Electromagnetism and Gravity with Continuous Spin

Kevin Zhou

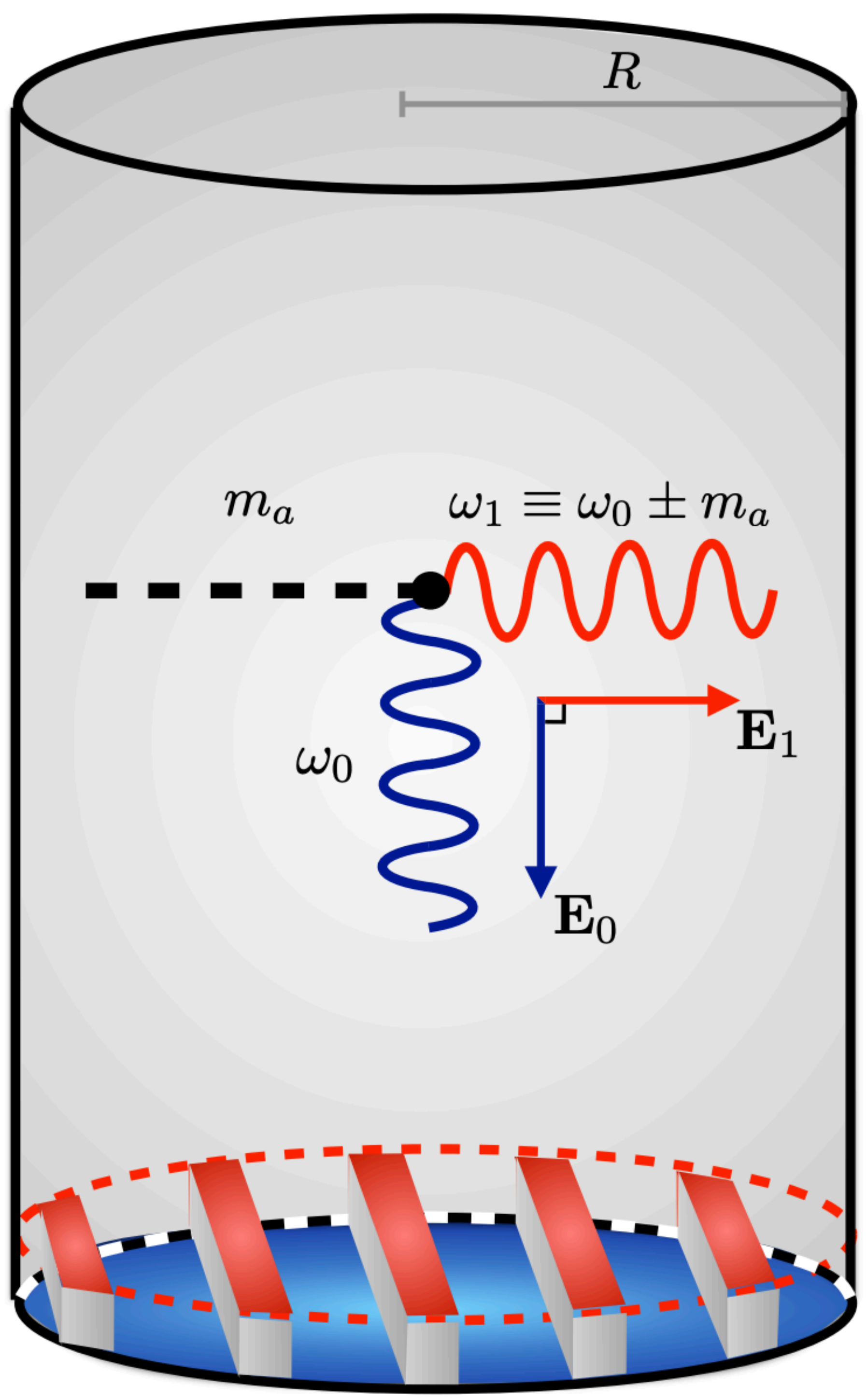
Stanford University

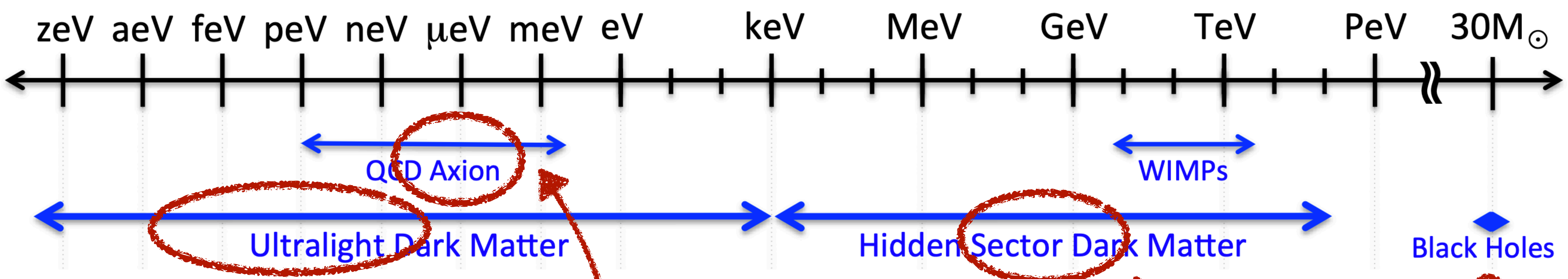
SLAC

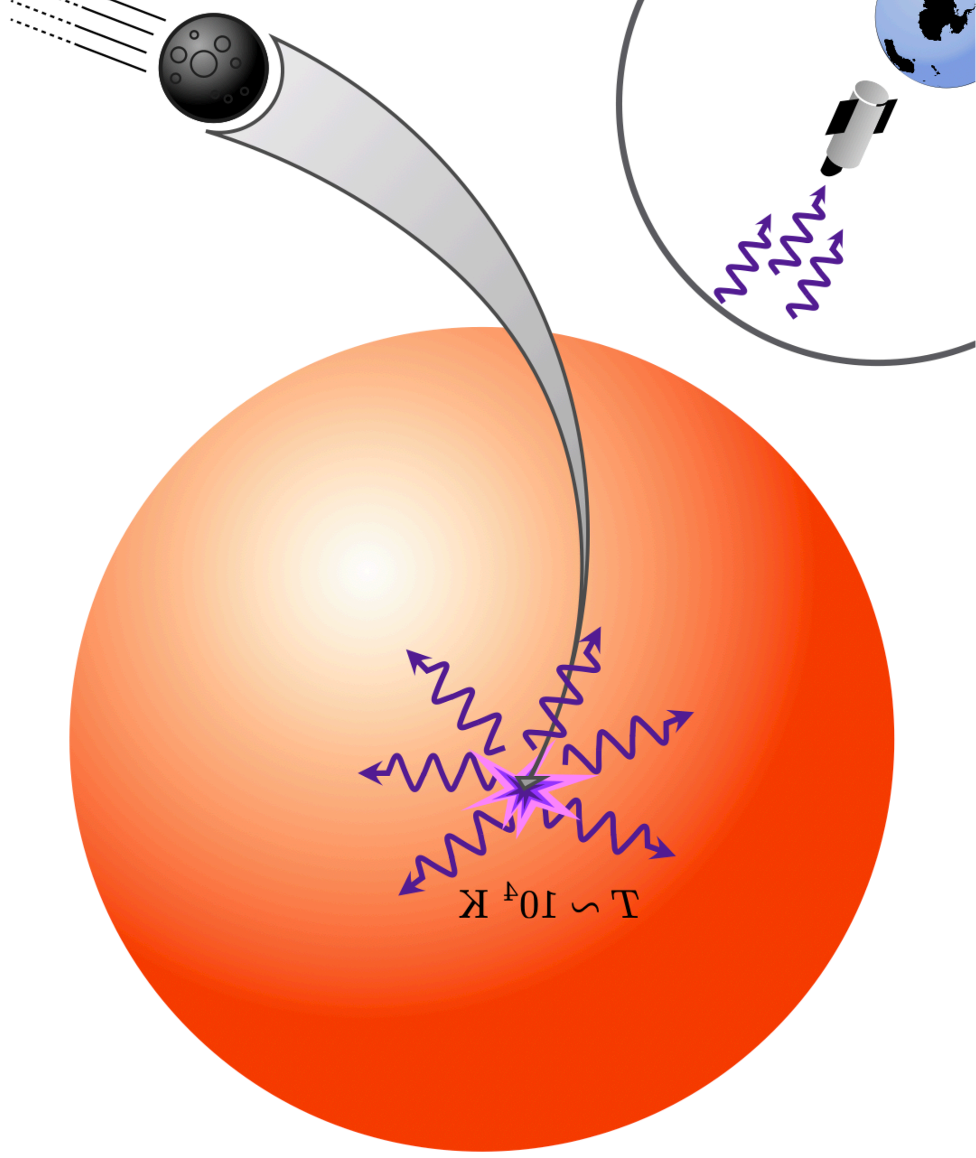
UC Davis QMAP Particles/Cosmology Seminar — April 11, 2023

arXiv:2303.04816, with Philip Schuster and Natalia Toro

My main focus is new experimental methods to search for dark matter.







