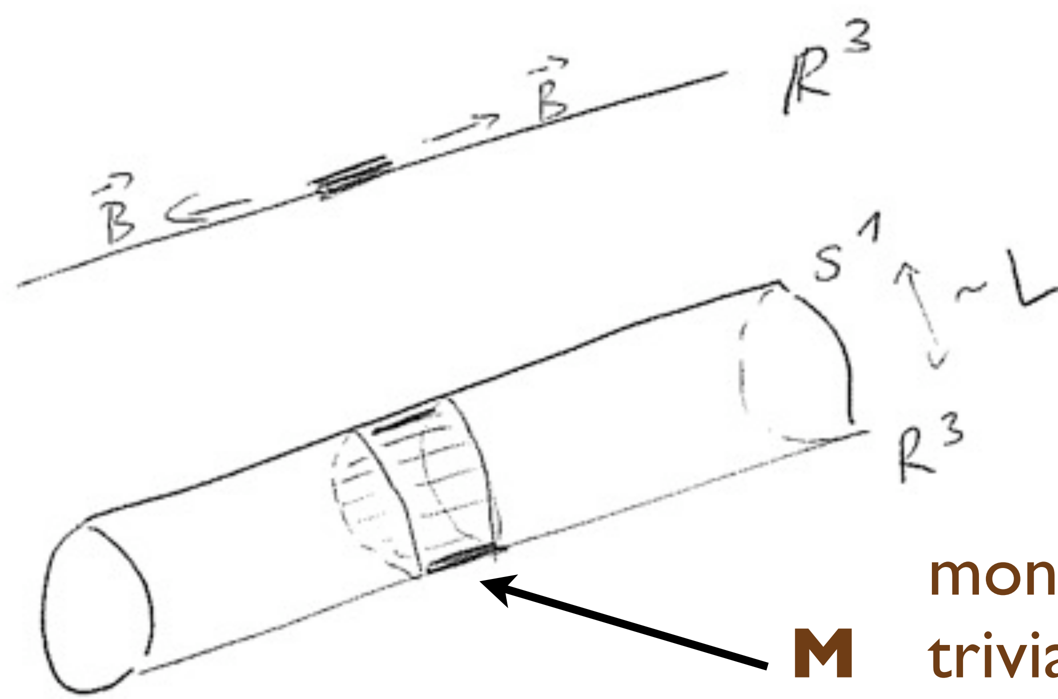
$$W = P\, e^{i \oint_{S_1} A_4\, dx^4}$$ if the expectation values are $\langle A_4^3 \rangle \sim \frac{\pi}{L}$

$$\langle W \rangle = \begin{pmatrix} e^{i\pi/2} & \\ & e^{-i\pi/2} \end{pmatrix}$$ then $\mathrm{tr}\langle W \rangle = 0$

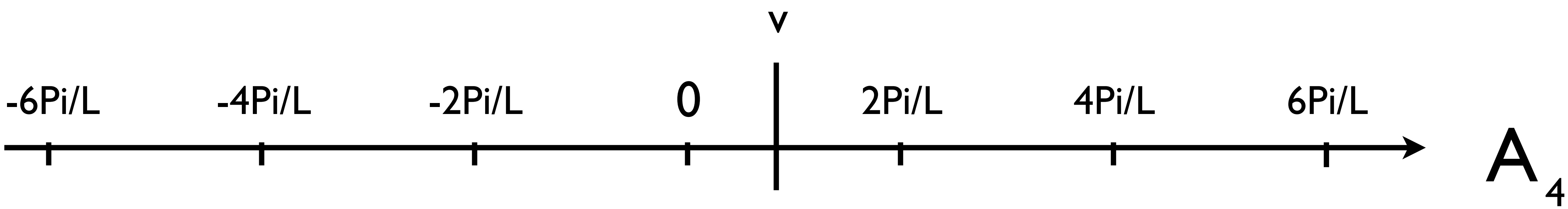
center symmetry is preserved $\mathrm{tr}\,W \to e^{i\pi}\,\mathrm{tr}\,W$ for SU(N): $e^{i\frac{2\pi}{N}}$

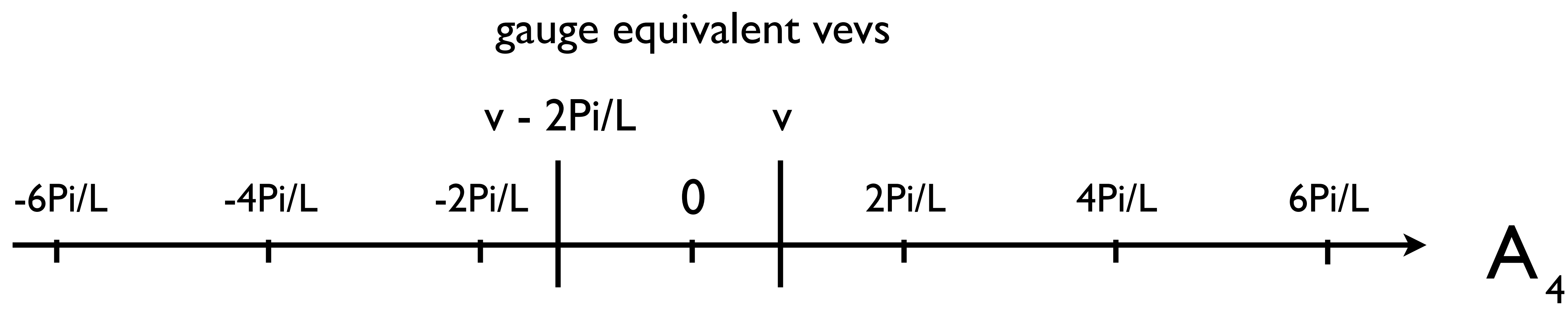
we are interested in **unbroken center (or "almost unbroken")**:
where <trW>=0 and SU(2) broken to U(1) at scale 1/L

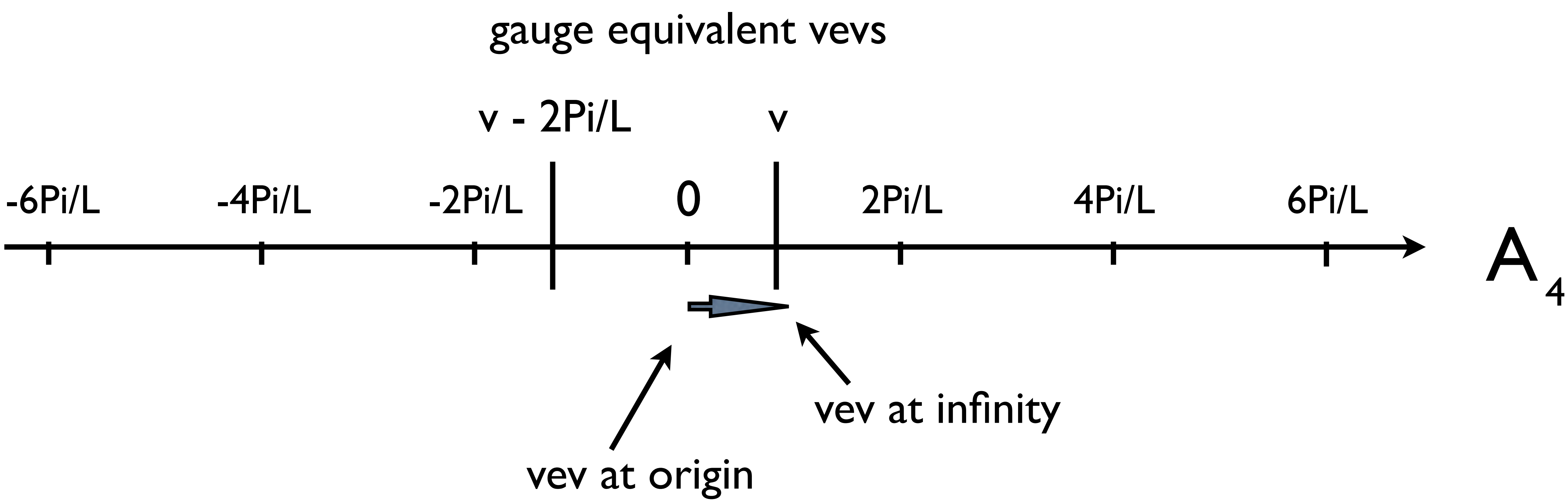
at small L, physics semiclassical and there are monopole-instantons, for example:



