Monodromy in the CMB: Chaotic Inflation & Gravity Waves in String Theory

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with: Liam McAllister & Eva Silverstein

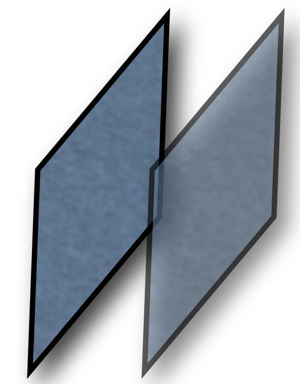
- **Moduli Stabilization met remarkable progress in recent years (mainly type II)**
- **Uses Fluxes (H_3, F_p, ...) and branes as essential tools**

- In IIB use $G_3 = F_3 - \tau H_3$ fluxes:
  gives warped geometry
  fixes dilaton S & complex structures U_I [GKP '01]

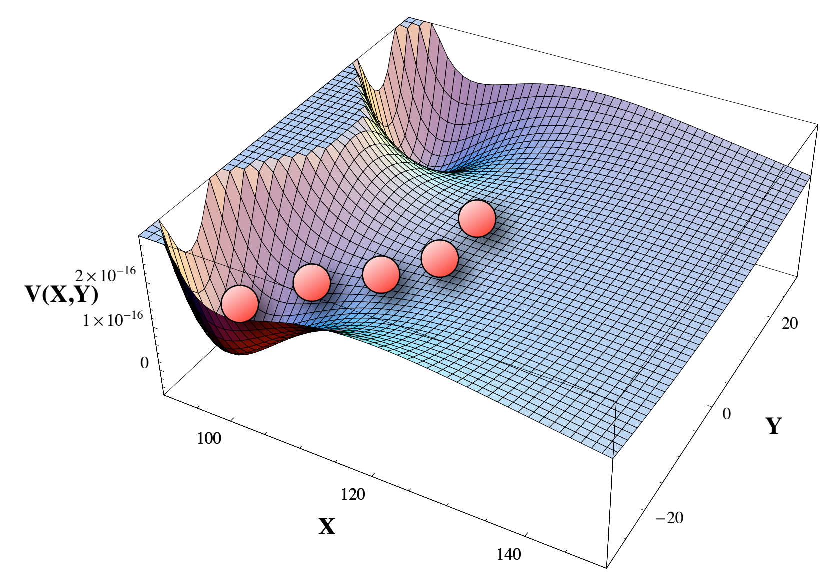
- non-perturbative effects (gauge or stringy instantons on D-branes) fix the Kahler moduli in AdS vacua; uplifting by anti-branes, *F*- or *D*-terms

  [KKLT '03, many others thereafter]

String(y) Inflation ...



- several IIB attempts in recent years ...
- position of moving D-branes as inflaton: e.g. realized in the D3-antiD3-brane model [KKLMMT ‘03], the D3-D7 model [Dasgupta et al. ’02, ...], or in DBI-inflation [Alishahiha, Silverstein & Tong ‘04] ...
- Modular inflation driven by Kähler moduli [Blanco-Pillado et al. ’04/’06, AW ’05, Conlon & Quevedo ’05, Bond et al. ’06, Linde & AW ‘07] ...



String(y) Inflation ...

- most *existing* IIB models are *small-field* models, driving inflation near a saddle or inflection point of the potential:
  [Blanco-Pillado et al. '04/'06, AW '05, Conlon & Quevedo '05, Bond et al. '06, Baumann et al. '07, Krause et al. '07, Linde & AW '07; Covi, Gomez-Reino, Gross, Louis, Palma & Scrucca '08 ...]

- thus no gravity waves due to Lyth bound:

$$r \equiv \frac{\mathcal{P}_T}{\mathcal{P}_S} \leq 0.003 \left(\frac{50}{N_e}\right)^2 \left(\frac{\Delta\phi}{M_{\rm P}}\right)^2$$

the reason ...

- string(y) inflatons: typically angular moduli, like (some) brane moduli or axions θ_a
  - shallower potential than for radial moduli
  - but have periodicity $\Delta\theta_a$ - limits field range

- example: axions

$$\int \sqrt{g} g^{\mu\nu} F_{\mu i_1 \cdots i_p} g^{i_1 i'_1} \ldots g^{i_p i'_p} F_{\nu i'_1 \cdots i'_p}$$

$$= \int \sqrt{g_4} f_a^2 (\partial_\mu \theta_a)^2 = \int \sqrt{g_4} (\partial_\mu \phi_a)^2$$

canonically normalized fields ϕ_a have periodicity $f_a \Delta\theta_a$

for compact spaces of size R :

[Banks, Dine, Fox & Gorbatov; Srvcek & Witten]

$$f_a \sim M_P \left(\frac{\sqrt{\alpha'}}{R} \right)^p \ll M_P$$

Wanted ...

- a *large field* model of inflation in string theory, i.e. "chaotic inflation"

- with control of the slow-roll parameters:

$$\epsilon \equiv -\frac{\dot{H}}{H^2} \simeq \frac{1}{2}\left(\frac{V'}{V}\right)^2 \quad , \quad \eta \equiv \frac{\dot{\epsilon}}{\epsilon H} \simeq \frac{V''}{V}$$

  over a super-Planckian field distance - avoid generic higher-dimension operators of effective field theory:

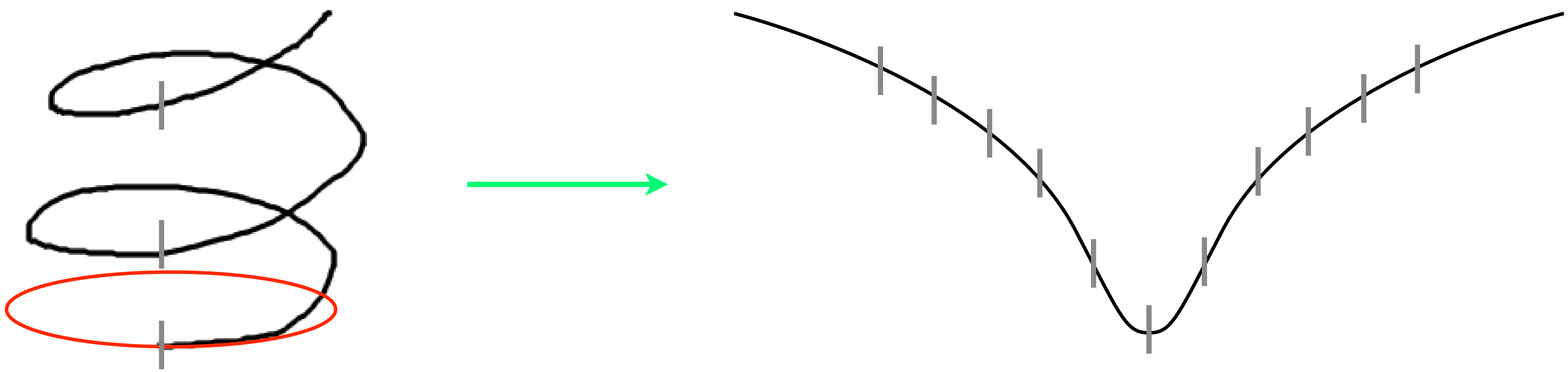
$$\delta V \sim V(\phi)\frac{(\phi-\phi_0)^2}{M_{\rm P}^2}$$

- idea: arrange for approximate shift symmetry of ϕ , broken only by the inflaton potential itself
  [Linde '83]

central idea ...