stellar shock transients

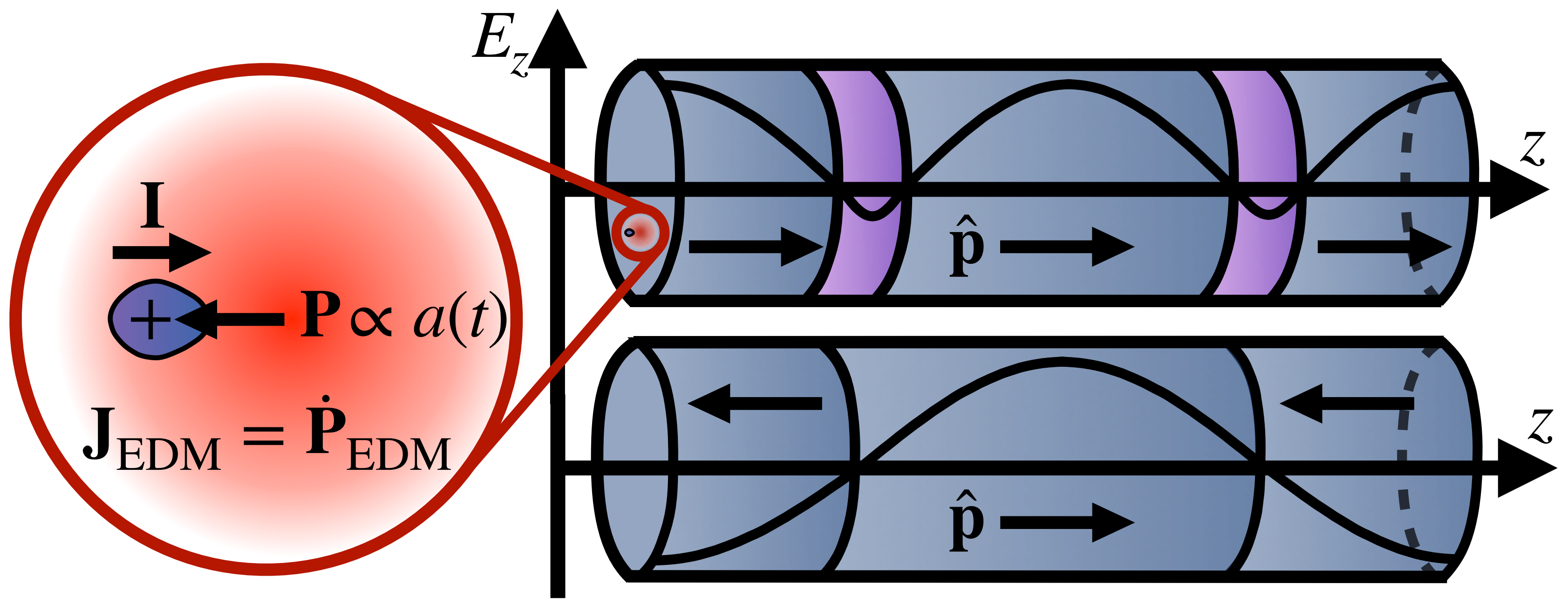
2106.09033

axion upconversion

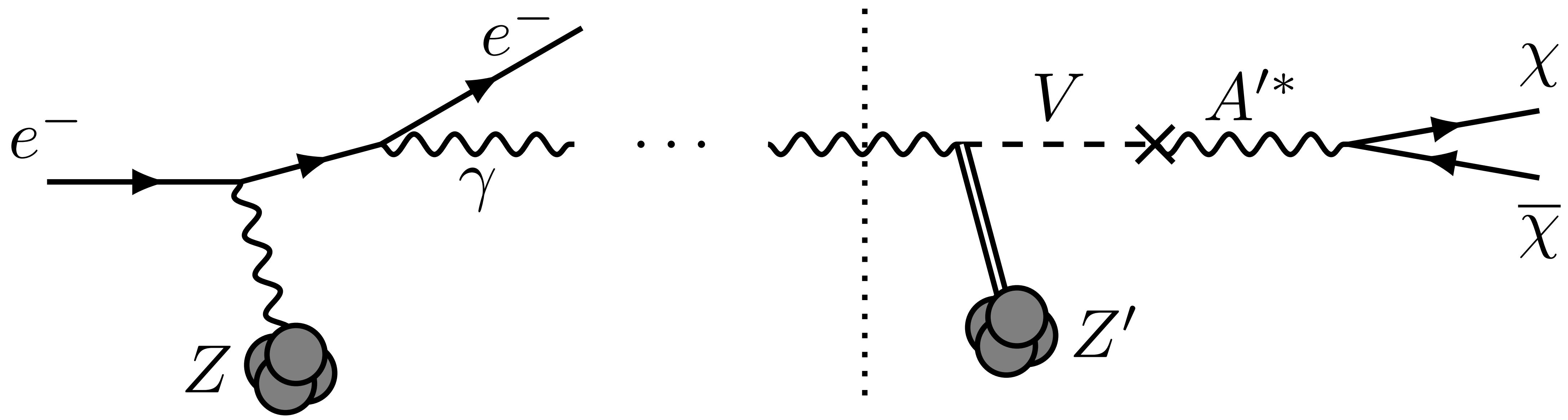
1912.11048, 2007.15656

polarization haloscopes

invisible meson decays

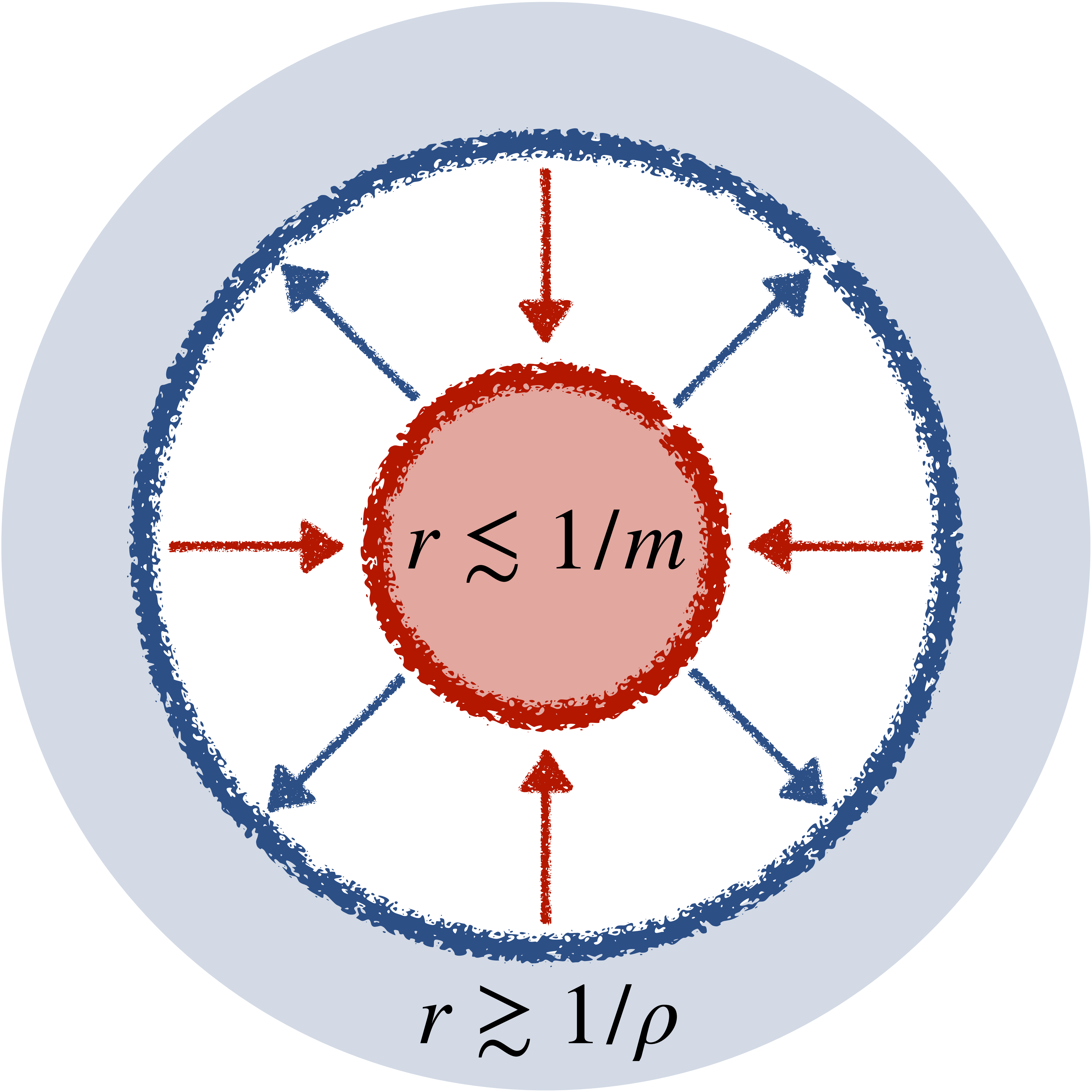


2209.12901



2112.02104

Today's talk is about a more fundamental question: where should we look for new physics?



Often assume long-distance physics is known, and look to smaller distances controlled by the mass scale m of new particles

But fundamental principles motivate new physics at larger distances controlled by the spin scale ρ of known particles!

Classifying Particles by Mass and Spin Scale

States transform under translations P^μ and rotations/boosts $J^{\mu\nu}$

Particle states with definite momentum obey $P^\mu \,|\, k, \sigma\rangle = k^\mu \,|\, k, \sigma\rangle$

Little group transformations $W^\mu = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} J_{\nu\rho} k_\sigma$ affect only internal state σ

Different types of particles classified by $P^2 = m^2$ and $W^2 = -\rho^2$

What is the physical meaning of the spin scale ρ?

Classifying Particles by Mass and Spin Scale

For $m^2 > 0$, representations are spin S massive particles

States are $|k, h\rangle$ for helicity $h = -S, \ldots, S$, which is not Lorentz invariant

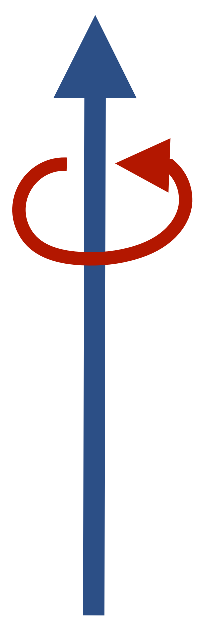
Boosts mix helicities by amount determined by $\rho = m\sqrt{S(S+1)}$

For $m^2 = 0$, states are still indexed by helicity $|k, h\rangle$

Spin scale again determines how helicity varies under boosts

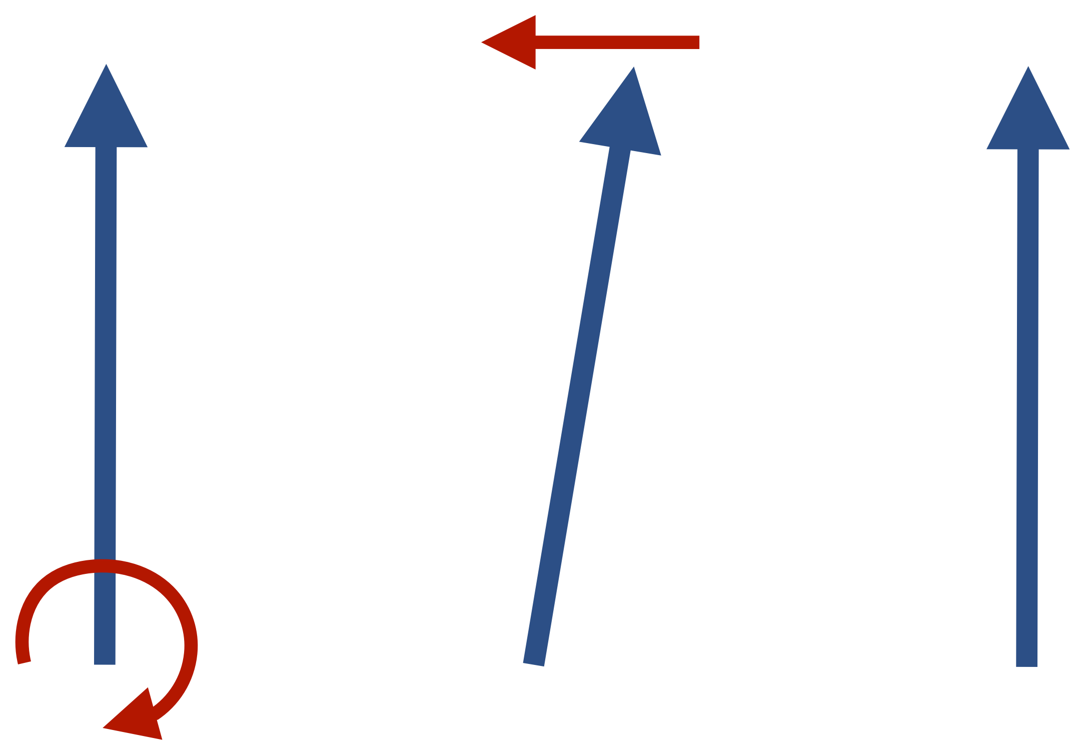
The Massless Little Group

For a massless particle, $k^\mu = (\omega, 0, 0, \omega)$, little group generators are



$$R = J_z$$

$$R\,|\,k, h\rangle = h\,|\,k, h\rangle$$



$$T_1 \propto J_x + K_y$$

$$T_2 \propto J_y - K_x$$

Defining $T_\pm = T_1 \pm iT_2$, commutation relations imply

$$T_\pm\,|\,k, h\rangle = \rho\,|\,k, h \pm 1\rangle$$

Generic result is an **infinite** ladder of integer-spaced helicities!

Allowed Helicities for Massless Particles

Generic massless particle representation has continuous-valued spin scale ρ

Since h is always integer or half-integer, gives two options, known since 1930s:

bosonic continuous spin		fermionic continuous spin
$\cdots$		$\cdots$
$h = 2$		$h = 3/2$
$h = 1$		$h = 1/2$
$h = 0$		$h = -1/2$
$h = -1$		$h = -3/2$
$h = -2$		$\cdots$
$\cdots$		

(plus supersymmetric, (A)dS, higher/lower dimension variants)

Allowed Helicities for Massless Particles

If we set $\rho = 0$, recover a single helicity h (related to $-h$ by CPT symmetry)

Focus on bosonic case, which can mediate long-range $1/r^2$ forces

$h = 0$	massless scalar (requires fine-tuning)
$\vert h \vert = 1$	photon (minimal coupling to conserved charge)
$\vert h \vert = 2$	graviton (minimal coupling to stress-energy)
$\vert h \vert = 3$	higher spin (no minimal couplings allowed)
$\ldots$	

Role of each $|h|$ in nature well-understood from general arguments from 1960s

Why Not Consider Continuous Spin?