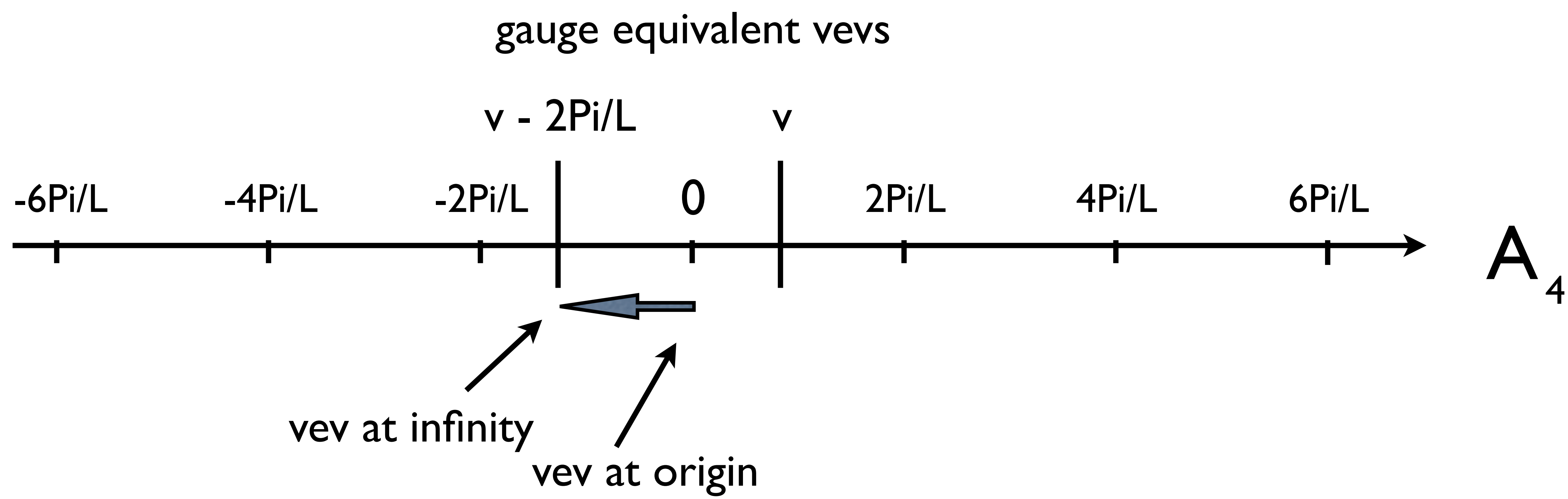
monopole trivially embedded in 4d


v
-6Pi/L
-4Pi/L
-2Pi/L
0
2Pi/L
4Pi/L
6Pi/L
A 4


gauge equivalent vevs
v - 2Pi/L
v
-6Pi/L
-4Pi/L
-2Pi/L
0
2Pi/L
4Pi/L
6Pi/L
A 4

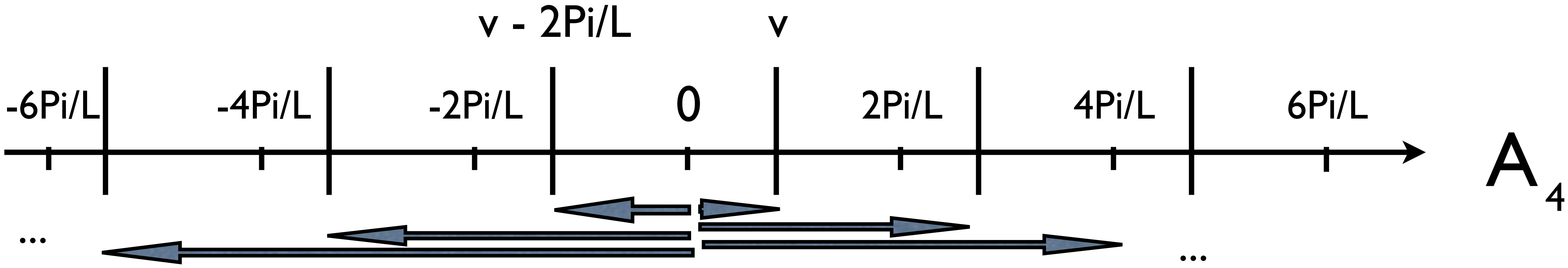


monopole-instanton of action ~ v/g_3^2



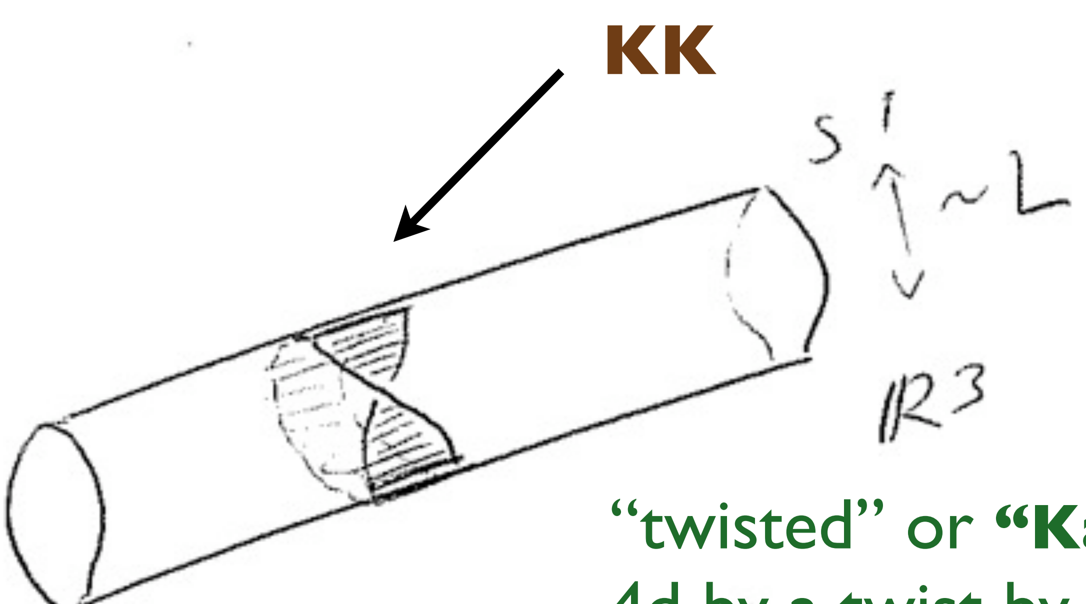
monopole-instanton of action ~ $|2\ Pi/L - v|/g_3^2$

gauge equivalent vevs



monopole-instanton tower; action ~ $|2\ k\ Pi/L - v|/g_3^2$

the lowest action member of the tower can be pictured like this (as opposed to the no-twist):



"twisted" or **"Kaluza-Klein":** monopole embedded in 4d by a twist by a "gauge transformation" periodic up to center - in 3d limit not there! (infinite action)

KK discovered by Kimyeong Lee, Piljin Yi, 1997, as

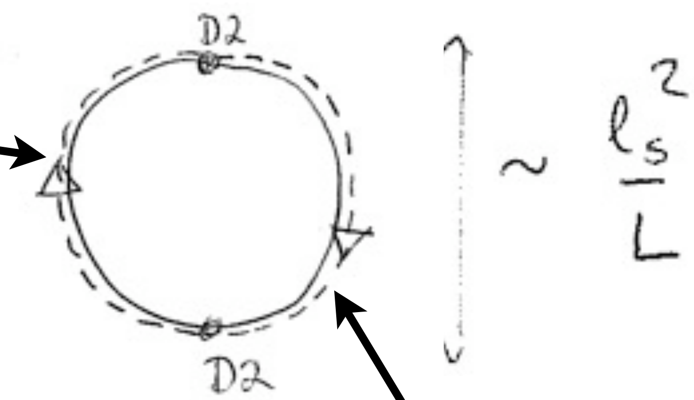
“Instantons and monopoles on partially compactified D-branes”

- can also understand in QFT, as alluded to above

- possibility mentioned - not studied - by Kronfeld, Schierholz, Wiese 1987

	magnetic	topological	suppression
M	+1	1/2	e^{-S_0}
KK	−1	1/2	e^{-S_0}
BPST	0	1	e^{-2S_0}

M
Euclidean D0-brane



KK
Euclidean D0-brane

lowest action M & KK are both self-dual objects, of opposite magnetic charges

+ their anti-”particles”

note, BPST instanton “ = M+KK ” (also see P. van Baal, 1998)

M & KK have 't Hooft suppression given by:

$$e^{-S_0} = e^{-\frac{4\pi v}{g_3^2}} \underset{\uparrow}{=} e^{-\frac{4\pi^2}{L g_3^2}} \underset{\uparrow}{=} e^{-\frac{4\pi^2}{g_4^2(L)}}$$

center-symmetric vev   coupling matching

$$SU(N): \quad e^{-S_0} = e^{-\frac{8\pi^2}{g_4^2(L) N}}$$

(large-N survive!)

M & KK have, in SU(N), 1/N-th of the 't Hooft suppression factor aka:
"fractional instantons", "instanton quarks", "zindons", "quinks", "instanton partons"... [collected by D. Tong]

Next, to understand the role M, KK, M* & KK* play in various theories of interest, need to know what happens to the operators they induce when there are fermions in the theory.

First, the key players:

3d Polyakov model & "monopole-instanton"-induced confinement — Polyakov, 1977

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the relevant index theorem — Nye, Singer, 2000; Unsal, EP, 2008

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First, the key players:

the relevant index theorem	Nye, Singer, 2000 Unsal, EP, 2008

- for some theories the answer for the number of zero modes in M or KK background had been guessed -
  correctly, e.g. in SUSY YM - Aharony, Hanany, Intriligator, Seiberg, Strassler, 1997

- while studying ISS(henker) proposal for SUSY breaking model [SU(2)+three-index symmetric tensor Weyl fermion] Unsal and I needed a general index theorem

- we found this:

Journal of Functional Analysis **177**, 203–218 (2000)
doi:10.1006/jfan.2000.3648, available online at http://www.idealibrary.com on IDEAL®

An L^2-Index Theorem for Dirac Operators on $S^1 \times \mathbb{R}^3$

Tom M. W. Nye and Michael A. Singer

where, in APPENDIX A. ADIABATIC LIMITS OF η-INVARIANTS

we found:

$$\text{ind}\,(D^+_{\mathbb{A}}) = \int_X \text{ch}(\mathbb{E}) + \frac{1}{\mu_0}\sum_{\mu} \epsilon_\mu c_1(E_\mu)[S^2_\infty]$$

$$= \int_X \text{ch}(\mathbb{E}) - \frac{1}{2}\overline{\eta}_{\text{lim}}$$

(last formula in paper)

two obvious questions:

1.) where does this come from?

2.) what number is it equal to in a given topological background (M,KK...)
& how does it depend on ratio of radius to holonomy?

for answers & more

see M. Unsal, EP 0812.2085

like on R^3 Callias $\xleftarrow{\text{physicist derivation}}$ E. Weinberg, 1970s, but on $R^3 \times S^1$, so must incorporate anomaly equation, some interesting effects

For this talk only consider 4d SU(2) theories with N_W adjoint Weyl fermions → *"applications":*

$N_W=1$ is N=1 SUSY YM

$N_W=4$

- "minimal walking technicolor"
- happens to be N=4 SYM without the scalars

M KK M* KK* each have $2N_W$ zero modes

disorder operators:

M: $e^{-S_0}\, e^{i\sigma}\, (\lambda\lambda)^{N_W}$

KK: $e^{-S_0}\, e^{-i\sigma}\, (\lambda\lambda)^{N_W}$

M*: $e^{-S_0}\, e^{-i\sigma}\, (\bar\lambda\bar\lambda)^{N_W}$

KK*: $e^{-S_0}\, e^{i\sigma}\, (\bar\lambda\bar\lambda)^{N_W}$

where:

$$(\lambda\lambda)^{N_W} = \underset{\underset{SU(N_W)}{\uparrow}}{\det_{I,J}}\ \lambda^{\overset{SU(2)}{\swarrow}a}_{\alpha I}\ \lambda^{a}_{\underset{\underset{SL(2,\mathbb{C})}{\uparrow}}{\beta} J}\ \epsilon^{\alpha\beta}$$

remarks:

- operator due to M+KK = 't Hooft vertex; independent of dual photon
- "our" index theorem interpolates between 3d Callias and 4d APS index thms.