- unwind a periodic field direction into a *monodromy*

  → e.g. by employing a wrapped brane



$$\mathbf{V}(\phi > \mathbf{M_P}) \sim \begin{cases} \phi^{2/3}\,, & \text{moving D4 – brane in a Nil manifold} \\ \phi\,, & \text{2 – form axions on a 5 – brane} \end{cases}$$

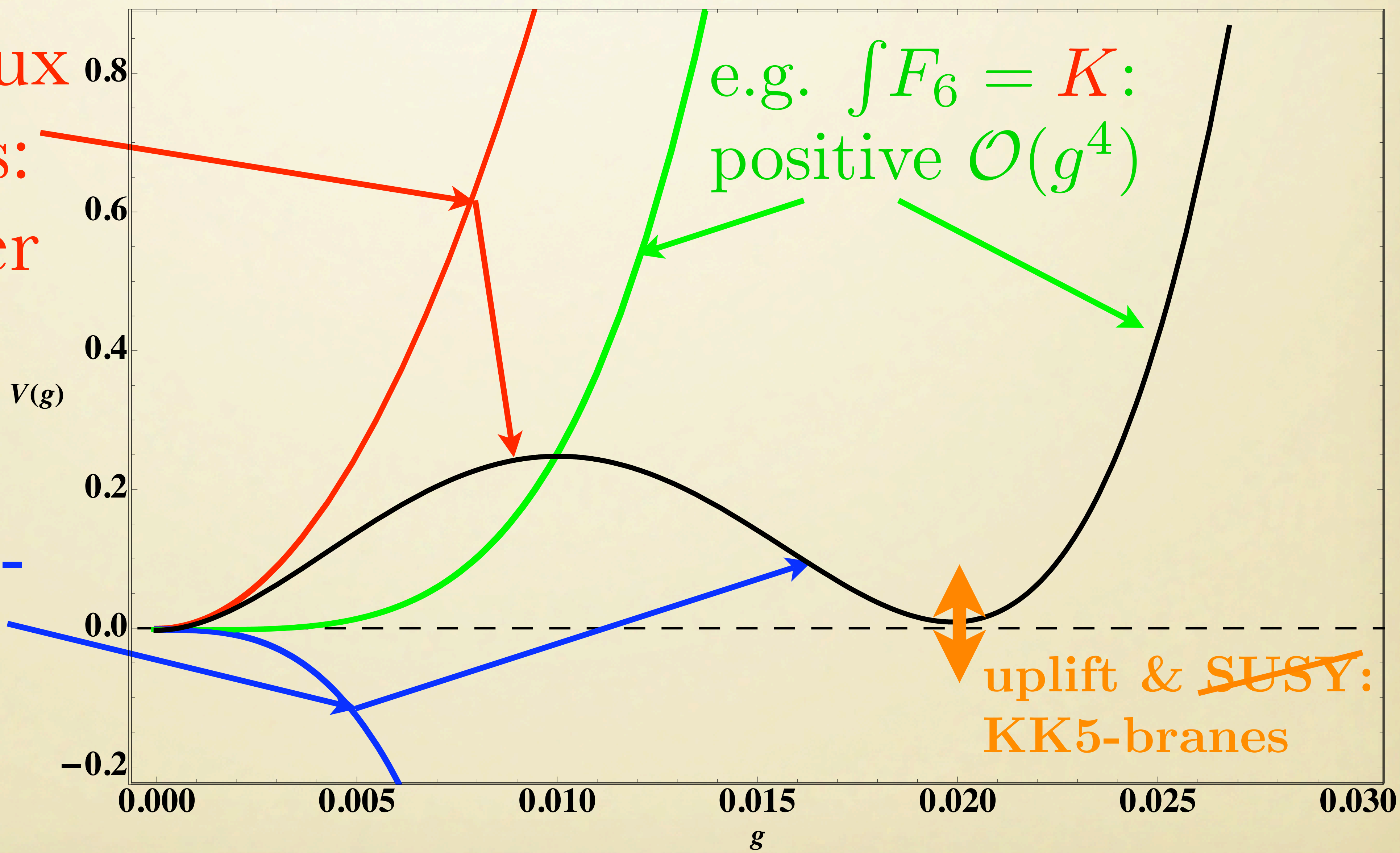
fast-track on IIA dS vacua [Silverstein '07]

$\mathcal{N}_3 \times \tilde{\mathcal{N}}_3$: Nil 3-fold with *negative* scalar curvature

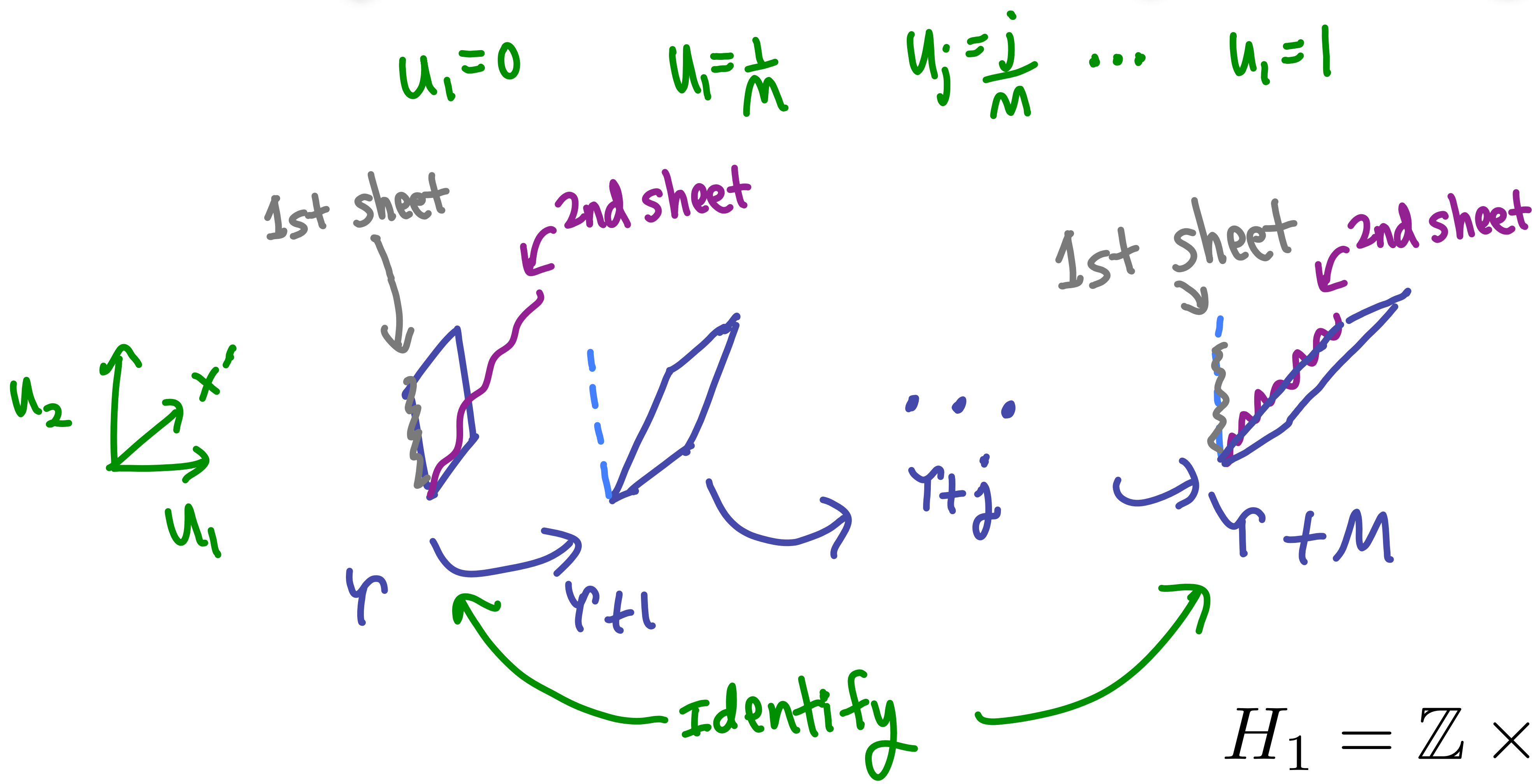
$$\left(ds^2 = L_{u_1}^2 du_1^2 + L_{u_2}^2 du_2^2 + L_x^2 \left(dx' + M u_1 du_2\right)^2 \right)^2$$