~~Ruled out by Weinberg soft theorems?~~

Theorems assume Lorentz invariant h

Generalize to good soft factors for $\rho \neq 0$

Schuster and Toro, JHEP (2013) 104/105

~~Incompatible with field theory?~~

Simple free gauge theory found

Schuster and Toro, PRD (2015)

~~Can't interact with anything?~~

Addressed in our paper!

~~Just way too complicated?~~

Addressed in our paper!

Infinite h leads to infinities in scattering/ cosmology/astrophysics/Hawking/Casimir/…?

Smooth $\rho \to 0$ limit where all but one $|h|$ decouples

Results even well-behaved for large ρ when many helicities relevant!

Why Consider Continuous Spin?

For the field theorist: “because it’s there”

Simple and directly motivated by the postulates of relativity and quantum mechanics, but still not fully understood

For the experimentalist: “because it’s testable”

Theory predicts ρ-dependent deviations from electromagnetism and general relativity

The value of ρ is unknown, and only experiment can determine it

Force in a radiation background: $$\frac{\mathbf{F}}{q} = \mathbf{E} + \mathbf{v} \times \mathbf{B} - \left(\frac{\rho v_\perp}{2\omega} \right)^2 \left(\mathbf{E}_\perp + \frac{\mathbf{E}}{2} \right) + \ldots$$

Significant correction when particles travel distance $\gtrsim 1/\rho$, total result always well-behaved

Why Consider Continuous Spin?

For the field theorist: "because it's there"

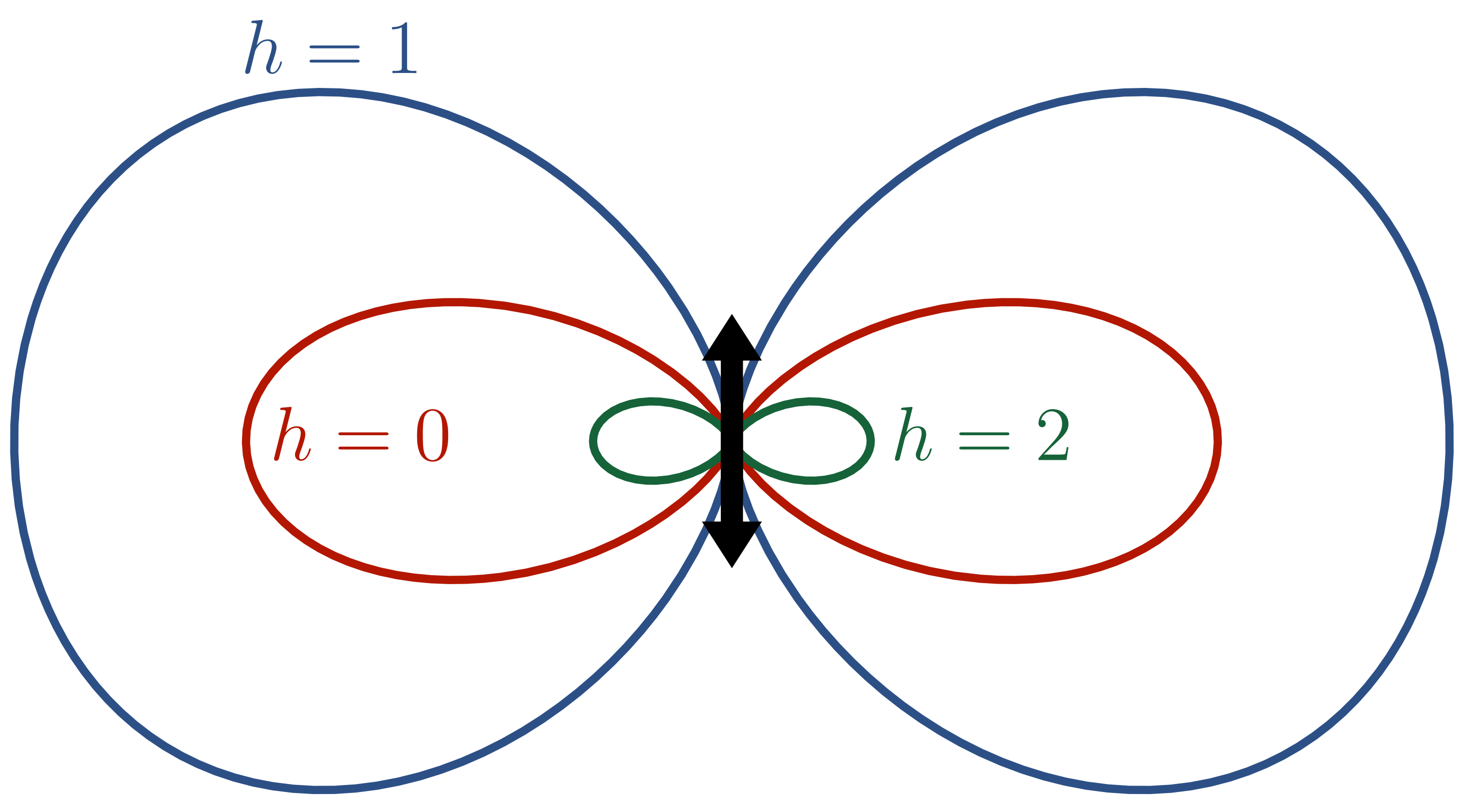
Simple and directly motivated by the postulates of relativity and quantum mechanics, but still not fully understood

For the experimentalist: "because it's testable"

Theory predicts ρ-dependent deviations from electromagnetism and general relativity

The value of ρ is unknown, and only experiment can determine it

Radiation from an oscillating particle:



$$P = \frac{q^2 \langle a^2 \rangle}{6\pi} \times \begin{cases} (\rho \ell)^2/40 & h = 0 \\ 1 - 3(\rho \ell)^2/20 & h = \pm 1 \\ (\rho \ell)^2/80 & h = \pm 2 \end{cases} + \ldots$$

Significant correction when particles travel distance $\gtrsim 1/\rho$, total result always well-behaved

Why Consider Continuous Spin?

For the field theorist: “because it’s there”

Simple and directly motivated by the postulates of relativity and quantum mechanics, but still not fully understood

For the experimentalist: “because it’s testable”

Theory predicts ρ-dependent deviations from electromagnetism and general relativity

The value of ρ is unknown, and only experiment can determine it

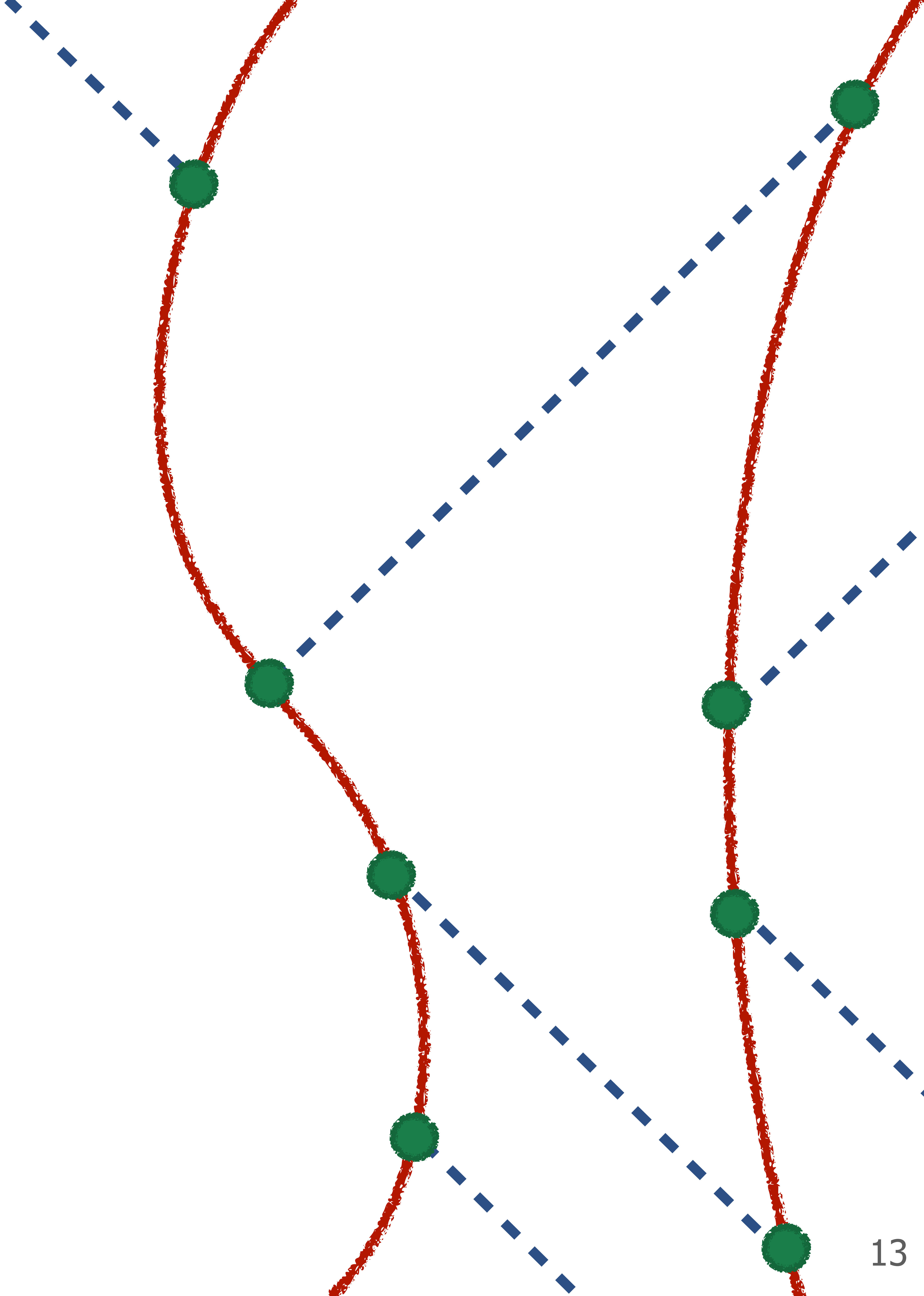
For the model builder: “because it’s novel”

A new infrared deformation of gauge theories, which may shed light on long-distance physics (dark matter, cosmic acceleration)

A new type of spacetime symmetry based on a bosonic superspace, possibly relevant for tuning problems (hierarchy, cosmological constant)

Outline

- **Free continuous spin fields**
- Coupling matter particles
- Physics with continuous spin



Free Fields for Massless Particles

Tricky even for $\rho = 0$, by mismatch of field and particle degrees of freedom

scalar $h = 0$	scalar field ϕ, no extra components
photon $h = \pm 1$	vector field A_μ, $4 - 2 = 2$ extra components
	must use action with gauge symmetry $\delta A_\mu = \partial_\mu \alpha$
graviton $h = \pm 2$	sym. tensor field $h_{\mu\nu}$, $10 - 2 = 8$ extra components
	must use action with gauge symmetry $\delta h_{\mu\nu} = \partial_{(\mu}\xi_{\nu)}$
higher spin $\lvert h \rvert > 2$	sym. tensor field $\phi_{\mu_1 \ldots \mu_h}$, many extra components

Given complexity of higher h, constructing a continuous spin field seems intractable!