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Shifman, Unsal, 2008
Unsal, Yaffe, 2008

"bions", "triplets", "quintets"... - new non-self-dual topological excitations and confinement

Unsal, 2007
Unsal, EP, 2009

First, the key players:

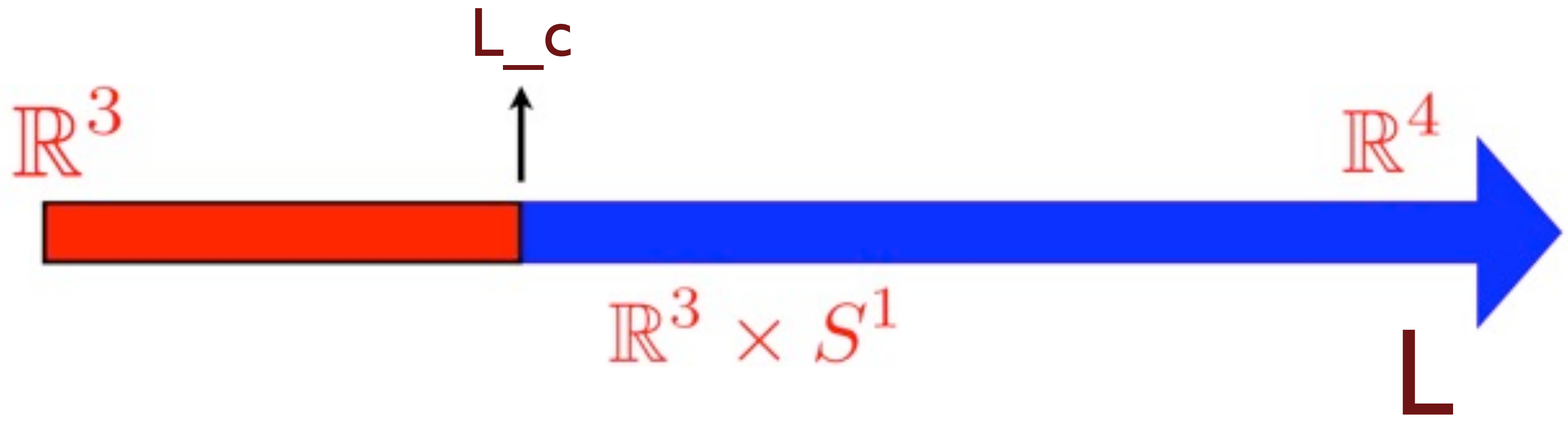
- Abelianization occurs only if there is a nontrivial holonomy (i.e., A_4 has vev)

$$\langle A_4 \rangle \sim \frac{\pi}{L}$$

- upon thermal circle compactifications, gauge theories with fermions do not Abelianize: center symmetry is broken at small circle size - transition to a deconfining phase - $A_4 = 0$, $\langle trW \rangle \neq 0$ - deconfinement - at high-T, 1-loop V_{eff}
  (Gross, Pisarski, Yaffe, early1980s)

center-symmetry on $R^3 \times S^1$ - adjoint fermions or double-trace deformations

Shifman, Unsal, 2008
Unsal, Yaffe, 2008



to ensure calculability at small L and smooth connection to large L in the sense of center symmetry: *can one find ways to avoid phase transition?*

I. non-thermal compactifications - periodic fermions ("twisted partition function")

- with N_W>1 adjoint fermions center symmetry preserved (Unsal, Yaffe 2007) as well as with other, "exotic" fermion reps (Unsal, EP 2009)
- in many supersymmetric theories, can simply choose center-symmetric vev

II. add double-trace deformations: force center symmetric vacuum at small L (also Shifman, Unsal 2008) - connection to large-N volume independence

In what follows, I'll assume center-symmetric vacuum - due to either I. or II. - will explicitly discuss only theory where center symmetry is naturally preserved at small L (**I**.)

First, the key players:

3d Polyakov model & "monopole-instanton"-induced confinement	Polyakov, 1977
"monopole-instantons" on $R^3 \times S^1$	K. Lee, P. Yi, 1997 P. van Baal, 1998
the relevant index theorem	Nye, Singer, 2000 Unsal, EP, 2008
center-symmetry on $R^3 \times S^1$ - adjoint fermions or double-trace deformations	Shifman, Unsal, 2008 Unsal, Yaffe, 2008

"bions", "triplets", "quintets"... - new non-self-dual topological excitations and confinement

Unsal, 2007
Unsal, EP, 2009

First, the key players:

Now ready to study the dynamics of theories with massless fermions on a small circle

in a vacuum with A_4 vev, Abelianization:

- in SU(2): (dual) photon massless + fermion components w/out mass from vev (neutral)
- monopoles + KK monopoles are the basic topological excitations

is there magnetic field screening in the vacuum?

the answer would appear to be "no":

M and KK have fermion zero modes

monopole operators do not generate potential for dual photon

so, **no screening & no confinement... ?**

"bions", "triplets", "quintets"... - new non-self-dual topological excitations and confinement

Unsal, 2007
Unsal, EP, 2009

but take a look at the symmetries first:

as an example, again
consider 4d SU(2) theories
with N_w adjoint Weyl fermions

classical global chiral symmetry is

$SU(N_w) \times U(1)$

but 't Hooft vertex $(\lambda\lambda)^{2N_w} e^{-\frac{8\pi^2}{g_4^2}}$ only preserves $\mathbb{Z}_{4N_w}: \lambda \to e^{i\frac{2\pi}{4N_w}} \lambda$

so, quantum-mechanically we have only $SU(N_w) \times Z_{4N_w}$ exact chiral symmetry

now **M, KK**(+*) operators all look like: $e^{-S_0} e^{i\sigma} (\lambda\lambda)^{N_w}$

hence $(\lambda\lambda)^{N_w} \xrightarrow{\mathbb{Z}_{4N_w}} e^{i\pi} (\lambda\lambda)^{N_w}$

invariance of **M**, **KK**(+*) operators under exact chiral symmetry means that

dual photon must transform under the exact chiral symmetry

i.e., topological shift symmetry is intertwined with chiral symmetry:

$$\mathbb{Z}_{4N_w}: \quad \sigma \to \sigma + \pi$$

$\sigma \to \sigma + \pi$ $\qquad \cancel{\cos \sigma} \qquad \cos(2\sigma)$ ✓

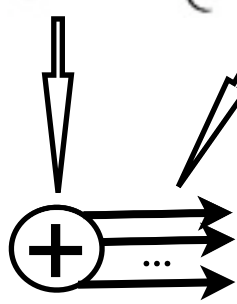
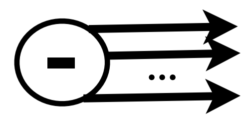
so the exact chiral symmetry allows a potential - **but what is it due to?**

$$e^{-S_0}\, e^{i\sigma} (\lambda\lambda)^{N_w}$$

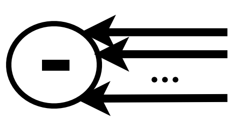
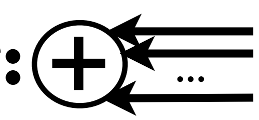
to generate $\cos(2\sigma)$ disorder operator, topological excitations must have

i. magnetic charge 2

ii. no fermion zero modes

M:  **KK:** 

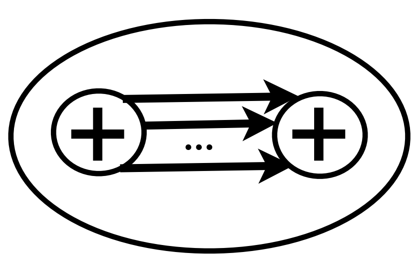
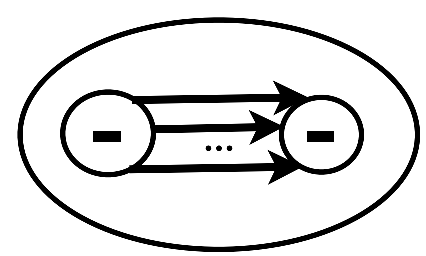
M + KK* bound state? (Unsal, 2007)

M*:  **KK*:** 

- same magnetic charge ~ 1/r-repulsion
- fermion exchange ~ log(r)-attraction

M + KK* = B - magnetic "bion"

size ~ $L/g_4^2(L) \gg L$ (our "lattice spacing")

B:  **B*:** 

$$e^{-2S_0}\left(e^{2i\sigma} + e^{-2i\sigma}\right)$$

dual photon mass is induced by magnetic "bions"- the leading cause of confinement in SU(N) with adjoints at small L (incl. SYM)

to summarize, in QCD(adj),

M + KK* = B - magnetic "bions" -

-carry magnetic charge

-no topological charge (non self-dual)

(locally 4d nature crucial: no KK in 4d)

-generate "Debye" mass for dual photon

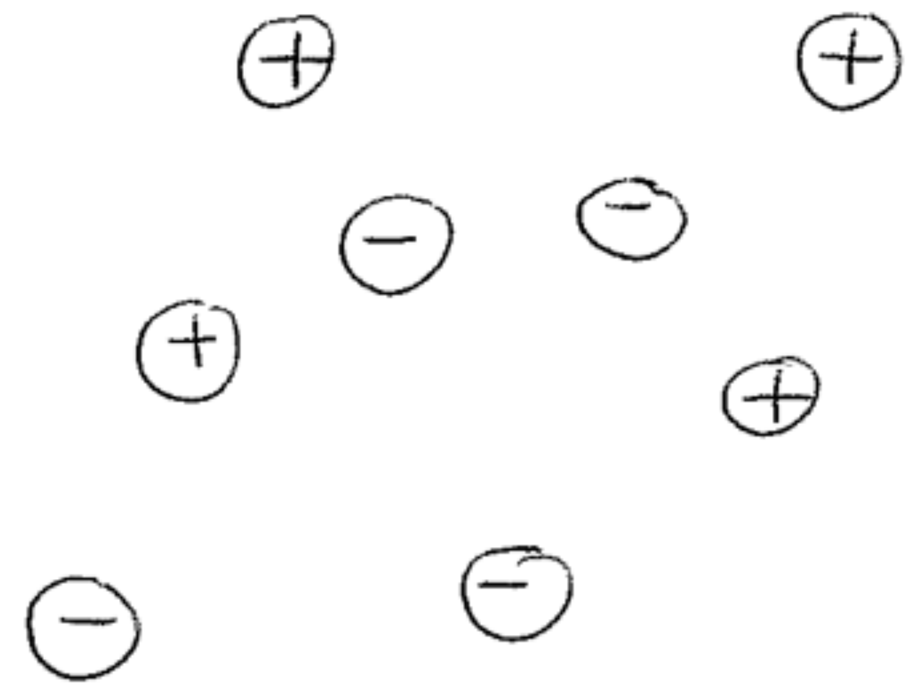
main tools used:

- intertwining of topological shift symmetry & chiral symmetry
- index theorem

topological objects generating magnetic screening depend on massless fermion content (not usually thought that fermions relevant)

to summarize, in QCD(adj),

3d pure gauge theory vacuum monopole plasma
Polyakov 1977



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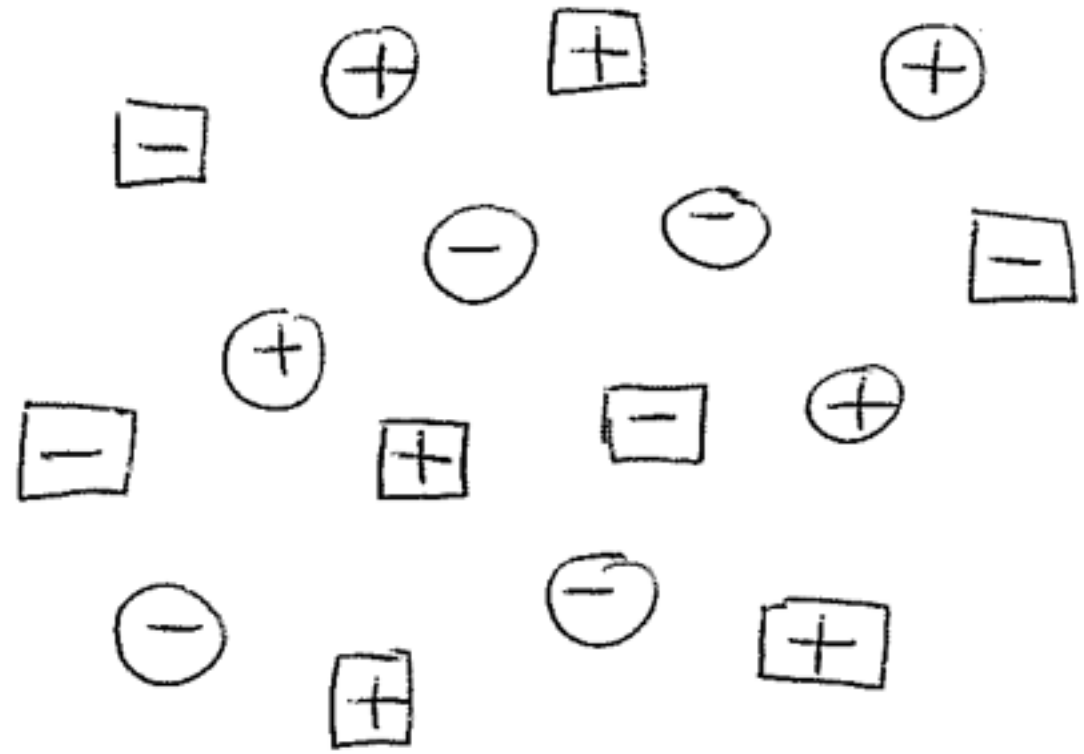
to summarize, in QCD(adj),

4d pure YM with "double-trace deformation" at small-L
Unsal-Yaffe 2008



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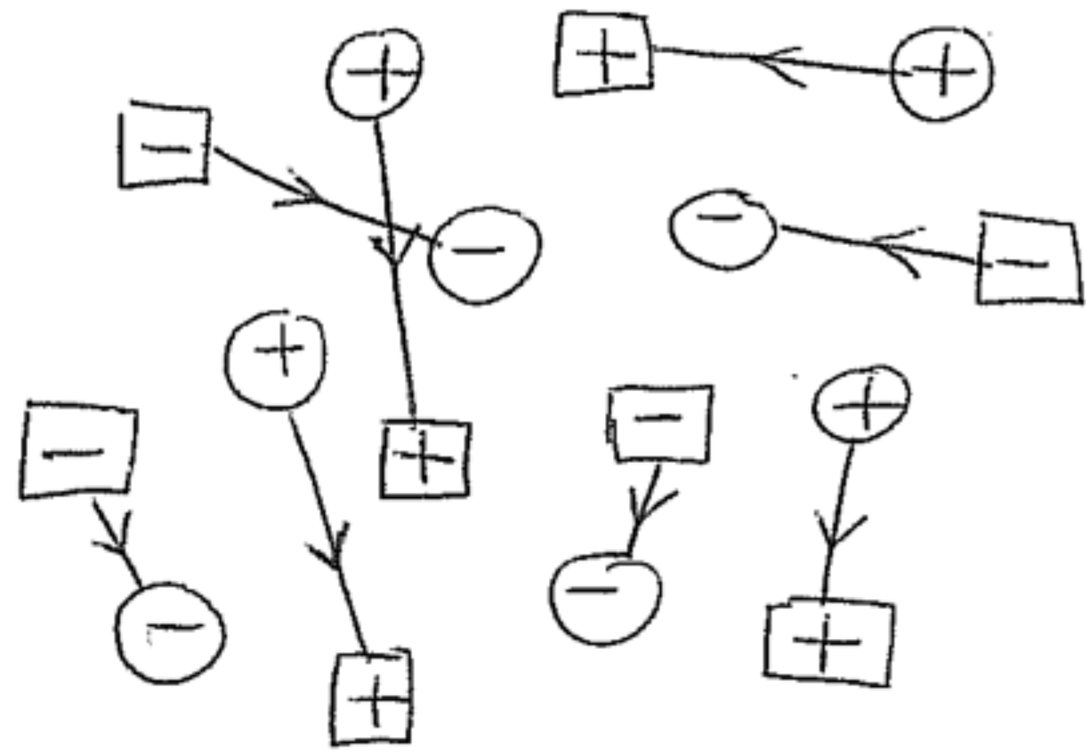
to summarize, in QCD(adj),

4d QCD(adj) fermion attraction M-KK at small-L
Unsal 2007, ...



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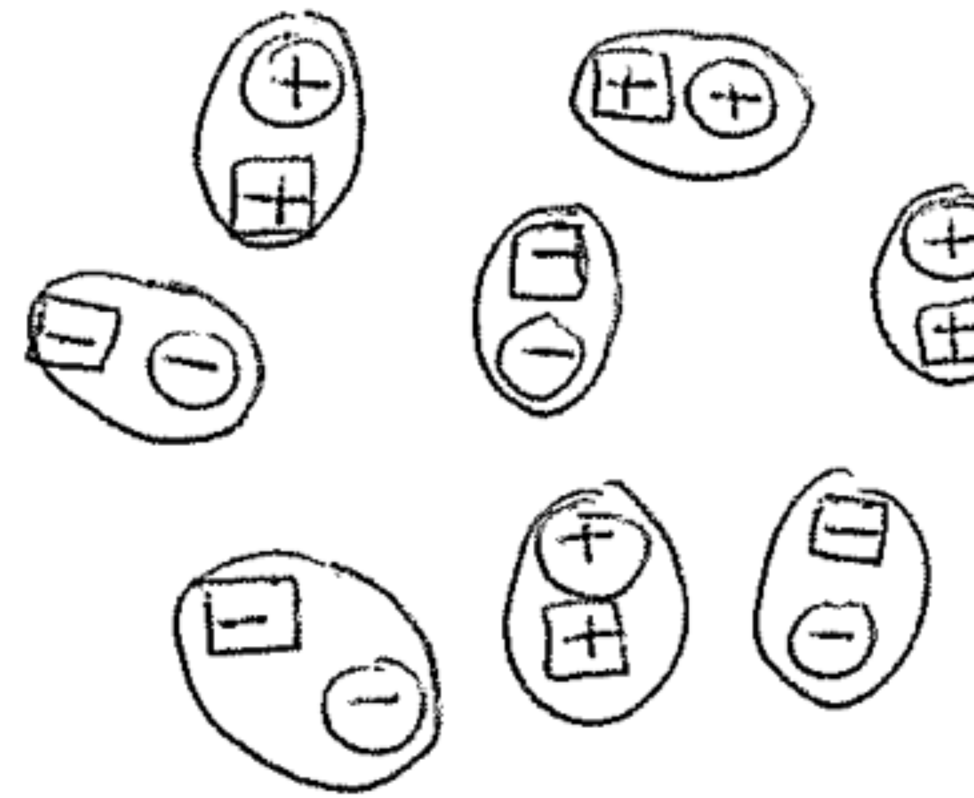
4d QCD(adj) bion plasma at small-L
Unsal 2007, ...



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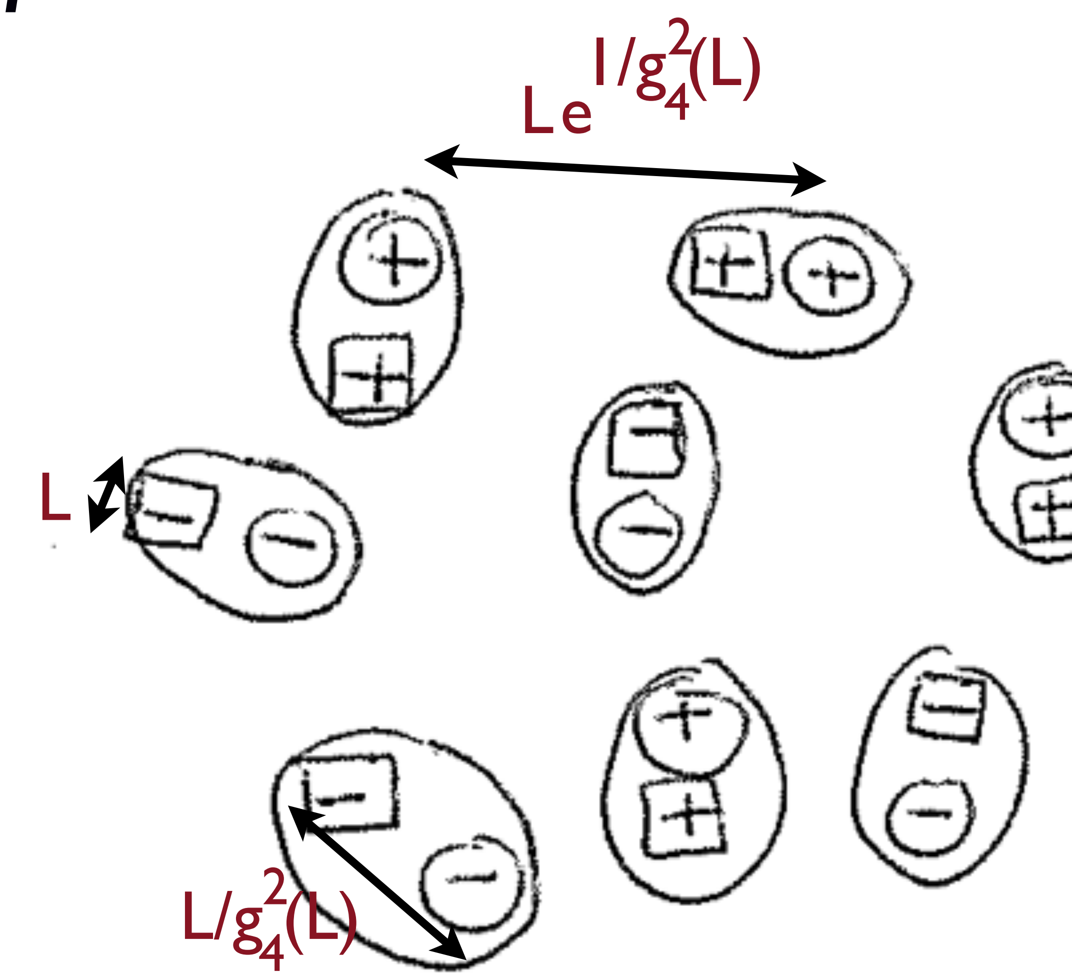
Unsal 2007, ...