$R < 0$, H-flux & 5-branes: $O(g^2)$ barrier

$O6$-plane: negative $O(g^3)$-contribution



wrap a D4-brane on a cycle along u_2 ...



$$H_1 = \mathbb{Z} \times \mathbb{Z} \times \mathbb{Z}_M$$

i.e. the D4-brane experiences a <u>monodromy</u> when moving around the u_1-circle:

D4 at $u_1 = j/M < 1$ wraps homologically different 1-cycle in (x', u_2) & brane tension grows without end

kinematics ... arbitrary large field range

- can see the monodromy in the D4's DBI-action:

$$S_{\rm D4} = -\int d^5\xi\, e^{-\Phi}\sqrt{\det G_{MN}\,\partial_\alpha X^M \partial_\beta X^N} + S_{CS} + loops$$

$$S_{{\rm D4},\mathcal{R}} = \frac{1}{g_s}\int d^4x\sqrt{-g_4}\left(\frac{L_u^2}{2\beta}\dot{u}_1^2\sqrt{\beta L_u^2 + L_x^2 M^2 u_1^2} - \sqrt{\beta L_u^2 + L_x^2 M^2 u_1^2}\right)$$

Nil geometry

define ratio:

$$\beta = \frac{L_{u_2}}{L_{u_1}}$$

$$L_u^2 = L_{u_1} L_{u_2}$$

fixed by SUSY breaking KK5-branes

curvature induced inflaton potential - 2 regimes

$$S_{\mathrm{D4},\mathcal{R}} \sim \frac{1}{g_s}\int d^4x\sqrt{-g_4}\left(\frac{L_u^2}{2\beta}\sqrt{\beta L_u^2 + L_x^2 M^2 u_1^2}\,\dot{u}_1^2 - \sqrt{\beta L_u^2 + L_x^2 M^2 u_1^2}\right)$$

regime 1 $\quad u_1 < u_{1,\mathrm{crit}} = \sqrt{\beta}\frac{L_u}{L_x M} \quad :$

$$S_{\mathrm{D4},\mathcal{R}} \sim \frac{1}{g_s}\int d^4x\sqrt{-g_4}\left(\frac{L_u^3}{2\sqrt{\beta}}\,\dot{u}_1^2 - \frac{L_x^2 M^2}{\sqrt{\beta}L_u}u_1^2\right)$$

$$\sim \dot{\phi}^2 - m^2\phi^2 \quad for \quad \phi \sim u_1$$

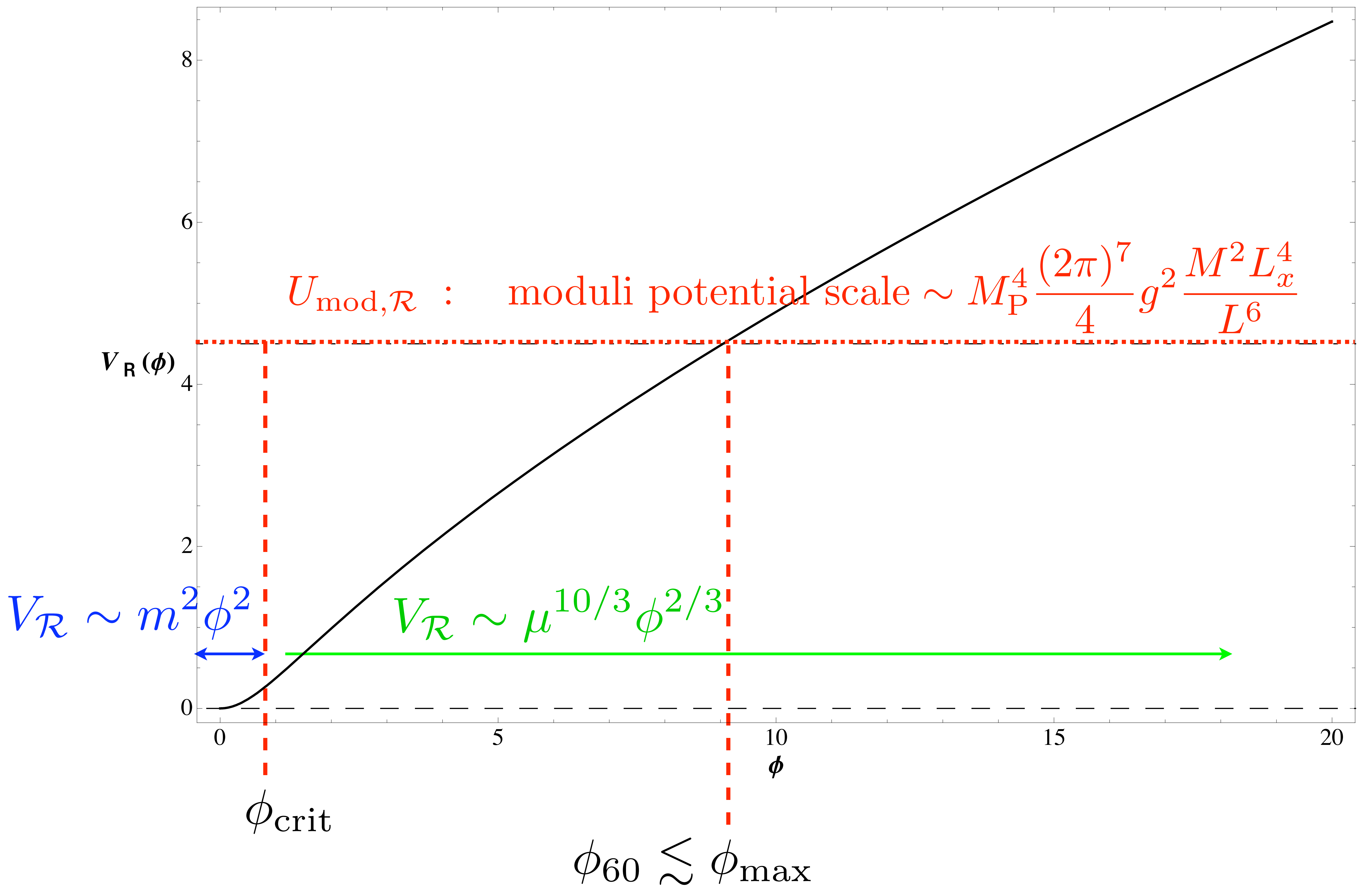
$$\Rightarrow u_{1,\mathrm{crit}} \sim \phi_{\mathrm{crit}} \lesssim M_{\mathrm{P}}$$

regime 2 $\quad u_1 > u_{1,\mathrm{crit}} \quad :$

$$S_{\mathrm{D4},\mathcal{R}} \sim \frac{1}{g_s}\int d^4x\sqrt{-g_4}\left(\frac{L_u^2}{2\beta}L_x M u_1 \dot{u}_1^2 - L_x M u_1\right)$$

$$\sim \dot{\phi}^2 - \mu^{10/3}\phi^{2/3} \quad for \quad \phi \sim \frac{1}{\sqrt{\beta}}u_1^{3/2}$$

curvature induced inflaton potential ...