Introducing Vector Superspace

A field in "vector superspace" (x^μ, η^μ) has tensor components of all ranks

$$\Psi(\eta, x) = \phi(x) + \sqrt{2}\,\eta^\mu A_\mu(x) + (2\eta^\mu\eta^\nu - g^{\mu\nu}(\eta^2+1))\,h_{\mu\nu}(x) + \dots.$$

Simple expression has free Lagrangian for each tensor field simultaneously!

$$\mathscr{L}[\Psi] = \frac{1}{2}\int_\eta \delta'(\eta^2+1)(\partial_x\Psi)^2 + \frac{1}{2}\delta(\eta^2+1)(\Delta\Psi)^2 \qquad \Delta = \partial_x \cdot \partial_\eta$$

Integration produces tensor contractions

$$\int_\eta \delta(\eta^2+1) = \int_\eta \delta'(\eta^2+1) \equiv 1$$

$$\int_\eta \delta(\eta^2+1)\,\eta^\mu\eta^\nu = -\frac{1}{4}g^{\mu\nu}$$

$$\int_\eta \delta'(\eta^2+1)\,\eta^\mu\eta^\nu = -\frac{1}{2}g^{\mu\nu}$$

Recovering Familiar Actions

$$\mathscr{L}[\phi] = \frac{1}{2}\int_\eta \underbrace{\delta'(\eta^2+1)}_{\text{gives } 1}\underbrace{(\partial_x\Psi)}_{\partial_x\phi}{}^2 + \frac{1}{2}\,\delta(\eta^2+1)(\partial_x\cdot\underbrace{\partial_\eta\,\Psi}_{\partial_\eta\phi=0})^2\Bigg|_{\Psi=\phi} = \frac{1}{2}\,(\partial_x\,\phi)^2$$

$$\mathscr{L}[A_\mu] = \frac{1}{2}\int_\eta \delta'(\eta^2+1)\underbrace{(\partial_x\Psi)^2}_{(\sqrt{2}\,\eta_\mu\partial_x A^\mu)^2} + \frac{1}{2}\,\delta(\eta^2+1)\underbrace{(\partial_x\cdot\partial_\eta\,\Psi)^2}_{(\sqrt{2}\,\partial_\mu A^\mu)^2}\Bigg|_{\Psi=\sqrt{2}\eta^\mu A_\mu} = -\frac{1}{2}(\partial_\mu A_\nu)^2 + \frac{1}{2}(\partial_\mu A^\mu)^2$$

More generally, we recover the linearized Einstein-Hilbert action, and higher-rank Fronsdal actions, with no mixing

Recovering Familiar Dynamics

One equation of motion contains Maxwell, linearized Einstein, Fronsdal:

$$\delta'(\eta^2+1)\,\partial_x^2\,\Psi - \frac{1}{2}\,\Delta\,(\delta(\eta^2+1)\,\Delta\Psi) = 0$$

One gauge transformation contains $U(1)$ gauge transformations, diffeomorphisms, …

$$\delta\Psi = (\eta\cdot\partial_x - \frac{1}{2}(\eta^2+1)\Delta)\,\epsilon(\eta, x)$$

One mode expansion contains modes of arbitrary integer helicity:

$$\Psi_{k,h} = e^{-ik\cdot x}\,(\eta\cdot\epsilon_\pm)^{|h|}$$

Turning on the Spin Scale

All previous results can be generalized to arbitrary ρ by taking $\Delta = \partial_x \cdot \partial_\eta + \rho$

Still get one mode of each helicity, but now the action, equation of motion, gauge symmetric, and plane waves all mix tensor ranks, e.g.

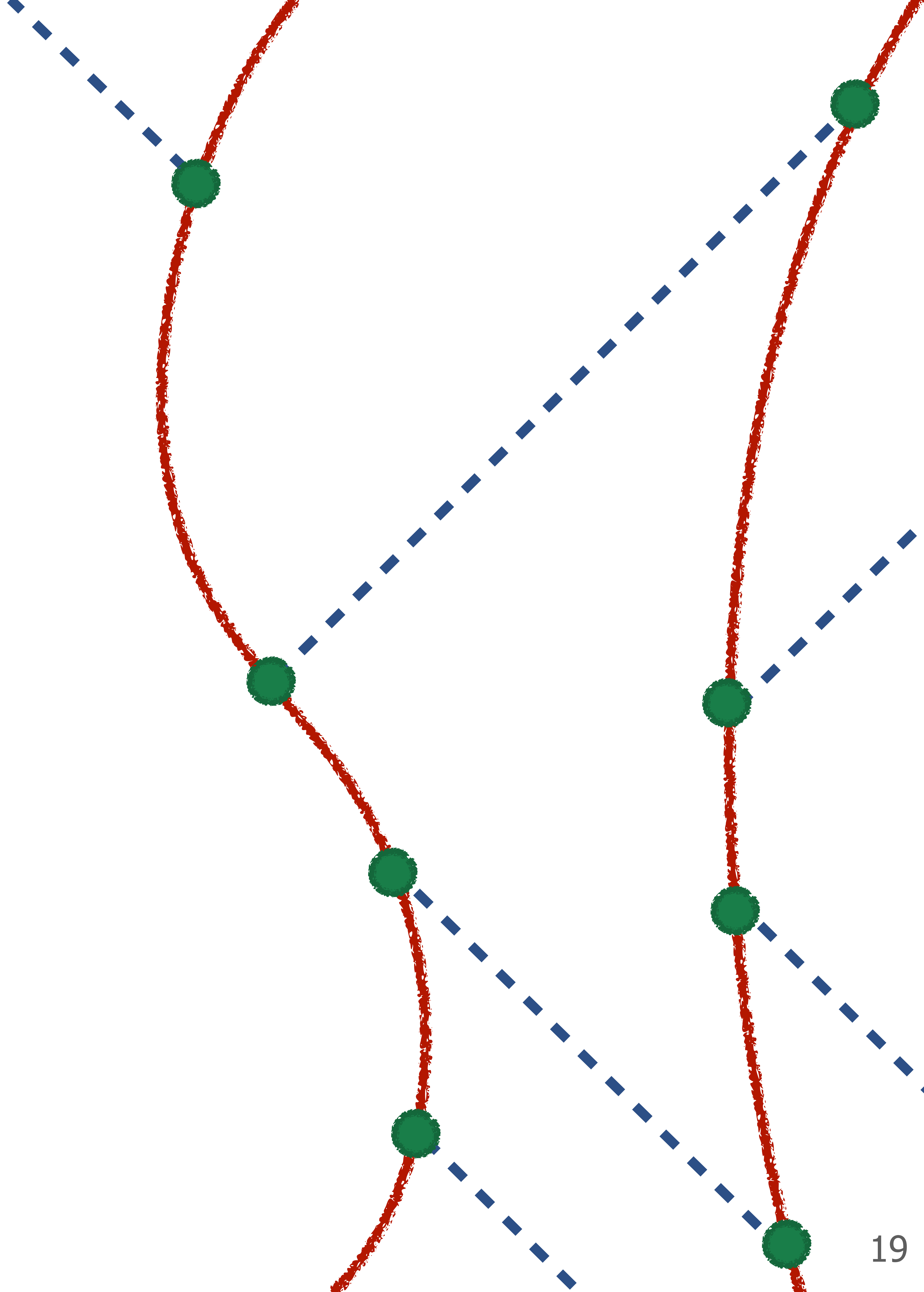
$$\mathscr{L} \supset \frac{\rho}{\sqrt{2}} \phi \, \partial_\mu A^\mu$$

$$\Psi_{k,h} = e^{-ik\cdot x} \, e^{-i\rho\eta\cdot q} \, (\eta \cdot \epsilon_\pm)^{|h|} \qquad\qquad q \cdot k = 1$$

Because of mixing, tensor expansion is complicated and physically opaque, while vector superspace description remains simple

Outline

- Free continuous spin fields
- **Coupling matter particles**
- Physics with continuous spin



Coupling Currents to Fields

Couple the continuous spin field to a current by

$$\mathscr{L}_{\rm int} = \int_\eta \delta'(\eta^2+1)\, J(\eta,x)\, \Psi(\eta,x) = \phi J - A_\mu J^\mu + h_{\mu\nu} T^{\mu\nu} + \ldots$$

Recover familiar results by tensor decomposition

$$J(\eta,x) = J(x) - \sqrt{2}\,\eta^\mu J_\mu(x) + (2\eta^\mu\eta^\nu + g^{\mu\nu})\, T_{\mu\nu}(x) + \ldots$$

Gauge invariance of the coupling gives a “continuity condition”

$$\delta(\eta^2+1)\Delta J = 0 \longrightarrow \partial_\mu J^\mu \sim \rho J$$

Tensor currents not conserved, reflecting mixing of tensor fields!

Currents From Matter Particles

In familiar theories, the current from a matter particle is local to its worldline $z^\mu(\tau)$

scalar-like current $\longrightarrow$ $$J(x) = g \int d\tau\, \delta^4(x - z(\tau))$$

vector-like current $\longrightarrow$ $$J^\mu(x) = e \int d\tau\, \delta^4(x - z(\tau)) \frac{dz^\mu}{d\tau}$$

tensor-like current $\longrightarrow$ $$T^{\mu\nu}(x) = m \int d\tau\, \delta^4(x - z(\tau)) \frac{dz^\mu}{d\tau} \frac{dz^\nu}{d\tau}$$

These correspond to minimal couplings in field theory, and physically interesting continuous spin currents should reduce to them in the $\rho \to 0$ limit

Locality and Causality

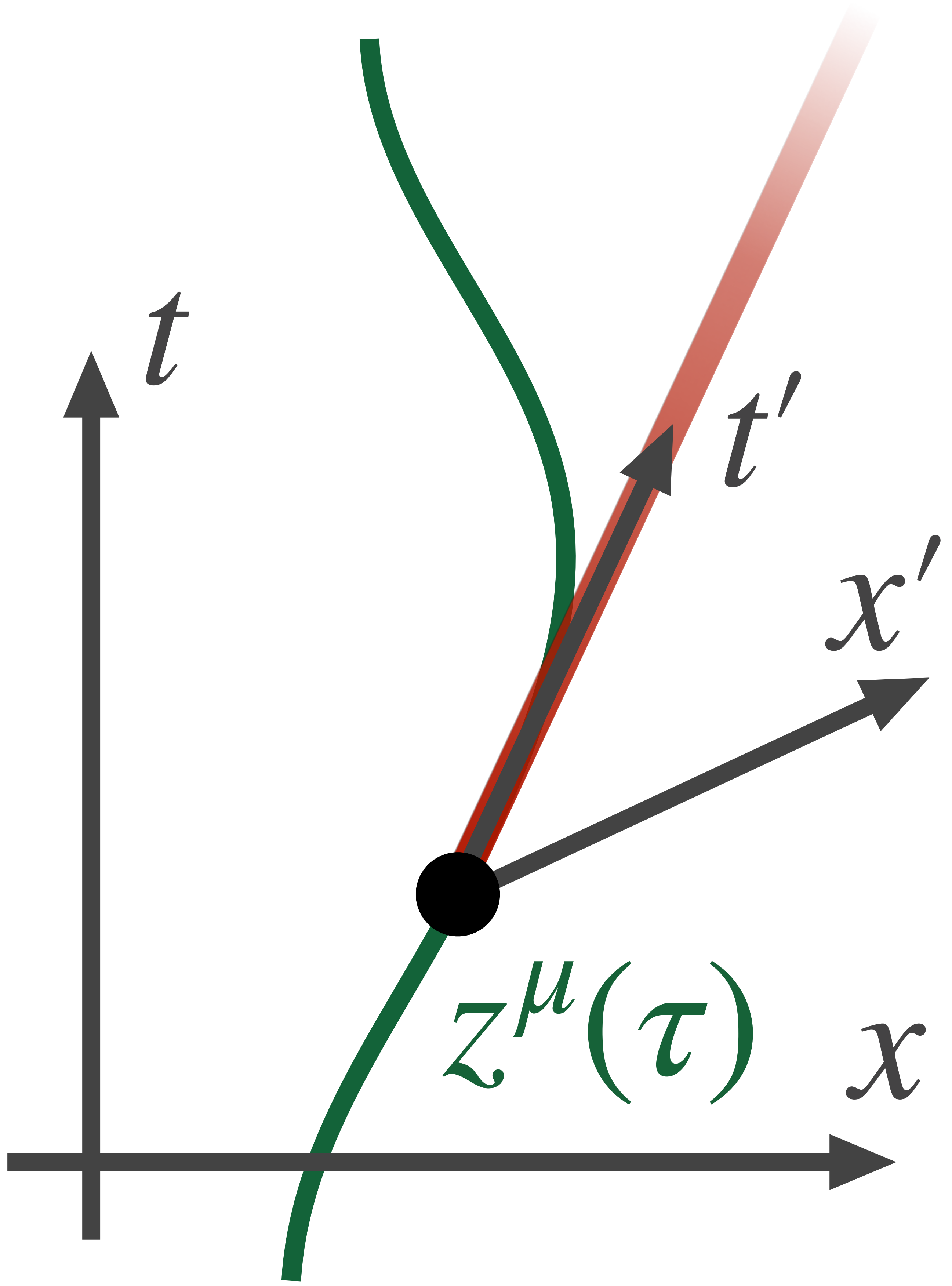
The current and continuity condition in Fourier space are

$$J(\eta, k) = \int d\tau \, e^{ik\cdot z(\tau)} f(\dot{z}, k, \eta) \qquad (-ik \cdot \partial_\eta + \rho) f \approx 0$$

One simple example solution is $f = e^{-i\rho\eta\cdot\dot{z}/k\cdot\dot{z}}$

Our currents are generically **not** localized to the worldline!