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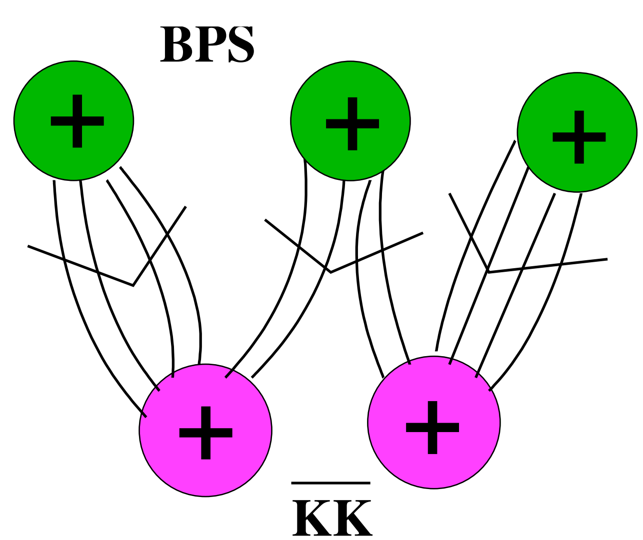
topological objects generating magnetic screening depend on massless fermion content (not usually thought that fermions relevant)

using these tools, one can analyze any theory...

in the last couple of years, many theories have been studied...

vectorlike | chiral

Theory all SU(N)	Confinement mechanism on $\mathbb{R}^3 \times S^1$	Index for monopoles $[\mathcal{I}_1, \mathcal{I}_2, \ldots, \mathcal{I}_N]$ Nye-M.Singer '00; PU '08	Index for instanton $I_{inst.} = \sum_{i=1}^{N} I_i$ Atiyah-Singer	$(\text{Mass Gap})^2$ units $\sim 1/L^2$
YM Y,U '08	monopoles	$[0, \ldots, 0]$	0	e^{-S_0}
QCD(F) S,U '08	monopoles	[illegible]	[illegible]	[illegible]
SYM /QCD(Adj) U '07	magnetic bions			
QCD(BF) S,U '08	magnetic bions			
QCD(AS) S,U '08	bions a[illegible] monopoles			
QCD(S) P,U '09	bions a[illegible] triplets			
SU(2)YM $I = \frac{3}{2}$ P,U '09	magnetic quintets	$[4, 6]$ SUSY version: ISS(henker) model of SUSY [non-]breaking	10	e^{-5S_0}
chiral $[SU(N)]^K$ S,U '08	magnetic bions	$[2, 2, \ldots, 2]$	$2N$	e^{-2S_0}
$AS + (N-4)\overline{\text{F}}$ S,U '08	bions and a monopole	$[1, 1, \ldots, 1, 0, 0]$ + $[0, 0, \ldots, 0, N-4, 0]$	$(N-2)\text{AS} + (N-4)\overline{\text{F}}$	$e^{-2S_0}, e^{-S_0},$
$S + (N+4)\overline{\text{F}}$ P,U '09	bions and triplets	$[1, 1, \ldots, 1, 2, 2]$ + $[0, 0, \ldots, 0, N+4, 0]$	$(N+2)\text{S} + (N+4)\overline{\text{F}}$	$e^{-2S_0}, e^{-3S_0},$



cartoon of the "magnetic quintet:" the leading cause of mass gap for the dual photon in non-SUSY chiral SU(2) with I=3/2

Table 1. Topological excitations which determine the mass gap for gauge fluctuations and chiral symmetry realization in vectorlike and chiral gauge theories on $\mathbf{R}^3 \times \mathbf{S}^1$. Unless indicated otherwise,

name codes:

U=Unsal
S=Shifman
Y=Yaffe
P=the speaker

+ SO(N),SP(N) - S.Golkar 0909.2838 (?) Argyres, Unsal - in progress; mixed-/higher-index reps.-P,U 0910.1245

So, I have now introduced all the key players:

3d Polyakov model & "monopole-instanton"-induced confinement

Polyakov, 1977

"monopole-instantons" on $R^3 \times S^1$

K. Lee, P.Yi, 1997
P. van Baal, 1998

the relevant index theorem

Nye, Singer, 2000
Unsal, EP, 2008

center-symmetry on $R^3 \times S^1$ - adjoint fermions or double-trace deformations

Shifman, Unsal, 2008
Unsal, Yaffe, 2008

"bions", "triplets", "quintets"... - new non-self-dual topological excitations and confinement

Unsal, 2007
Unsal, EP, 2009

The upshot is the **dual lagrangian of QCD(adj)** on a circle of size L:

$$\frac{g^2(L)}{2L}(\partial\sigma)^2 - \frac{b}{L^3}\, e^{-2S_0}\cos 2\sigma + i\bar{\lambda}^I\gamma_\mu\partial_\mu\lambda_I + \frac{c}{L^{3-2N_f}}\, e^{-S_0}\cos\sigma(\det_{I,J}\lambda^I\lambda^J + \text{c.c.})$$

B, B*

M, KK+*

leading-order perturbation theory; perturbative corrections ~ $g_4(L)^2$ omitted

$$m_\sigma \sim \frac{1}{L} e^{-S_0} = \frac{1}{L} e^{-\frac{8\pi^2}{N_c g_4^2(L)}}$$

to leading exponential accuracy

$$m_\sigma = \frac{1}{L}(\Lambda L)^{\frac{\beta_0}{N_c}} = \Lambda(\Lambda L)^{\frac{\beta_0}{N_c}-1} = \Lambda(\Lambda L)^{\frac{8-2N_W}{3}}$$

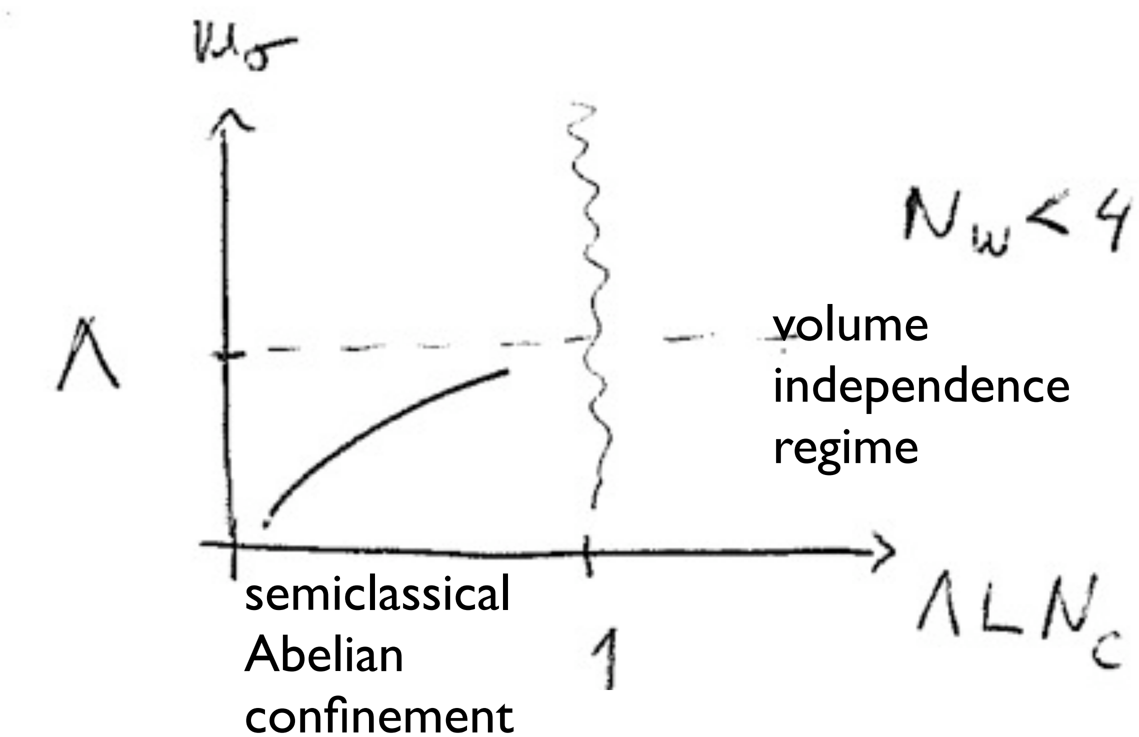
behavior of mass gap

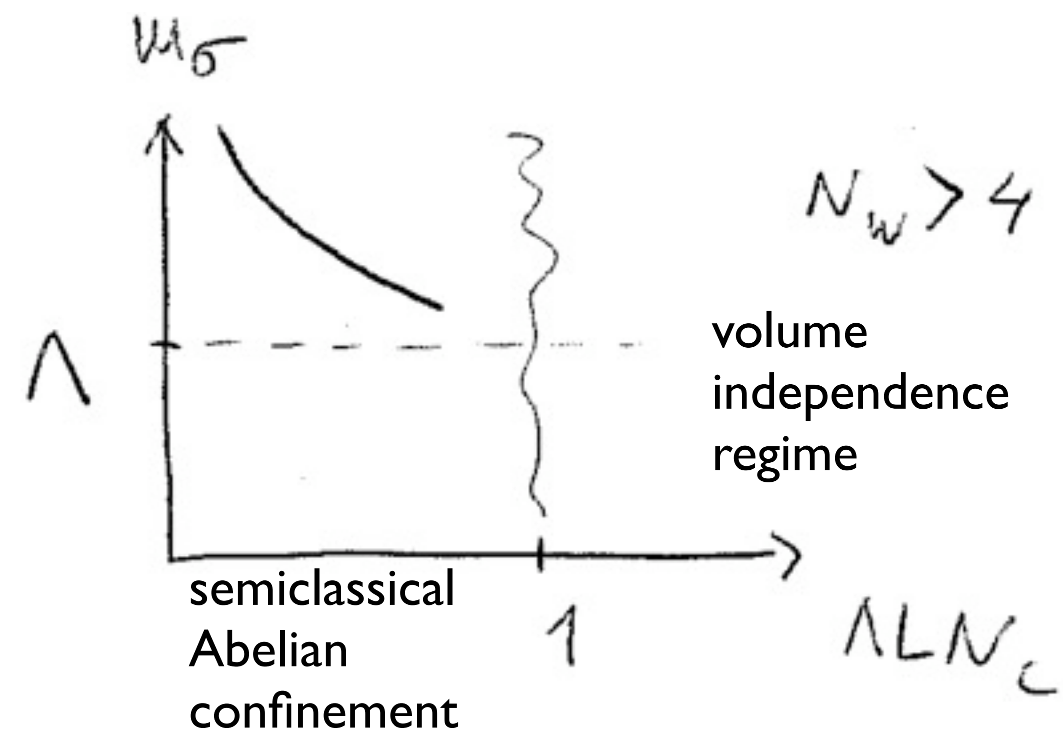
as L changes at fixed Λ ... $N_W^* = 4$?