I: consistency with moduli stabilization ...

- ϕ^2-regime destabilizes moduli for $\phi > M_P$:

$$V_{\mathcal{R}}(\phi < \phi_{\rm crit}) = \frac{1}{2}m^2\phi^2 \sim \left(\frac{\phi}{M_{\rm P}}\right)^2 M_{\rm P}^4 \frac{(2\pi)^7}{4} g^2 \frac{M^2 L_x^4}{L^6} \sim \left(\frac{\phi}{M_{\rm P}}\right)^2 \mathcal{U}_{mod,\mathcal{R}}$$

scale of moduli potential, “barrier”

$$L^6 \equiv 6 - \text{volume} = L_x^2 L_u^4 \sim \int F_6 \sim K$$

- $\phi^{2/3}$-regime: determine $\phi_{\max}$ from

$$V_{\mathcal{R}}(\phi > \phi_{\rm crit}) = \mu^{10/3}\phi^{2/3} < U_{mod,\mathcal{R}}$$

II: back-reaction & string corrections ...

- then in concrete IIA setup of [Silverstein '07] can arrange for:

$\Rightarrow$

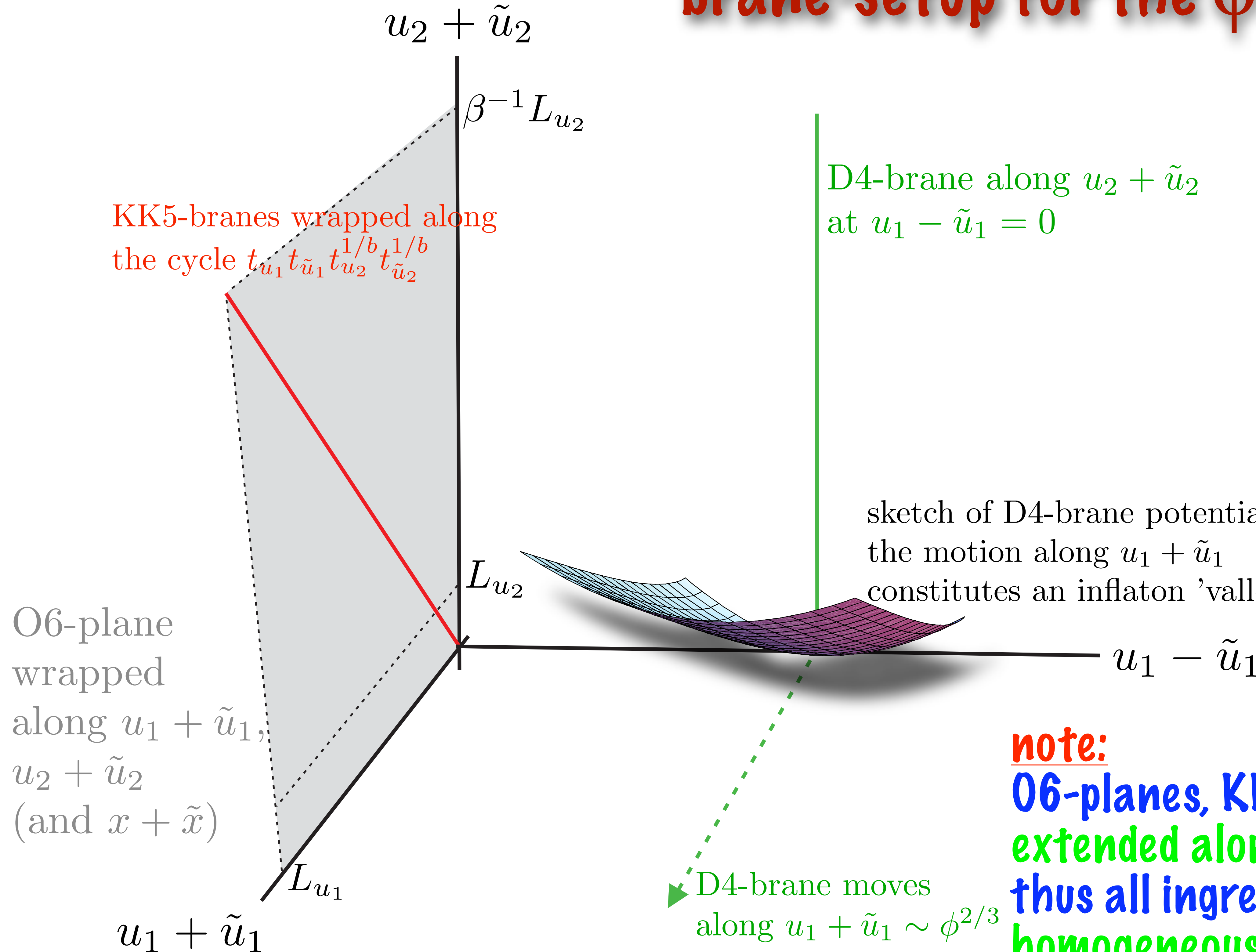
M=1, $\int F_6 \sim K \sim 10^6$

& stabilize ratio $\beta = L_{u2}/L_{u1} \sim 0.05$ with KK5-branes (see below)

$$\phi_{\rm crit} < M_{\rm P} \quad \& \quad \phi_{\rm max} \gtrsim \phi_{60} \simeq 9 M_{\rm P}$$