But choosing appropriate boundary conditions in equations of motion yields causal particle dynamics



A Universality Result

Our key technical result: all currents can be decomposed as

$$f = e^{-i\rho\eta\cdot\dot{z}/k\cdot\dot{z}}\,\hat{g}(k\cdot\dot{z}) + \mathscr{D}X$$

where the free equation of motion is $\delta'(\eta^2+1)\mathscr{D}\Psi = 0$. All terms in X are "contact" interactions whose coupling to Ψ can be removed by field redefinition!

Most physical observables are determined by $\hat{g}$ and thereby **universal**, with

$$\hat{g} = \begin{cases} g & \text{scalar-like current} \\ e\,k\cdot\dot{z} & \text{vector-like current} \\ m\,(k\cdot\dot{z})^2 + \ldots & \text{tensor-like current} \end{cases}$$

All valid currents found in other works correspond to $\hat{g} = 0$

Extracting the Physics

From the action $S[\Psi, z_i^\mu(\tau)]$ we can compute any desired classical observable:

"Integrate out" Ψ (plug solution into action) → Matter interaction potential $V(\mathbf{r}_i, \mathbf{v}_i, \ldots)$

Depends on nonlocal contact terms

Find z_i equation of motion with given Ψ → Matter forces in background $\mathbf{F}(\Psi)$

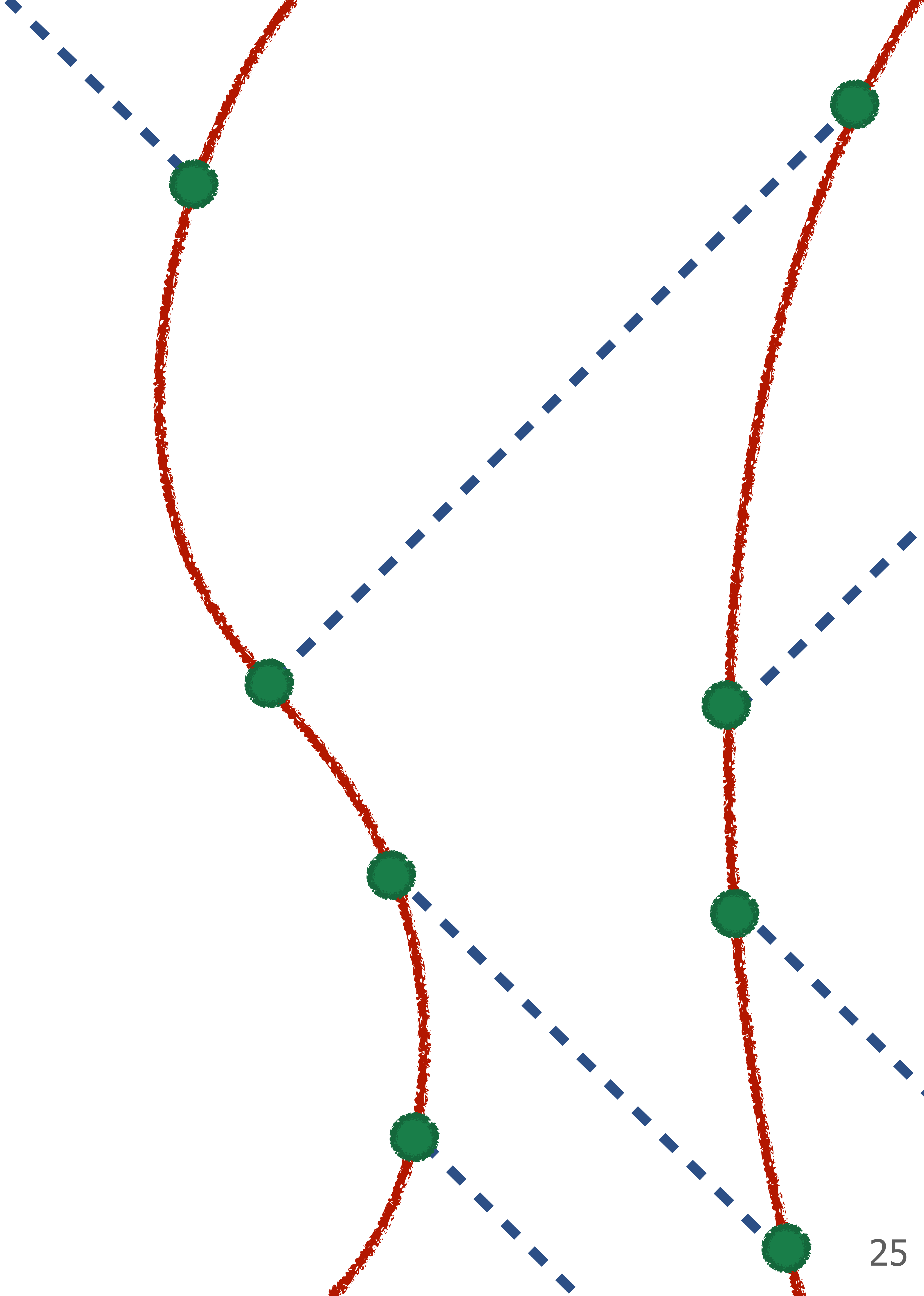
Universal

Solve Ψ equation of motion with given trajectories → Radiation emission $dP_h/d\Omega$

Universal

Outline

- Free continuous spin fields
- Coupling matter particles
- **Physics with continuous spin**



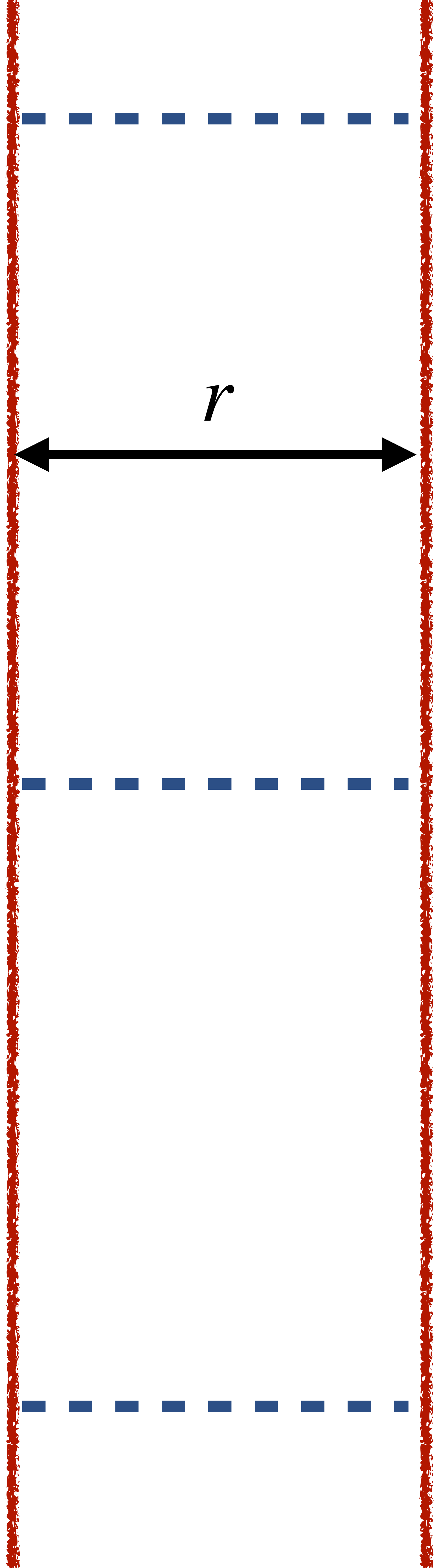
Static Potentials

Static potentials can exhibit deviations at long distances:

$$V(r) = \frac{g^2}{4\pi r}\left(1 - c_1 \rho r + c_2(\rho r)^2 + \ldots\right)$$

Coefficients depend on current: vanish for simplest currents, but for general currents can cause force to flip sign at large distances

Similar results for vector-like currents; can also find velocity-dependent potentials (e.g. corrections to magnetic interaction)



Forces in Background Fields

Force on particle with vector-like current in background of frequency ω, helicity h:

$$\frac{\mathbf{F}_{h=0}}{q} = \frac{\rho}{\omega}\frac{\dot{\phi}\,\mathbf{v}_\perp}{2} + \dots$$

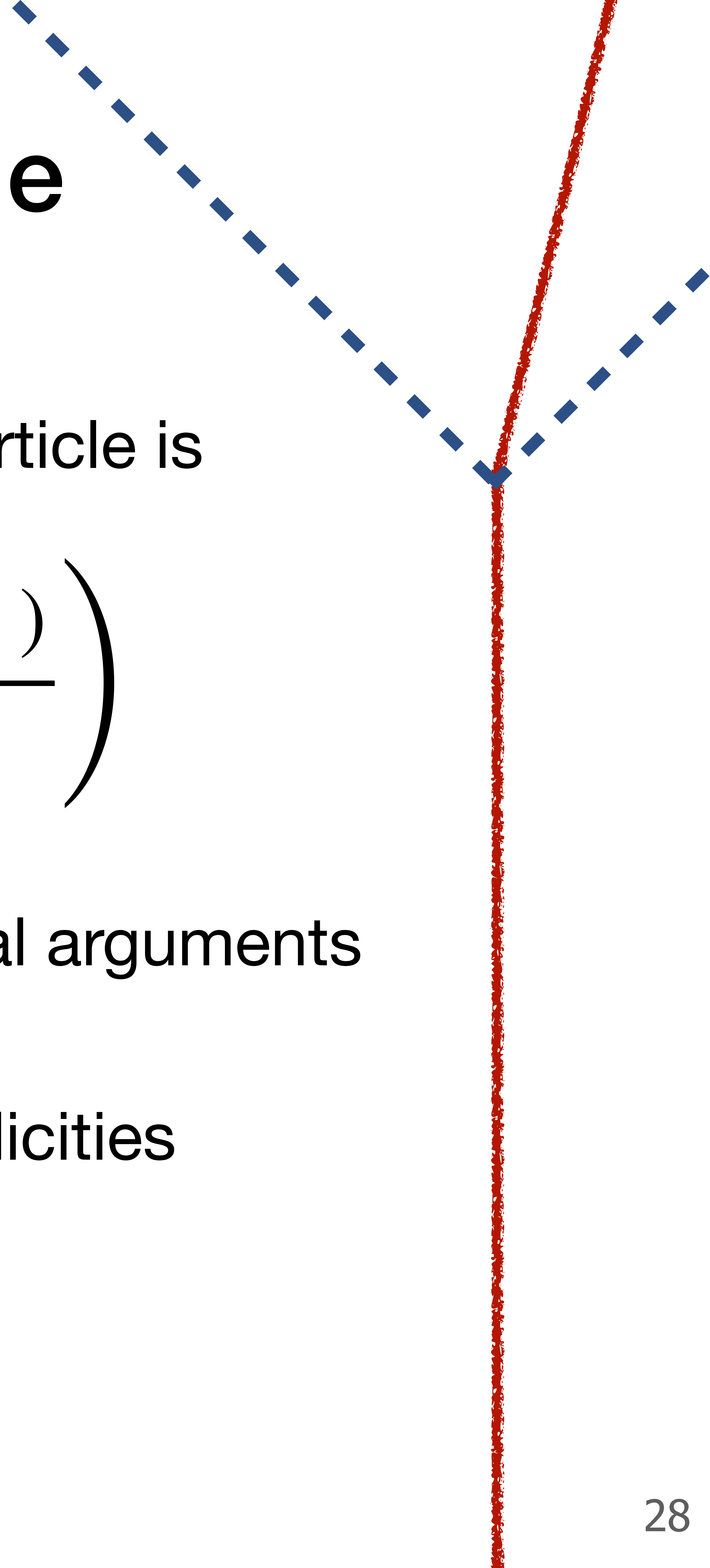
$$\frac{\mathbf{F}_{h=\pm 1}}{q} = \mathbf{E} + \mathbf{v}\times\mathbf{B} - \left(\frac{\rho}{\omega}\right)^2\left(\frac{\mathbf{v}_\perp(\mathbf{v}_\perp\cdot\mathbf{E})}{4} + \frac{v_\perp^2\,\mathbf{E}}{8}\right) + \dots$$