Staying honest, for now, recall region of validity of semiclassical analysis:

$$\Lambda L \ll 1 \qquad (N_c \Lambda L \ll 1 \text{, really})$$ as mass of W ~ 1/(NL)

$$M_\sigma \sim \Lambda (\Lambda L)^{(8-2N_w)/3}$$ × pre-exp. factors (M. Anber, EP, to appear); relevant for 4-adjoint case





analysis shows that this switch of behavior as number of fermion species is increased occurs in all theories - vectorlike or chiral alike

in each case we obtain a value for the critical number of "flavors" or "generations"... N_f^*

like $N_w^* = 4$ for QCD(adj)

Does it tell us anything about R^4 ?

Leaving honesty aside, it is very tempting to continue the lines in their "natural" direction...

... how **dare** you study
non-protected quantities?

It is very tempting to continue the lines in their “natural”direction;
not a defensible position, but hardly unique...

Some circumstantial “inspirational evidence” in earlier “twisted-EK-type” work:

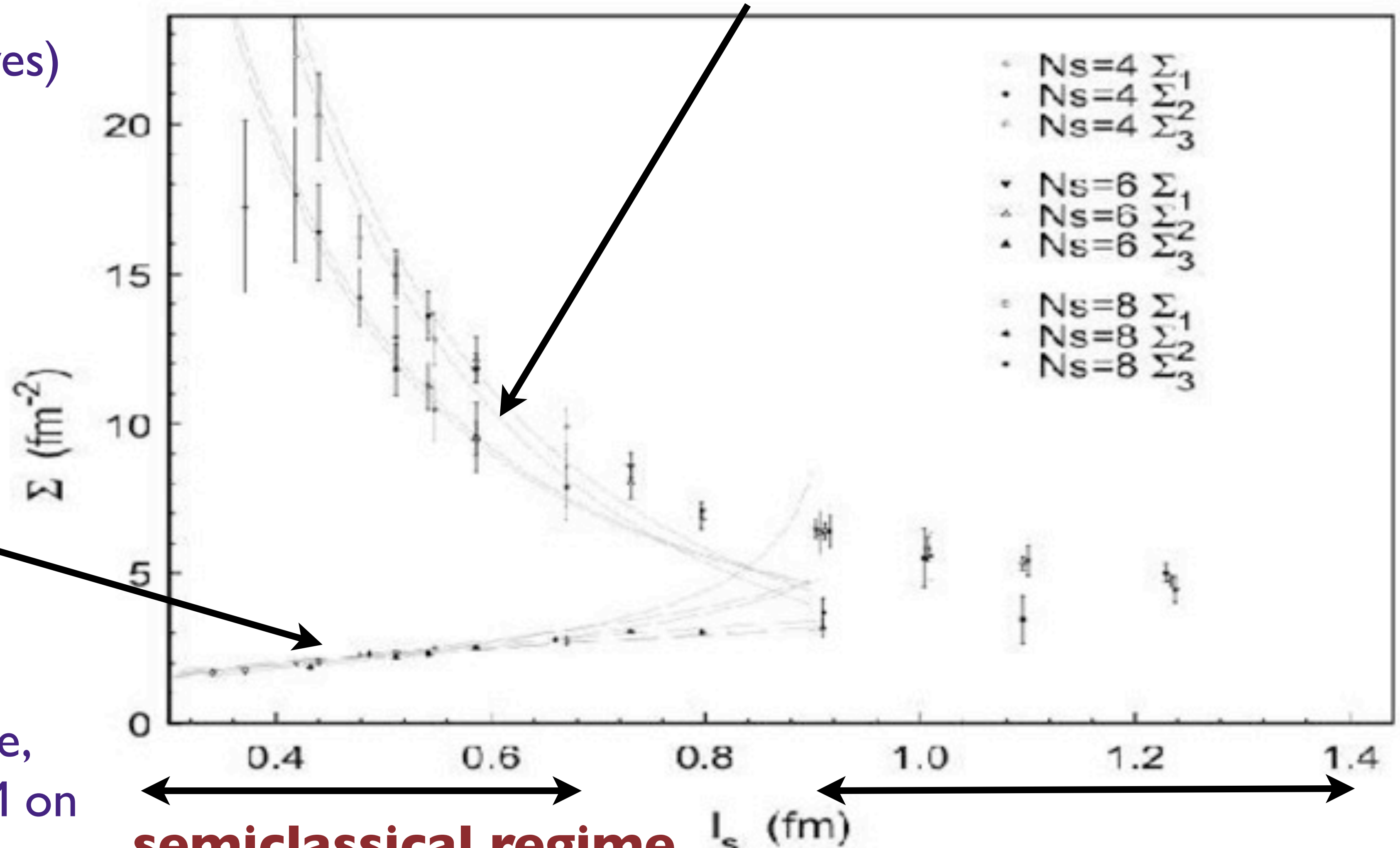
M. Perez, A. Gonzalez-Arroyo '93

pure YM - no fermions - on (small) T^3, twisted b.c. (center-symmetric!)

W-mass^2 ~1/L^2

semiclassical calculation (curves)
vs
lattice Monte Carlo (points)

string tension
~(ΛL)$^{5/6}$



same L-scaling as our (U.+Yaffe, actually) prediction for pure YM on a circle - since also due to fractional instantons, but now on T^3!

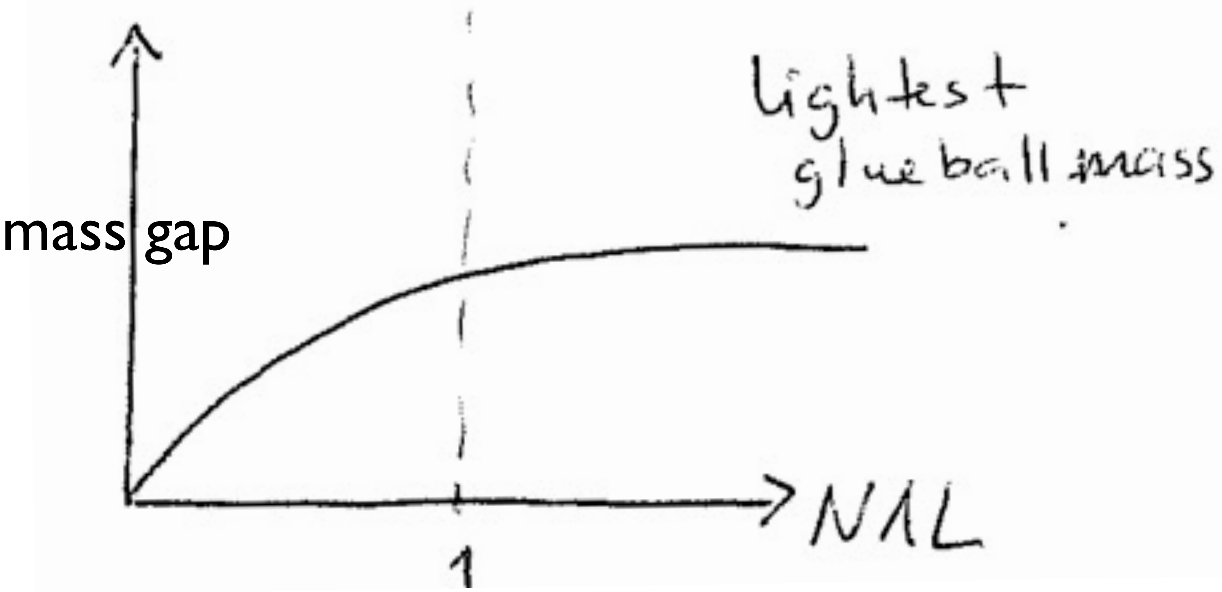
semiclassical regime (calculable)

strong-coupling regime

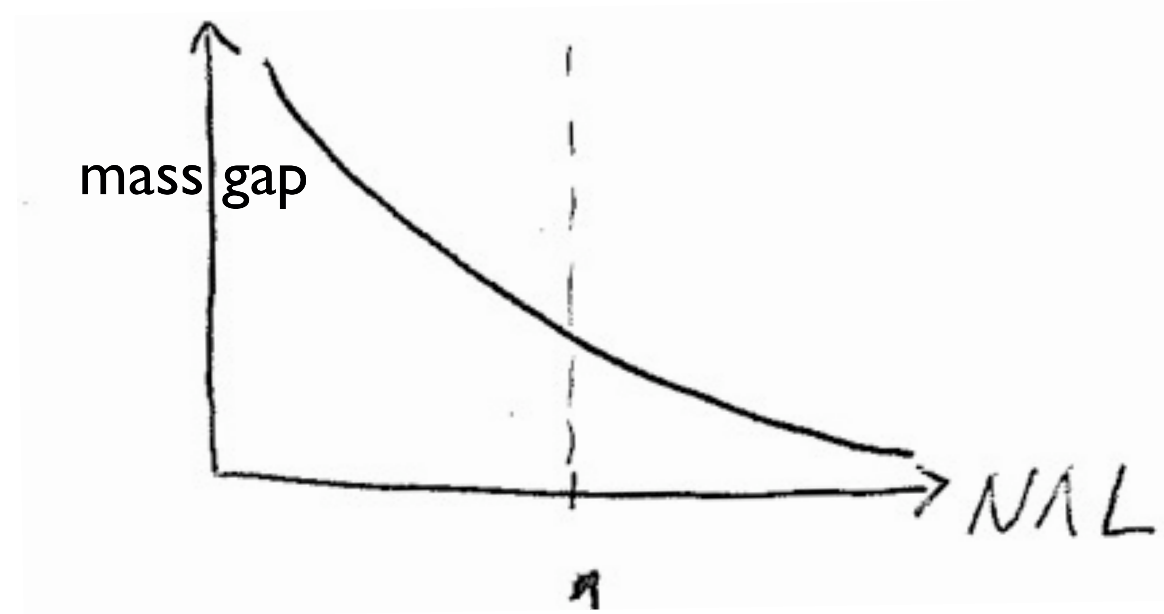
L << 1/strong scale

Back to SU(N) with Weyl adjoints [no deformation needed]:

less than 4 Weyl adjoints (and =4, Anber, EP, 5/2011)



> Weyl adjoints
mass gap = 0 at infinite L: conformal?