$$\text{and} \quad \frac{\delta\rho}{\rho}\bigg|_{60} \simeq 2 \times 10^{-5}$$

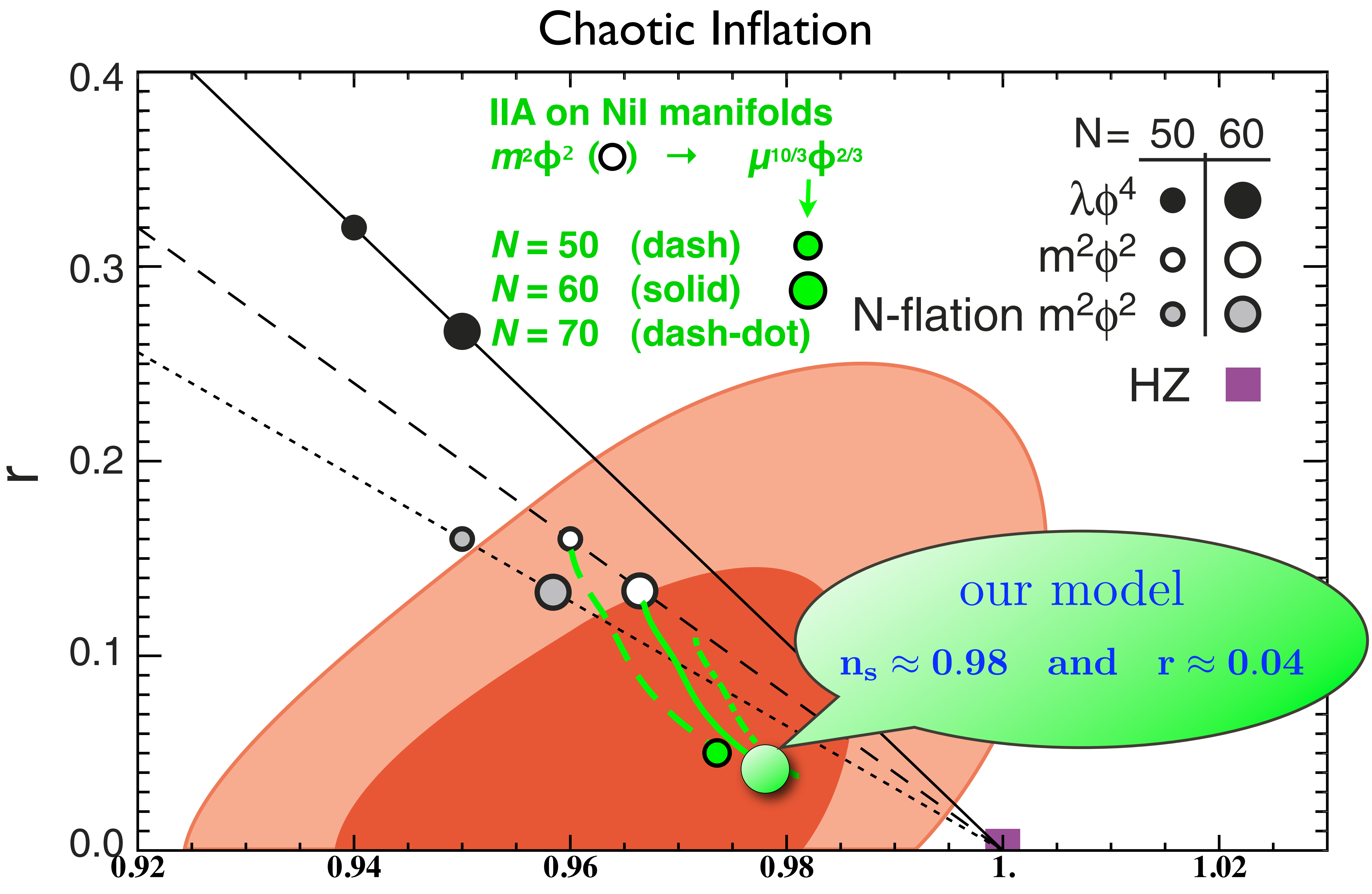
brane-setup for the $\phi^{2/3}$ ...



<u>note:</u>

O6-planes, KK5-brane & fluxes
extended along D4-motion ...
thus all ingredients are
homogeneous along inflaton
direction, with homogeneity
broken by D4-potential itself ...
thus approximate shift symmetry

Predictions for n_s and r:



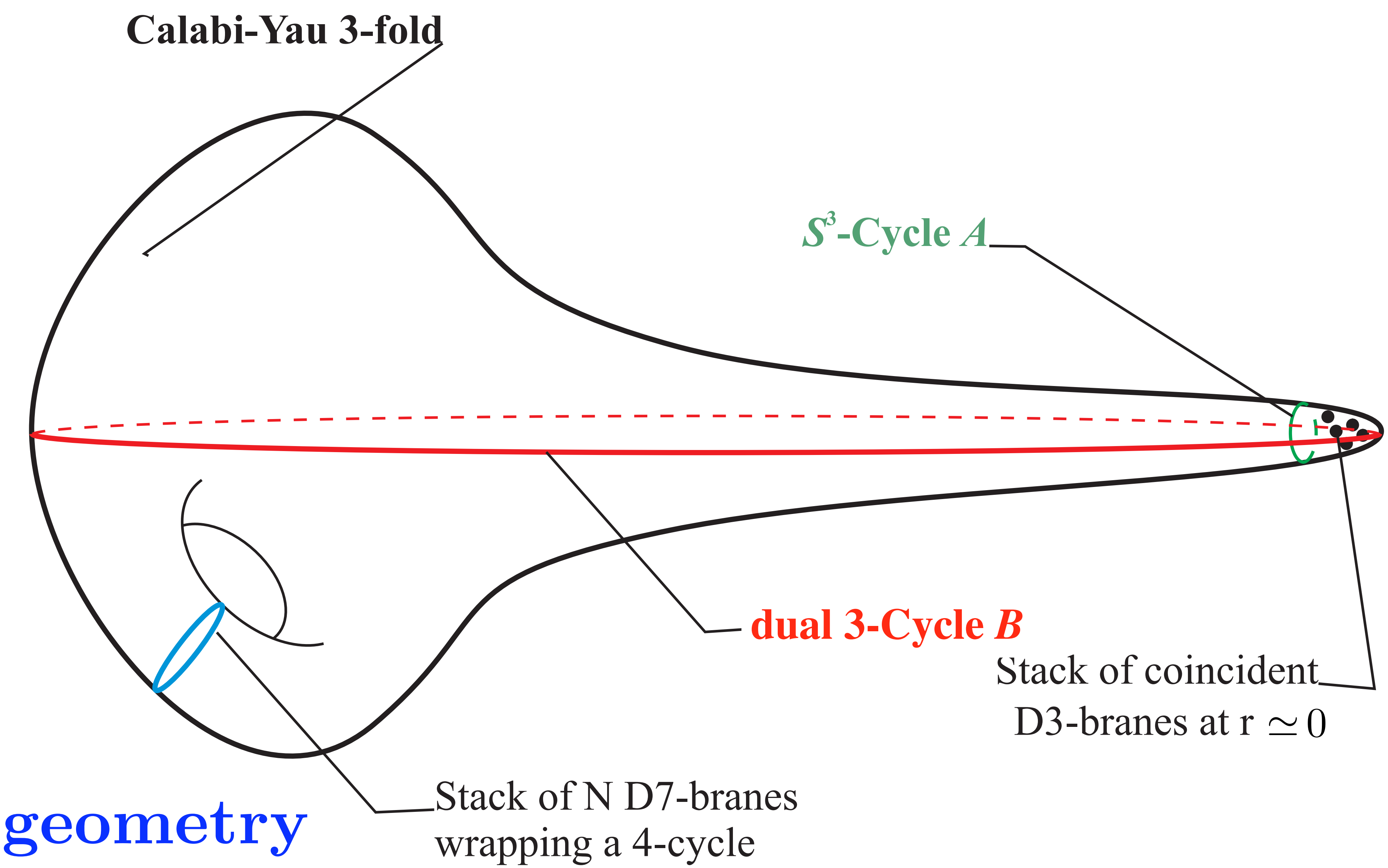
2nd example - type IIB: GKP-KKLT & linear axion inflation

- We would like to use monodromies in closed string moduli space on warped CYs in IIB