$$\frac{\mathbf{F}_{h=\pm 2}}{q} = \frac{\rho}{\omega}\frac{\dot{h}_+(v_x\,\hat{\mathbf{x}} - v_y\,\hat{\mathbf{y}})}{4} + \dots$$

Corrections controlled by $\rho v/\omega$, and as $\rho \to 0$ other helicities decouple

Full expressions are Bessel functions, convergent at large arguments

Radiation From Kicked Particle



For any scalar-like current, radiation amplitude from a kicked particle is

$$a_{h,k} \propto g\left(\frac{\tilde{J}_h(\rho\,|\,\epsilon_- \cdot p/k \cdot p\,|)}{k \cdot p} - \frac{\tilde{J}_h(\rho\,|\,\epsilon_- \cdot p'/k \cdot p'\,|)}{k \cdot p'}\right)$$

which exactly matches soft emission amplitudes fixed by general arguments

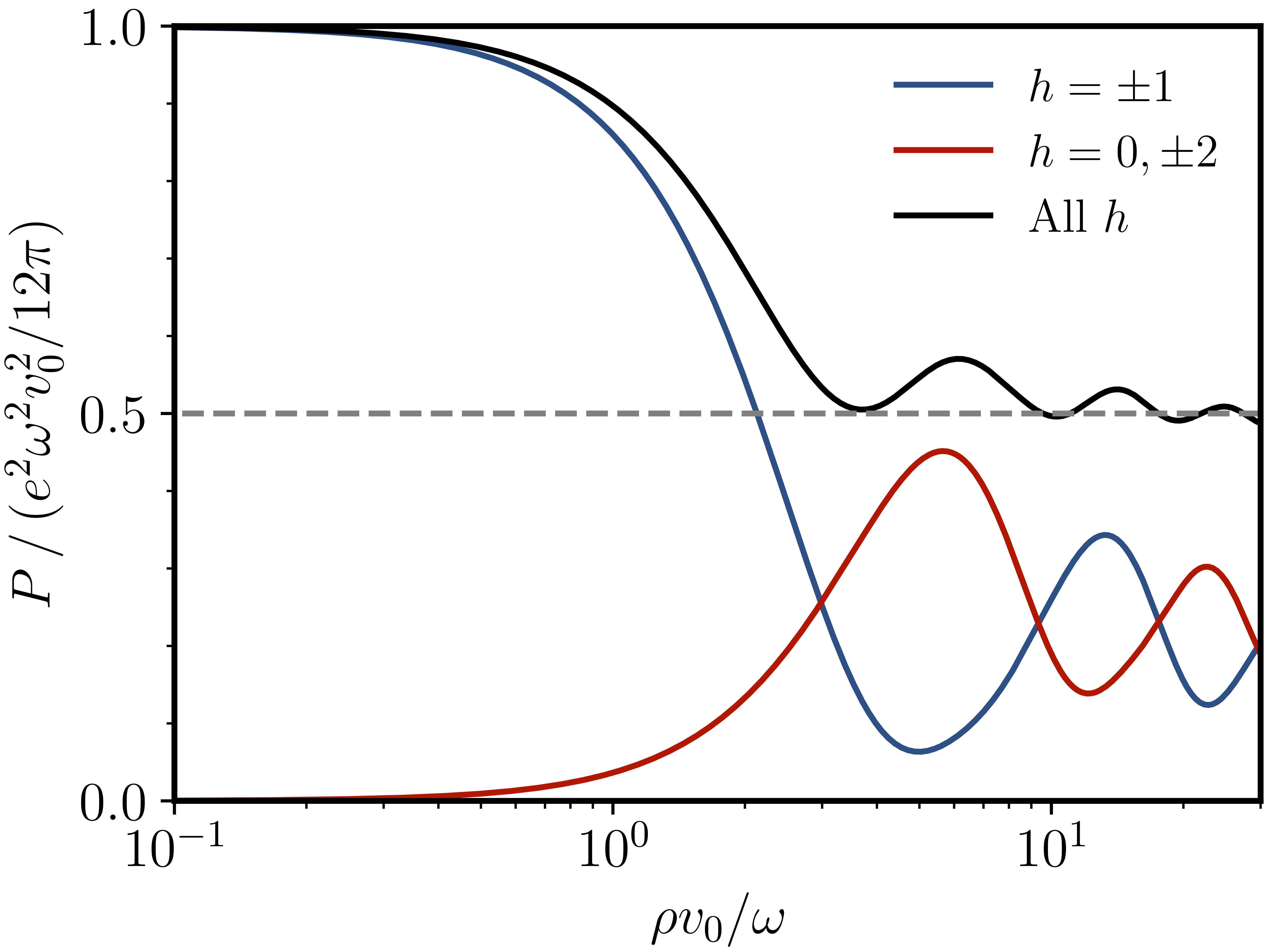
Same agreement for vector-like currents; in both cases other helicities decouple as $\rho \to 0$

Radiation From Oscillating Particle

Consider motion with amplitude $\ell = v_0/\omega$, and vector-like current

Radiation emitted in all helicities, and at large $\rho\ell$ many helicities contribute, but total power radiated is finite!

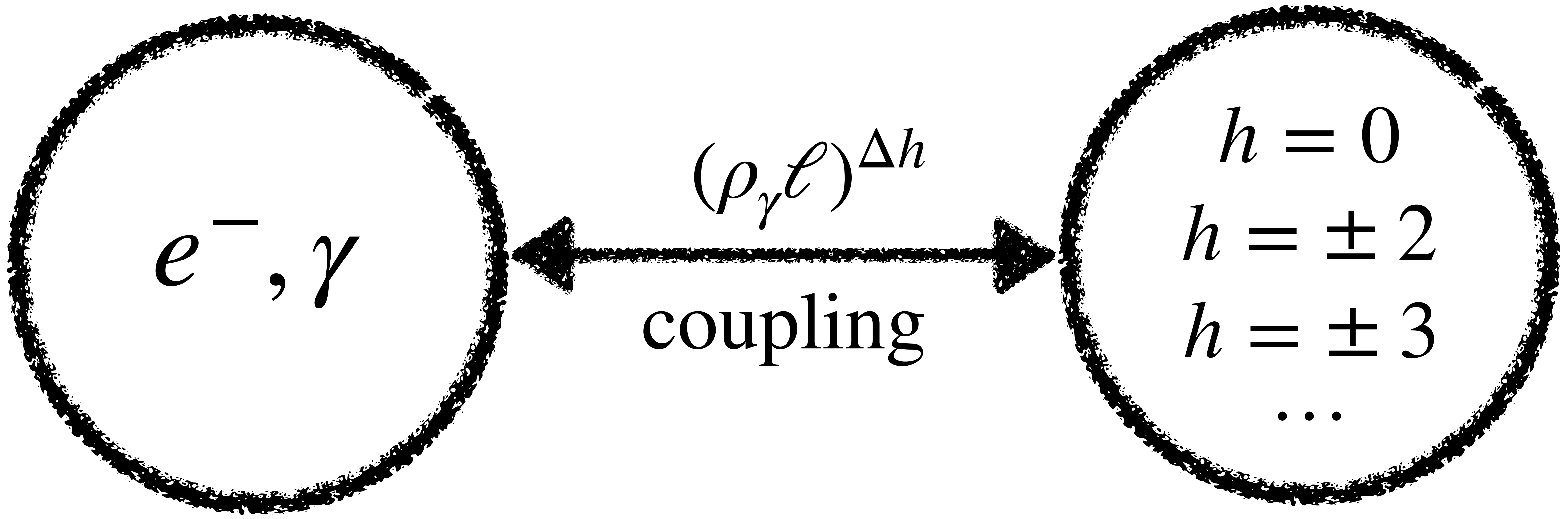
These are the calculations we've already done; from this point on things gradually get more speculative



Probing The Spin Scale of the Photon

For vector-like currents, $h = \pm 1$ could be observed photon

Other helicities are weakly coupled "dark radiation"