Note: perhaps defensible for 5 adjoints ~ "Banks-Zaks-ish"...

	our "estimate"	taken from Ryttov, Sanino: gap equation	beta function gamma=2/1	AF lost	"experiment"
any N	4	4.15	2.75/3.66	5.5	**4 ?**

Catterall et al; del Debbio, et al; Hietanen et al. all 2007-

all theoretical "determinations" rely on un-controlled extrapolations
hence, "error bars" unknown

lattice will eventually tell us whether curves really continue like this

Conclusions 1:

Compactifying 4d gauge theories on a small circle is a "deformation" where nonperturbative dynamics is under control - dynamics as "friendly" (OK, "almost"...) as in SUSY, e.g. Seiberg-Witten.
(regime of validity: $\Lambda L N \ll 1$ complimentary to EK: $\Lambda L N \gg 1$)

Confinement is due to various "oddball" topological excitations, in most theories non-self-dual.

Polyakov's "Debye screening" mechanism works on $R^3 \times S^1$ also with massless fermions, contrary to what many thought
- KK monopoles and index theorem-crucial ingredients of analysis.

Precise nature - monopoles, bions, triplets, or quintets - depends on the light fermion content of the theory.

U,P; 0812.2085, 0906.5156

- the above is more or less the moral of my talk -

Conclusions II:

Using these tools, we also gave "estimates" of conformal window boundary in vectorlike and chiral gauge theories (OK with "experiment" when available).

Conformality tied to relevance vs irrelevance of topological excitations. Perhaps of interest especially in theories where chiral symmetries do not break.

Unsal,EP; 0906.5156

Didn't have time to explain how:

In mixed-rep. theories with anomaly-free chiral global U(1), chiral symmetry broken at any radius:

-at small-L chiral symmetry breaking due to disorder operator vev
(correct interpretation of statements in SUSY literature - Davies et al, late '90s)

-at large-L due to fermion bilinear (large-L, as usual, not theoretically controllable, but likely true...)

Unsal,EP; 0910.1245

Conclusions III:

More things I didn't have time to talk about:

Circle compactification gives another calculable deformation of SUSY theories - not yet fully explored - in I=3/2 SU(2) Intriligator-Seiberg-Shenker model we argued that theory conformal, rather than SUSY-breaking

Unsal,EP; 0905.0634; agrees with different arguments of Shifman, Vainshtein `98; Intriligator `05; more recent: Vartanov `10 (index?)

High-T deconfinement in "deformed" YM - competition of electric and magnetic excitations near T_crit. - lead to theories with order and disorder variables

Simic, Unsal 2010 (earlier pheno. models of Liao/Shuryak 2006) ...

Stimulated by the observation that in N=1 SYM M&KK-induced potential fixes center symmetric ground state, Diakonov (2004-) has also made the point that M&KK also crucial near T_crit in pure ("undeformed") YM
- strictly not calculable; contributions not accounted for by Gross, Pisarsky, Yaffe

...also true in "undeformed" theories with fermions Unsal, EP, in progress

Conclusions/Questions IV:

Now, clearly,

on R^3x S^1 we only see the shadow of the real thing...

...except in special cases, chiral symmetry breaking still uncontrolled

can only note that monopole multifermion operators are clearly more relevant than instanton ones (fewer zero modes).

Natural question to ask is how this connects to 4d?

Is it so crazy to expect "relevance vs. irrelevance" (with changing Nf) of topological excitations also in R^4?

Conclusions/Questions V:

Lattice studies in pure YM (early ref.: Kronfeld, Schierholz, Wiese, 1987) **have found that confinement appears to be due to various topological excitations- center vortices, monopoles -** these are 't Hooft's "transient particles" (1978) that are revealed to us in wisely chosen gauges; the deconfinement transition is thought to be associated with them becoming irrelevant. Large body of literature, mostly pure YM.

To expect that massless fermions would affect the nature of topological excitations also on R^4 is thus quite natural.

What is harder (for me?) is how to make this precise on R^4.

confinement

Lesieur (1987) said that "~~turbulence~~ is a dangerous topic which is at the origin of serious fights in scientific meetings since it represents extremely different points of view, all of which have in common their complexity, as well as an inability to solve the problem. It is even difficult to agree on what exactly is the problem to be solved."

Clearly, it would be nice to get a better understanding...

While waiting for this to happen ... back to SUSY?

- theorists' "experiment"

Conclusions/Questions VI:

... back to SUSY?

We argued that "bions" are responsible for confinement in N=1 SYM at small L (a particular case of our Weyl adjoint theory).

This remains true if N=1 obtained from N=2 by soft breaking

Monopole and dyon condensation is responsible for confinement in N=2 softly broken to N=1 at large L (Seiberg, Witten `94)

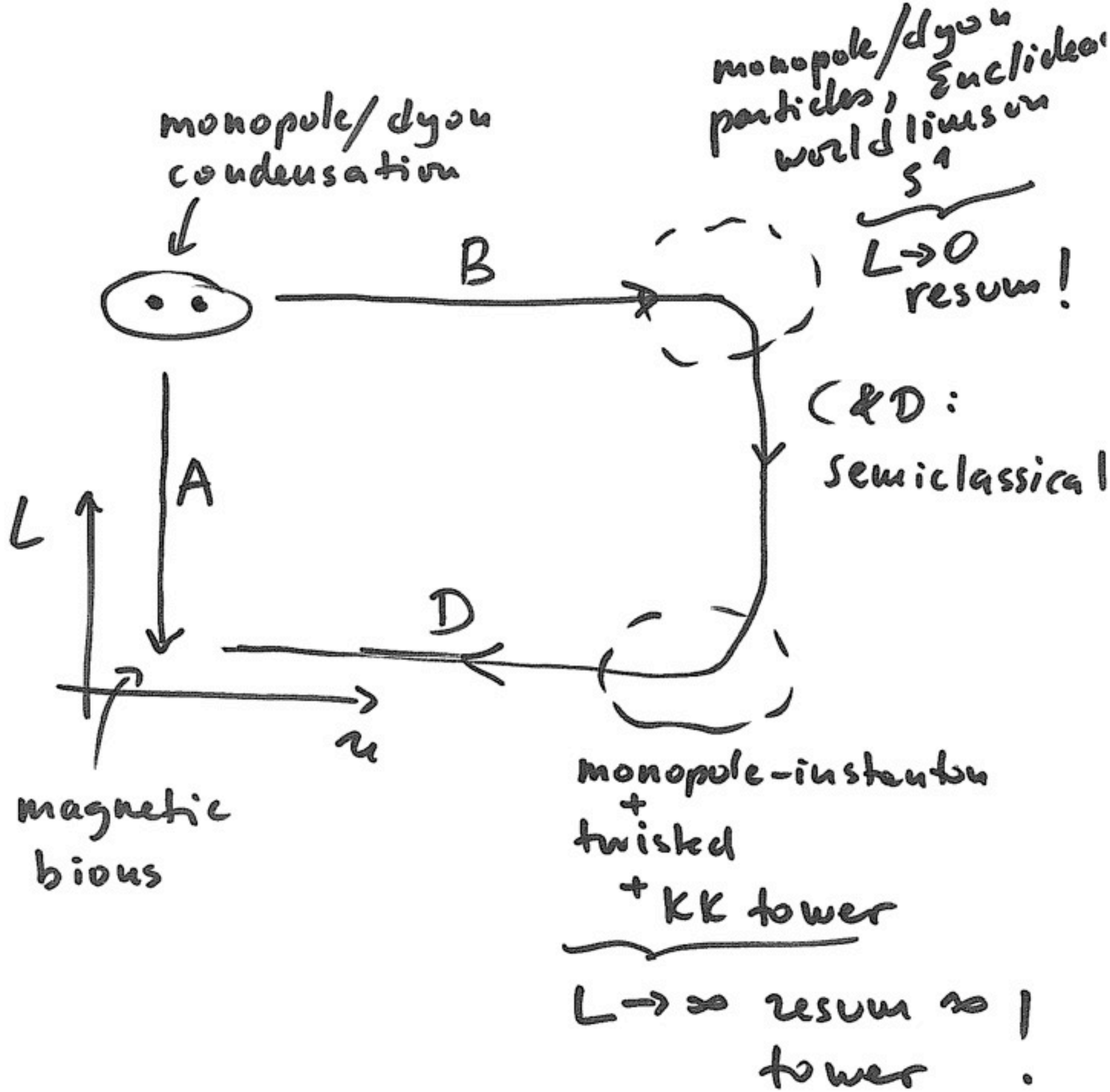
So, in different regimes we have different pictures of confinement in N=1 SYM (obtained as softly broken N=2).

Do they connect in an interesting way?

Unsal, EP - to appear

Preview:

... back to SUSY?