Turn on fluxes

$$\int_A F_3 = M$$

$$\int_B H_3 = -K$$



gives warped geometry

fixes dilaton S & complex structures U_I [GKP '01]

mondromies in IIB Kahler moduli space

- simple IIB-monodromy on a CY: B_2 or C_2

$b \equiv \int_{\Sigma_2} B_2^{\rm NS}$ or $c \equiv \int_{\Sigma_2} C_2^{\rm RR}$ on a 2-cycle Σ_2 wrapped by a 5-brane,

b and c are part of the $\mathcal{N}=2$ Kähler moduli multiplets

$\Rightarrow$ similar to monodromies of mirror-dual CY-pairs around the large complex structure point [e.g. Candelas et al. '91]

- explicitly visible: wrap a D5-brane on 2-cycle

$$S_{\rm DBI}^{D5} = -\frac{1}{(2\pi)^6 g_S \alpha'^3}\int_{\mathcal{M}_4\times\Sigma_2} d^6\xi\sqrt{\det(G+B)}$$

$$= -\frac{1}{(2\pi)^4 g_S \alpha'^2}\int d^4x\sqrt{-g}\sqrt{L_{\Sigma_2}^4 + b^2}$$

breaks perturbative shift symmetry in B_2

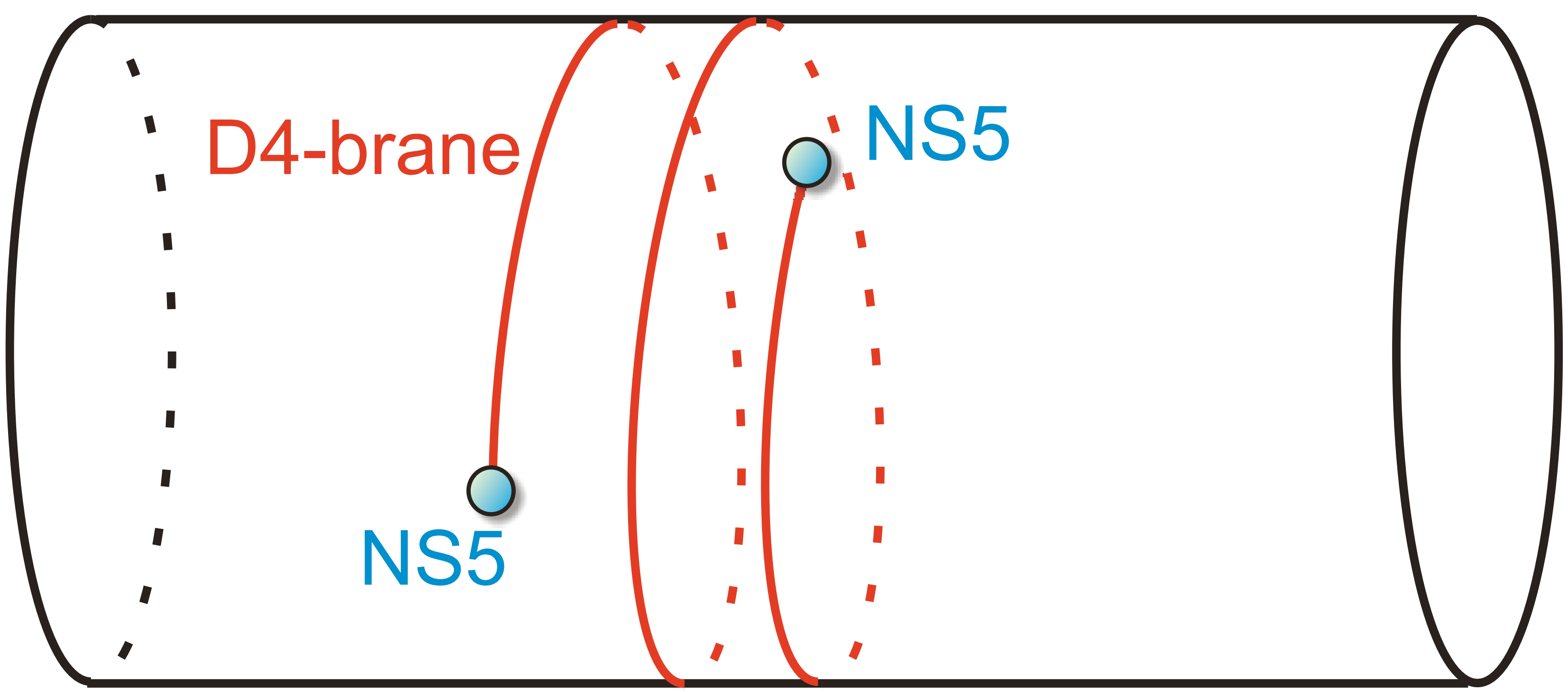
similarly, NS5-brane breaks C_2 shift symmetry

- possible realization:

$\mathbb{C}^2/\mathbb{Z}_2$ orbifold, local model to be glued onto a GKP-style warped CY has $\mathcal{N}=4$ SUSY broken to $\mathcal{N}=2$ at fix points

wrap D5 on shrunken blow-up orbifold 2-cycle, turn on B_2 there

described by a gauge group quiver - T-dual to a D4 stretched between 2 NS5's along a circle in IIA:



an η-problem for B_2 as the inflaton ...

structure of 4d N=1 multiplets poses a problem:

O7-orientifold splits 4d $\mathcal{N}=2$ Kahler moduli into chiral multiplets

$$T_\alpha = \frac{vol(\Sigma_4^{(\alpha)})}{g_s} + i\int_{\Sigma_4^{(\alpha)}} C_4 \quad \text{and} \quad G^a = \frac{b^a}{g_s} - ic^a$$

superpotential depends through instantons holomorphically on T_α, G^a [e.g. Grimm '07], thus stabilizes the fields T_α and G^a separately

consider most simple case:

one field T, one field G

then the Einstein frame Calabi-Yau volume $\mathcal{V}_E$ is:

even-odd intersection number: κ_{TGG}

$$\mathcal{V}_E \sim \left[T + \bar{T} - \kappa g_s (G + \bar{G})^2\right]^{3/2}$$

- **alternative: wrap an NS5 with C_2 on 2-cycle**

  D5-brane is a BPS-state

  S-duality thus links D5- and NS5-tension, implying:

  for B_2, C_2 large

$$S_{\rm NS5} \;=\; -\frac{1}{(2\pi)^4 g_S^2 \alpha'^2}\int d^4x\sqrt{-g}\sqrt{L_{\Sigma_2}^4 + g_S^2 c^2}$$