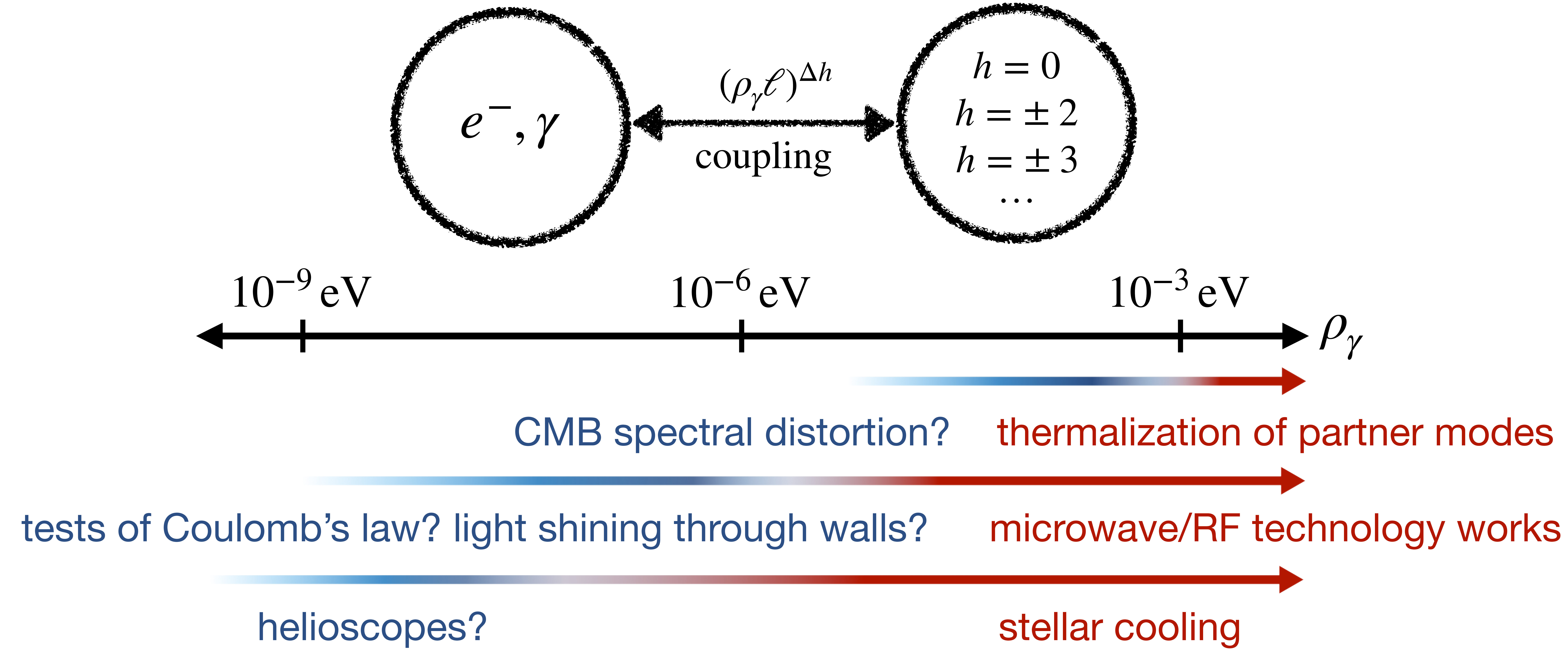


Familiar electron and photon

Photon partner polarizations

Sensitivity of various probes can be readily calculated

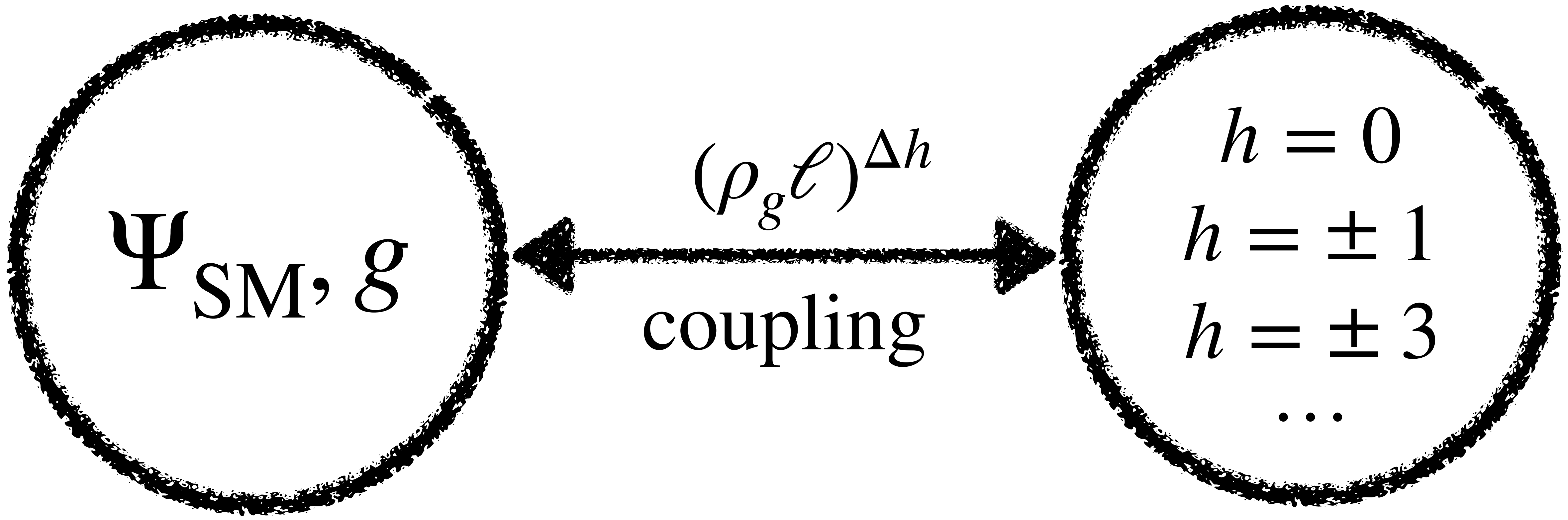
Probing The Spin Scale of the Photon



Very rough, preliminary estimates!

Probing The Spin Scale of the Graviton

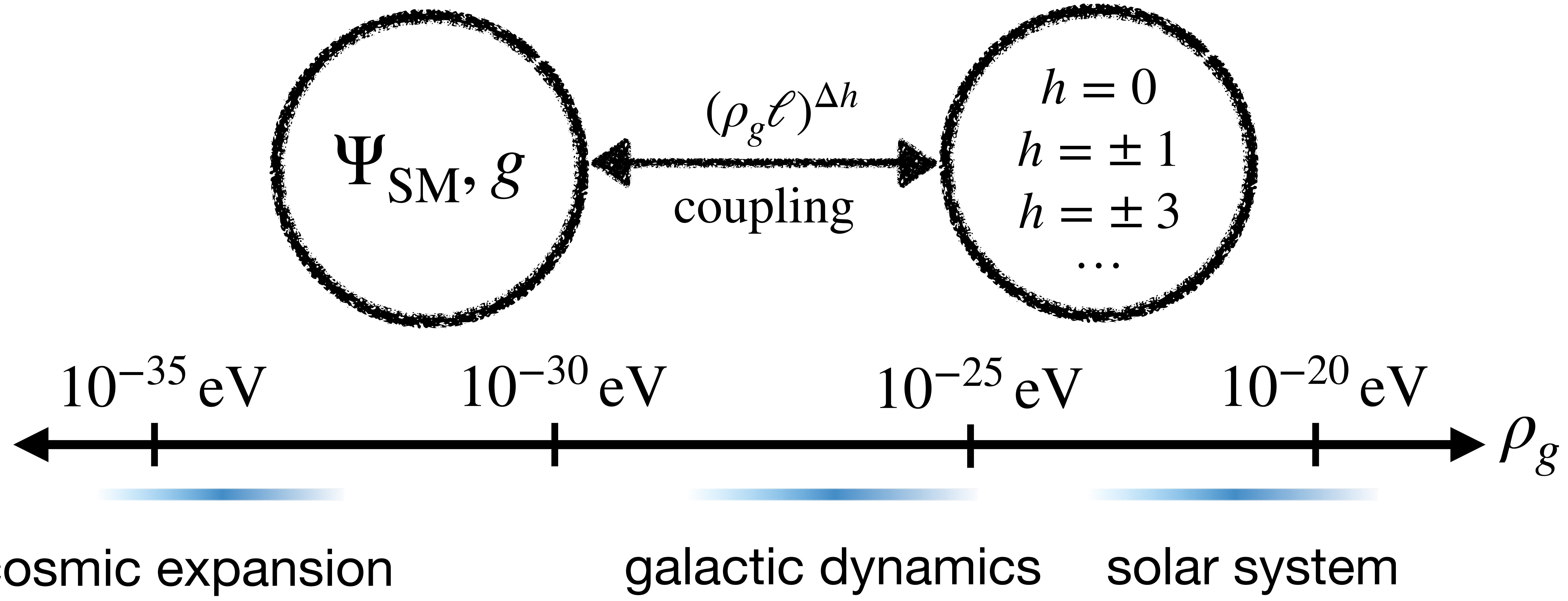
Natural next step, requires resolving some gauge subtleties of tensor-like currents



Linearized theory enough for many observables, but full treatment requires understanding nonlinear continuous spin gauge symmetry

(related but independent question: embedding interacting continuous spin fields in background curved spacetime)

Probing The Spin Scale of the Graviton

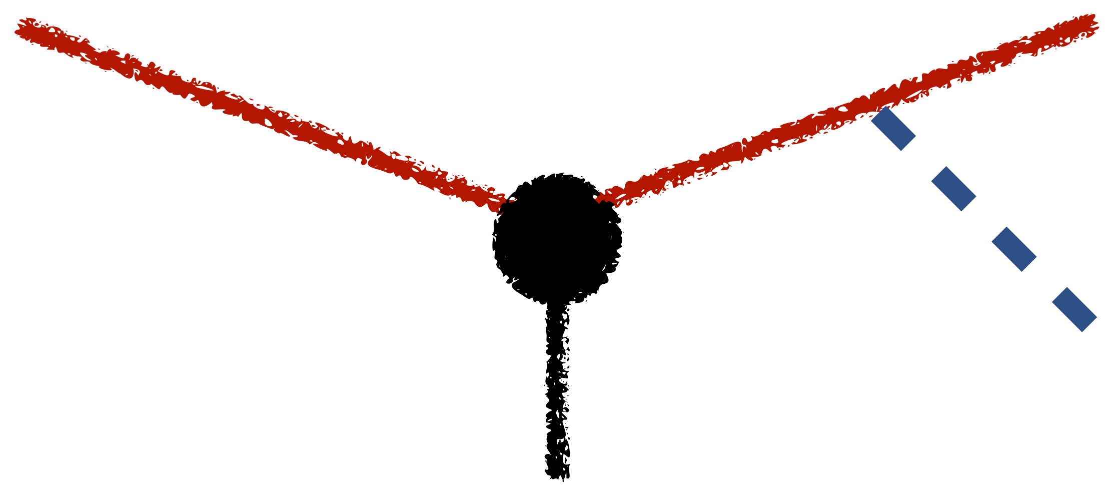


Certain scales motivated by potential deviations from inverse square force

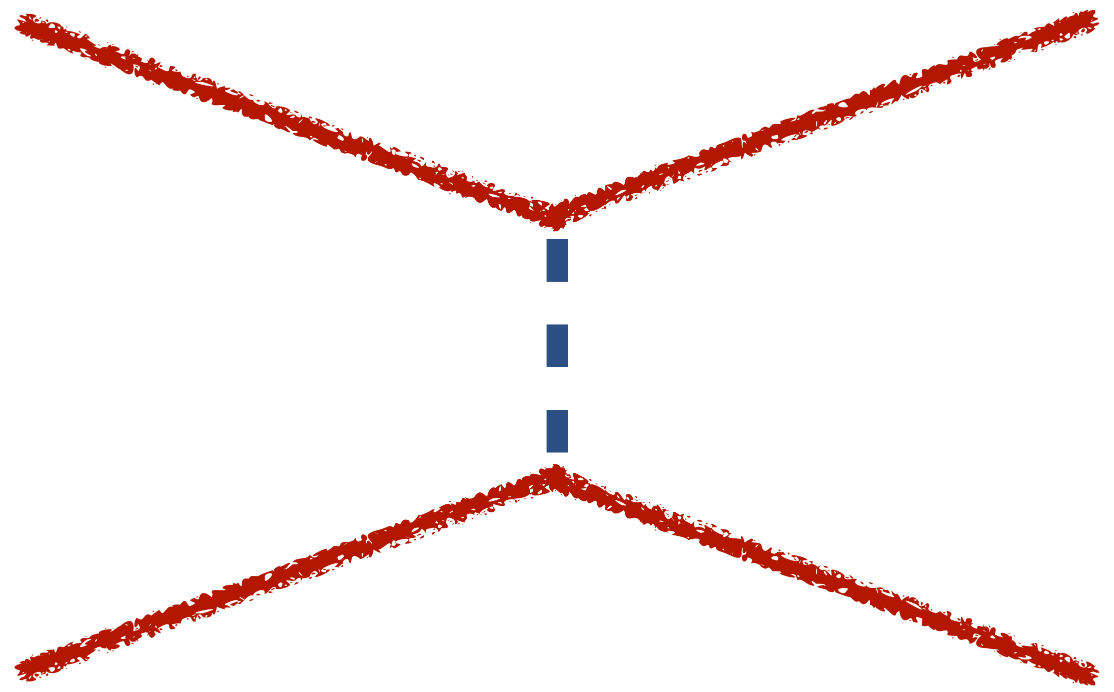
Can also probe universal deviations from gravitational radiation physics