Preview:

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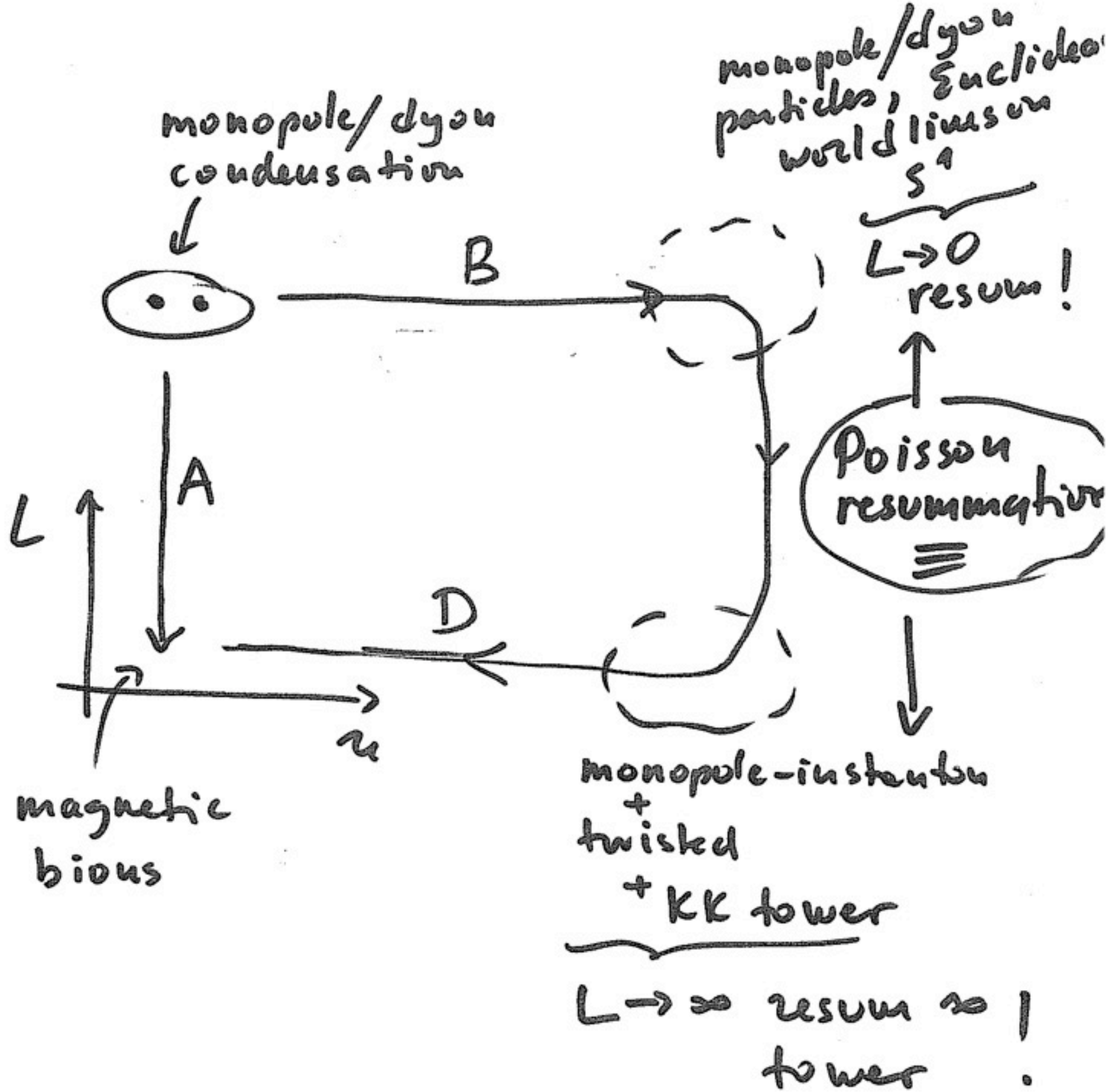

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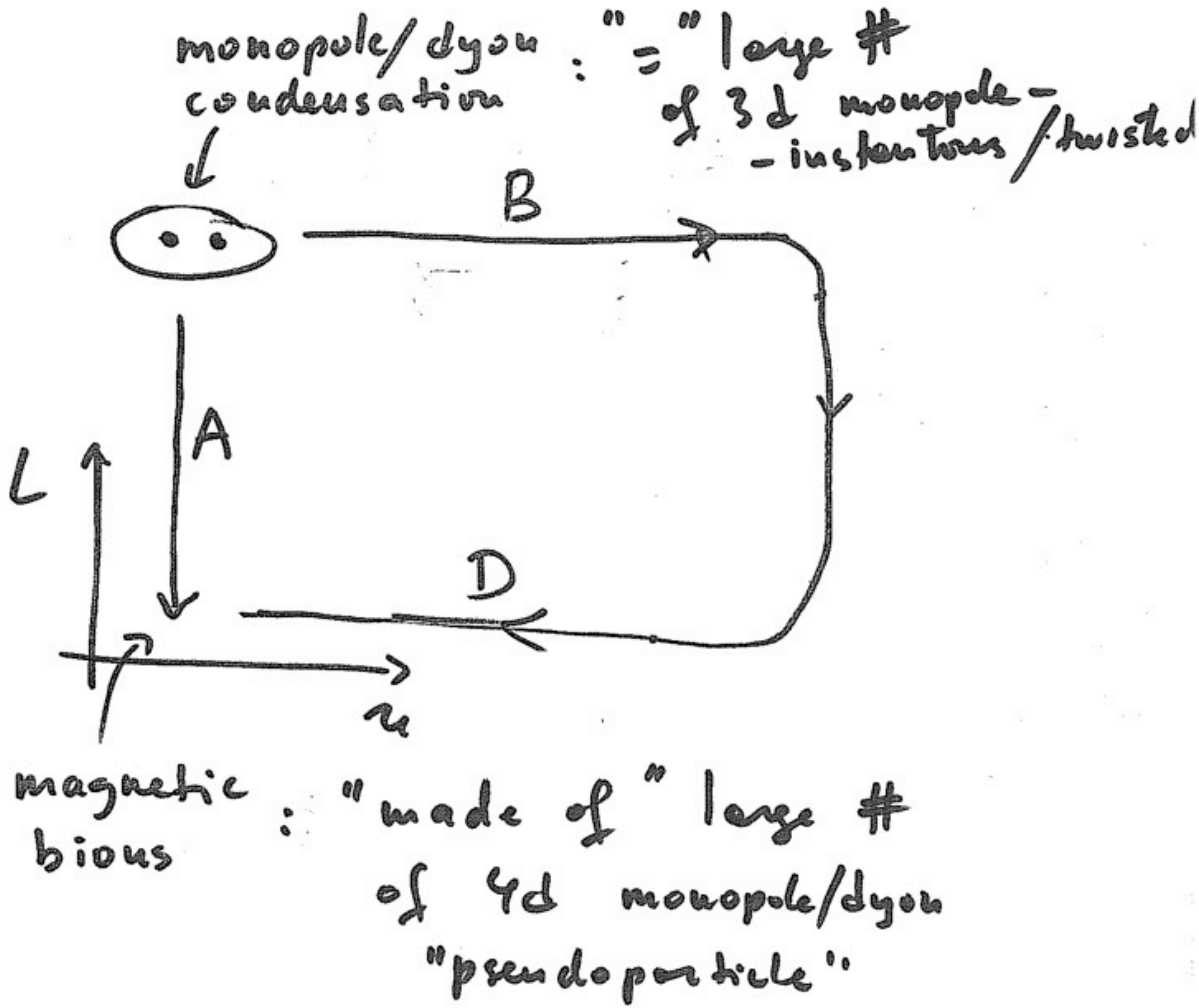

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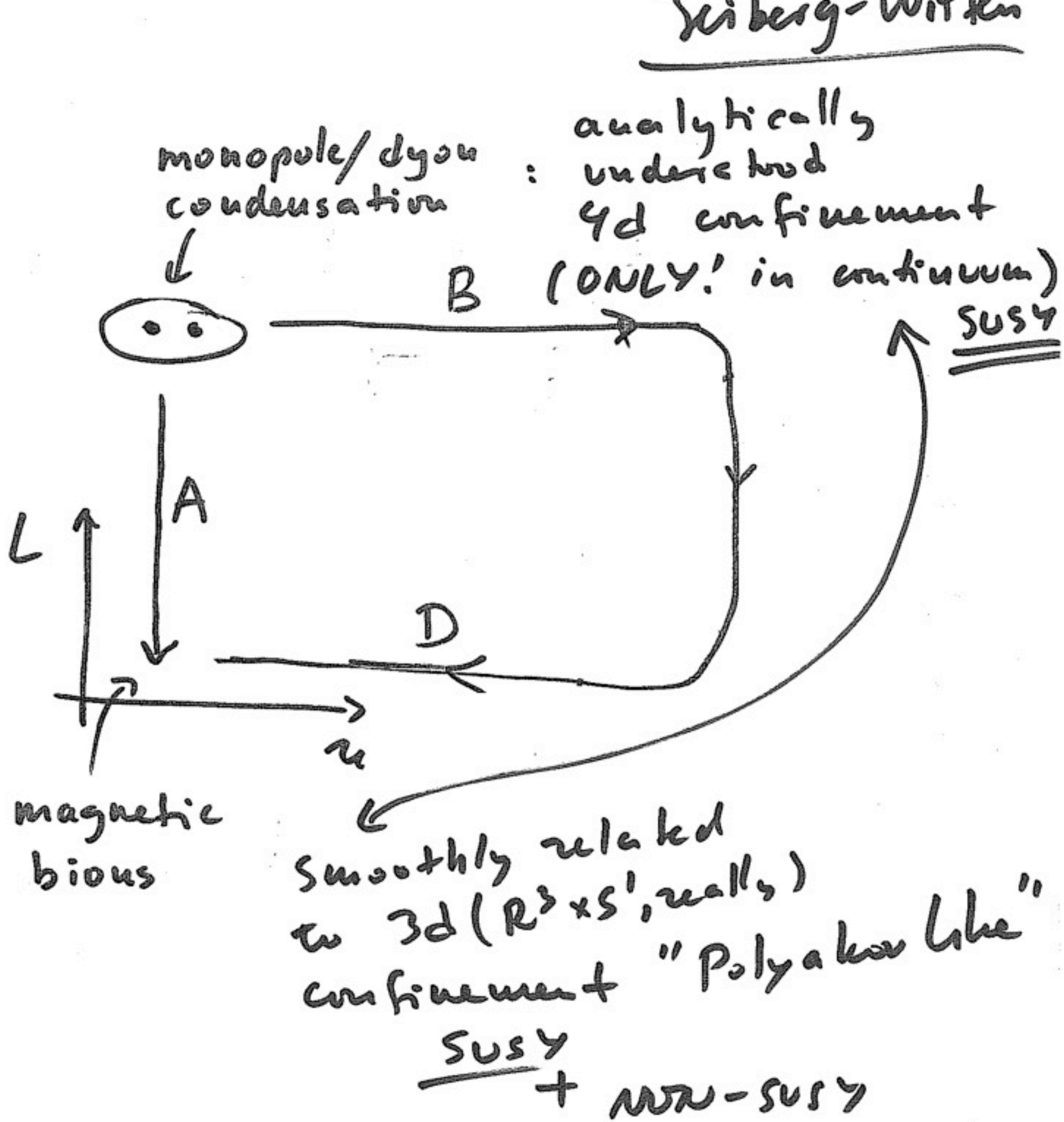

problem for the future...

... repeat for non-susy QCD(adj)...