$$\Rightarrow \quad V(c)\sim c$$

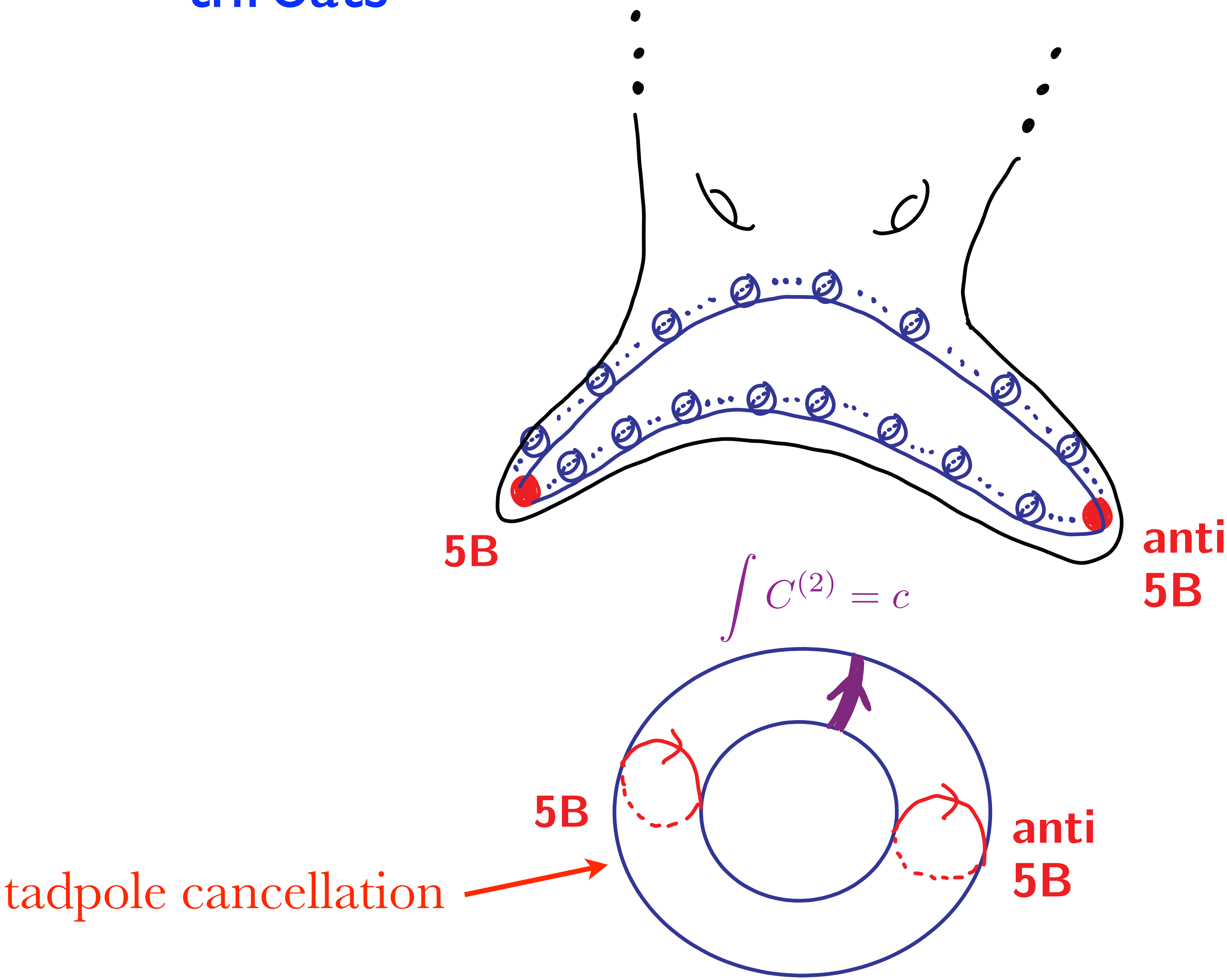
- **thus *kinematically* unbounded field range - but limited dynamically:**

$$V_{D5,NS5} < U_{mod} \;,\; U_{mod} \text{ scale of moduli potential}$$

warp 5-brane tension down by placing it down a throat

make GUT scale (for $\delta\rho/\rho$)

- a setup: family of 2-cycles (anti)invariant under a freely acting O7, reaching down into warped throats



way out: use C_2 on an NS5-brane ...

- shift symmetry in C_2 preserved in tree-level K

but potentially dangerous:

$ED1$ instantons: in K only by holomorphy

$ED3$ instantons / with dissolved D1-branes

$$\begin{aligned} K &= -3 \ln \left[\underbrace{T + \bar{T}}_{\sim v^2} + e^{-2\pi v - G/(2\pi)} \right] \\ \Delta W_c &= e^{-T} \left[1 + e^{-G/(2\pi)^2} \right] \end{aligned}$$

gives dangerous $\cos(c)$-dependences in potential

suppressing instantons ...

- can avoid this by stabilizing *T*'s with D7-branes:

$$\Delta W_c = e^{-\frac{2\pi}{N} f(T)}$$

gaugino condensation from stack of D7-branes

holomorphy and *N*=1 supersymmetry constrain *f(T)*

$$f(T) = T + \text{ 1-loop} + \text{instantons}$$

- in absence of B_2 then by holomorphy & large-volume asymptotics: [Witten '95]

$$f(T) = T + e^{-T-G/(2\pi)^2}$$

$$\Rightarrow \quad \Delta W_c = e^{-T} + e^{-2T} e^{-G/(2\pi)^2} + \ldots$$

suppressing instantons ...

- using D7-brane gaugino condensation to stabilize:

$$\mathrm{Re}\, T \sim v^2 > 1$$

implies then

$$\begin{aligned} \Delta K &\sim e^{-2\pi v} \ll K_0 = -3\ln(2\,\mathrm{Re}\, T) \\ \Delta W_c &= e^{-T}\left[1 + \mathcal{O}(e^{-T})\right] \end{aligned}$$

- thus, stabilizing the *T*'s at finite volume with D7-brane stacks gives natural exponential suppression of the cos(*c*) modulation

a lot of consistency conditions to meet ...

- the strongest: avoiding excessive backreaction

$$r^4_{core,D3} \sim \alpha'^2 g_s N_w < R^4_\perp \quad , \quad N_w \equiv N_{D3}$$

$$\Rightarrow \quad N_w < \frac{R^4_\perp}{g_s \alpha'^2} \quad , \quad N_w \sim \frac{\phi_a}{f_a (2\pi)^2} \sim \frac{R^2_\perp}{\alpha'}$$