Scattering Amplitudes

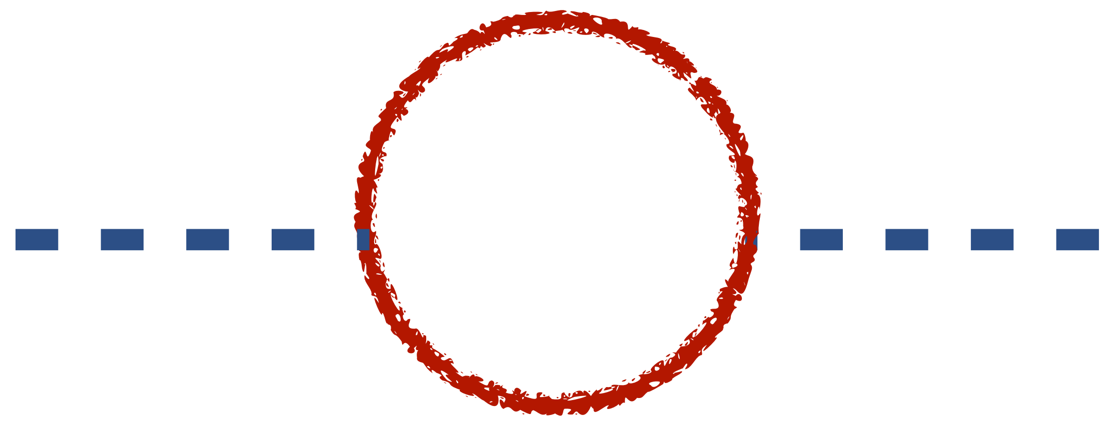
Starting from our action, can compute scattering amplitudes with path integral



CSP emission straightforward, recovers soft factors

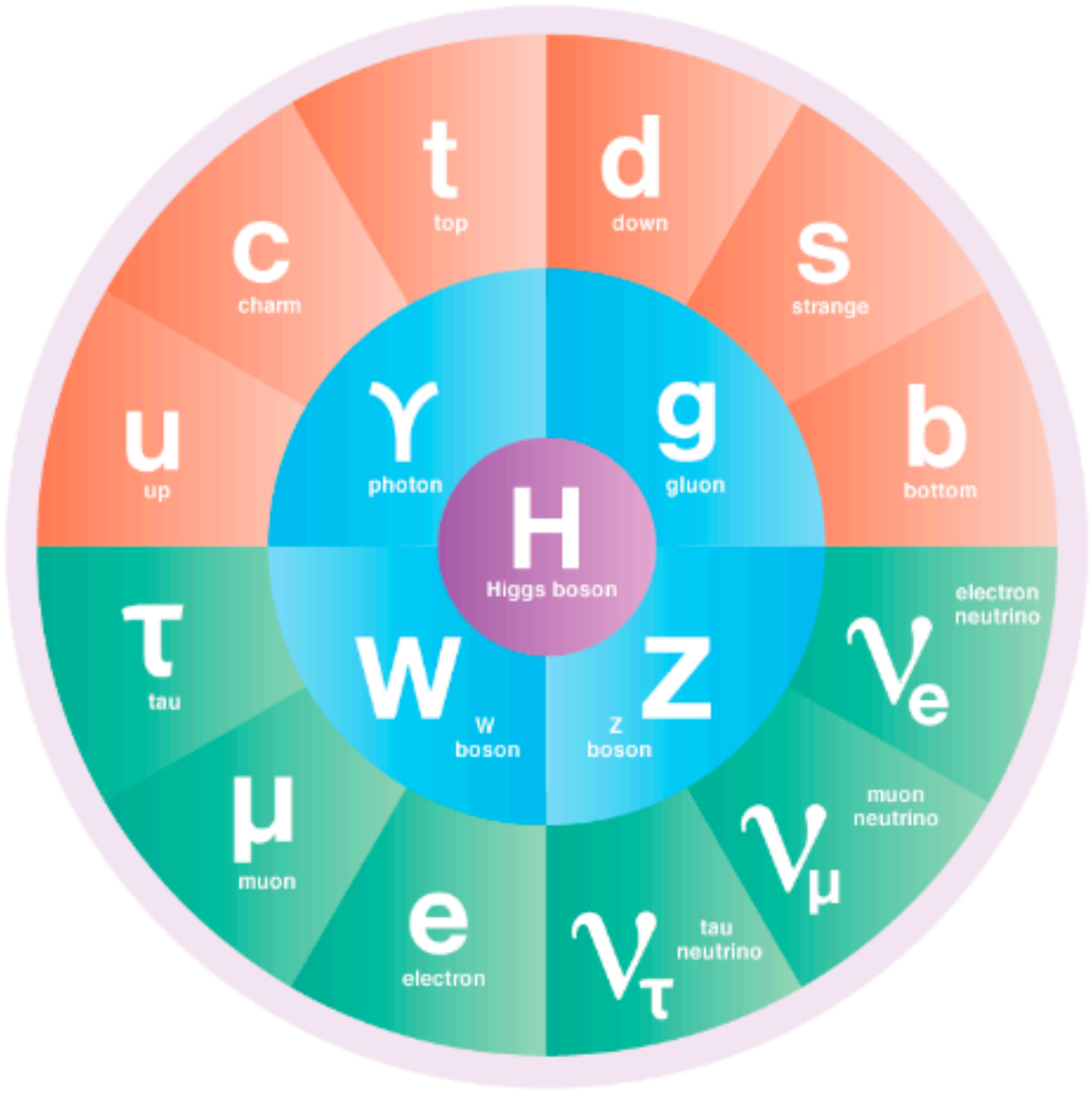


CSP exchange obeys tree-level unitarity,
due to completeness relation for helicity modes



Consistency at loop level unknown,
may place constraints on current

A Continuous Spin Standard Model?



Would ultimately want to embed a continuous spin photon within the electroweak sector

Need to give continuous spin fields mass

Need nonabelian continuous spin gauge symmetry

As a first step, consider a Stuckelberg mass term $m^2\Psi^2/2$

Yields massive weakly coupled partner polarizations; natural dark matter candidate?

Connections to the Hierarchy Problem

One framing: scalar particles cannot naturally mediate $1/r^2$ forces

Minimally coupled massless scalar receives large mass corrections $\delta m^2 \sim \Lambda_{\mathrm{UV}}^2$

Light axions have mass protected by shift symmetry — but requires derivative couplings, no $1/r^2$ forces

Continuous spin fields with scalar-like currents can mediate $1/r^2$ forces, and their mass is protected by their gauge symmetry!

How is this possible?

Protecting Scalar Masses: Bottom Up

One way to see how continuous spin protects the mass of a minimally coupled scalar: truncate the tensor expansion at order ρ

$$\mathscr{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\rho}{\sqrt{2}}(\phi\, \partial_\mu A^\mu) + \phi J - A_\mu J^\mu + \ldots$$

For scalar like current, J dominates and $A^\mu, J^\mu \propto \rho$ with $\partial_\mu J^\mu = -\rho J/\sqrt{2}$

Vector field can couple to nonconserved current due to its mixing with ϕ

Action has gauge symmetry $\delta A_\mu = \partial_\mu \epsilon/\sqrt{2}$, $\delta\phi = \rho\epsilon$

Forbids a scalar mass term for $\rho \neq 0$, but allows a minimal coupling ϕJ!

Our theory extends this to consistency at all orders in ρ

Protecting Scalar Masses: Top Down

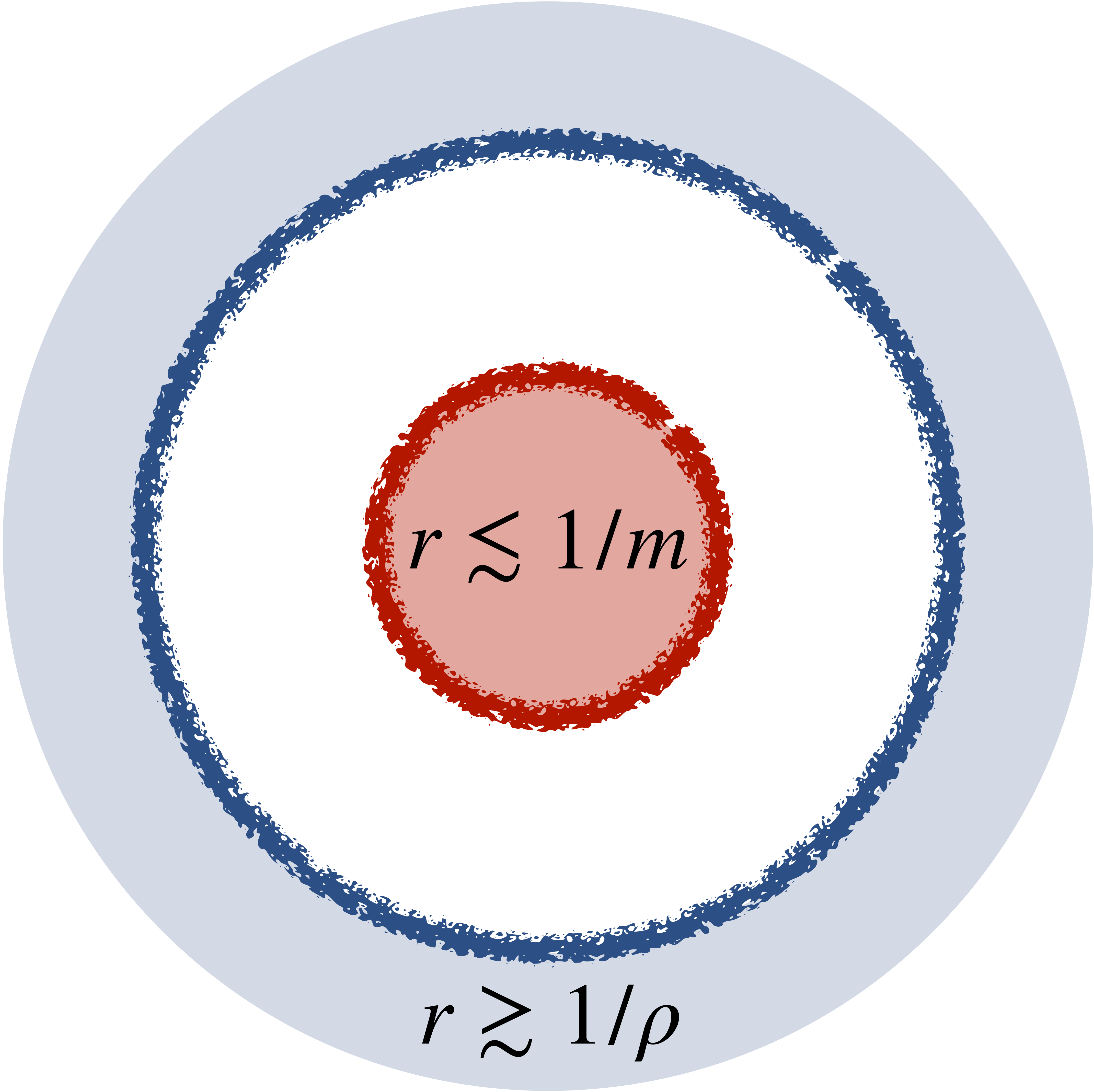
A deeper perspective: the action for our theory

$$S = \frac{1}{2} \int_{x,\eta} \delta'(\eta^2 + 1)(\partial_x \Psi)^2 + \frac{1}{2} \delta(\eta^2 + 1)(\Delta \Psi)^2 + \delta'(\eta^2 + 1) \Psi J$$

is symmetric under the bosonic superspace translation $\delta x^\mu = \omega^{\mu\nu} \eta_\nu$

Corresponds to tensorial conserved charge $i\eta^{[\mu} \partial_x^{\nu]}$ which mixes modes separated by **integer** helicity — a new exception to Coleman-Mandula

Motivates further development, to see if this can protect the mass of the Higgs at the quantum level



Continuous spin motivates viewing the Standard Model as an effective theory on both long and short distance scales

Nonzero spin scale produces universal, testable effects — and can shed light on a variety of fundamental problems

Much more work to be done!