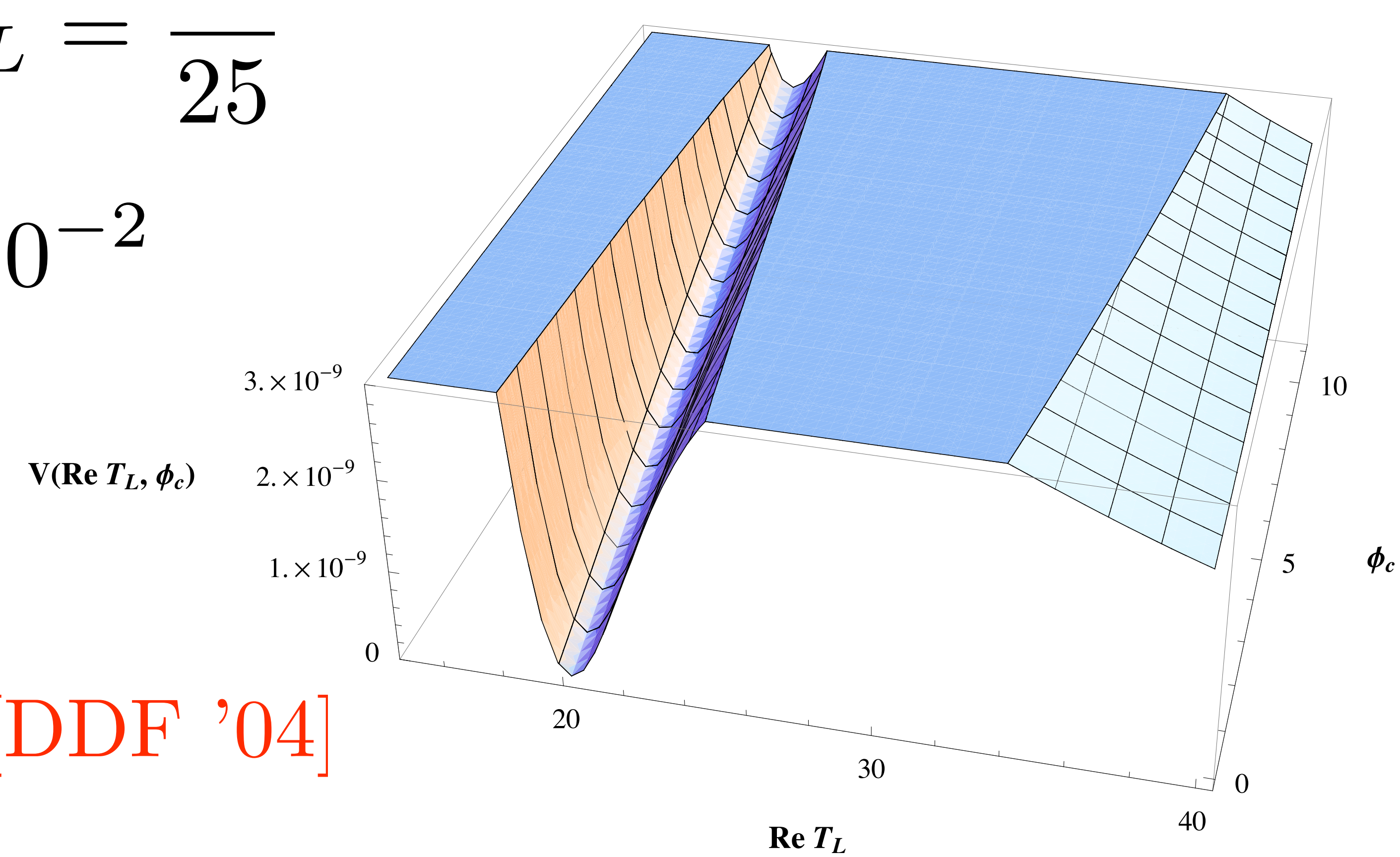
a semi-explicit toy model ... satisfying all constraints

$$K = -2\ln \mathcal{V}_E = -2\ln\left\{(T_L+\bar{T}_L)^{3/2} - \left[T_+ + \bar{T}_+ + \frac{3}{8}g_s c_{+--}\,(G_- + \bar{G}_-)^2\right]^{3/2}\right\}$$

$$W = W_0 + A_L e^{-a_L T_L} + A_+ e^{-a_+ T_+}$$

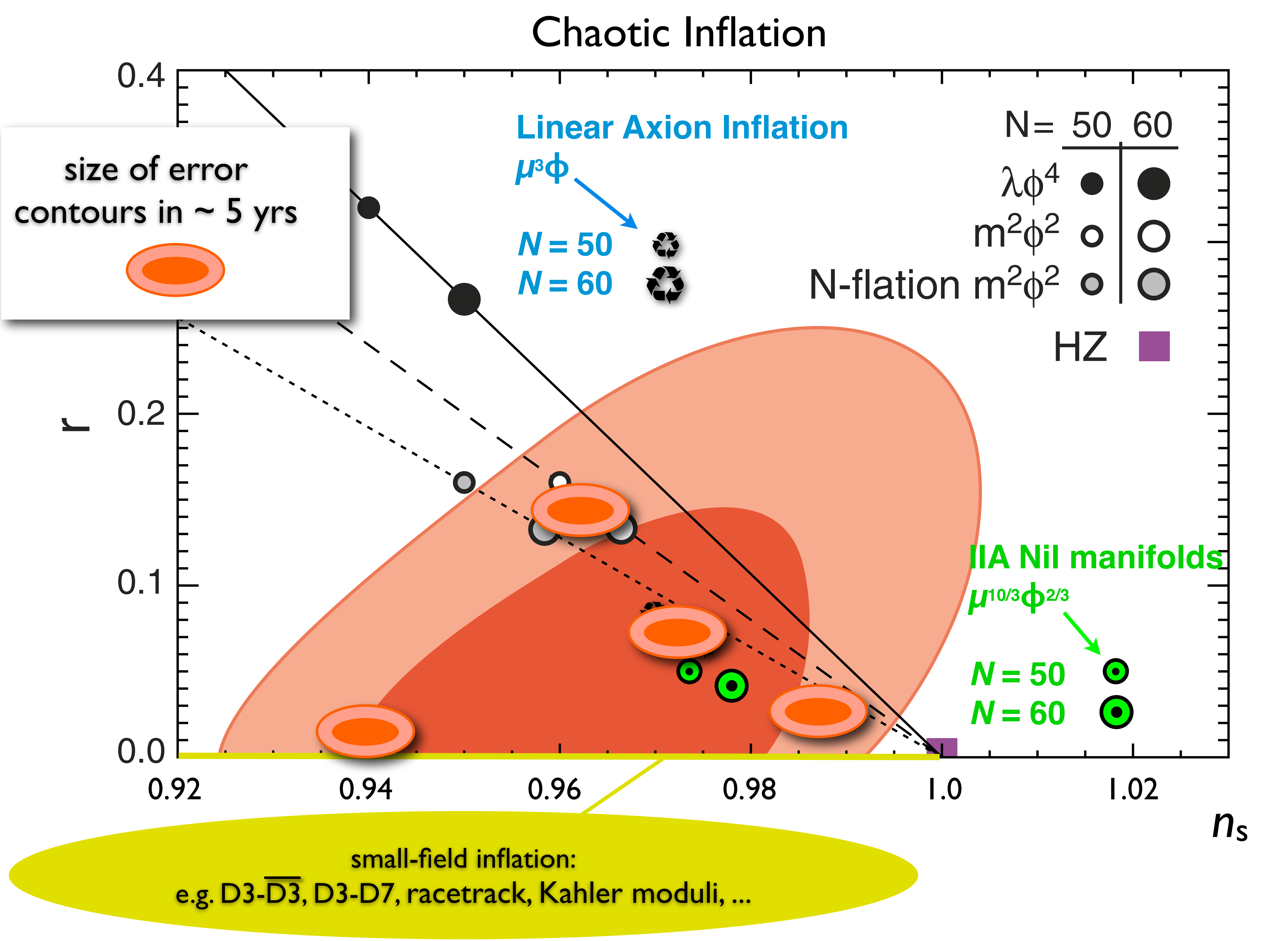
$$A_L = -1\,,\ A_+ = 1\,,\ a_L = \frac{2\pi}{25}$$

$$a_+ = \frac{2\pi}{3}\,,\ W_0 = 3\times 10^{-2}$$



very similar to e.g. $\mathbb{P}_{11169}$ in [DDF '04]

$$\Rightarrow \quad T_L \sim 20 \quad , \quad T_+ \sim 4 \quad , \quad b \sim 0$$


Chaotic Inflation
size of error contours in ~ 5 yrs
Linear Axion Inflation
μ³φ
N = 50
N = 60
N= 50 60
λφ⁴
m²φ²
N-flation m²φ²
HZ
IIA Nil manifolds
μ^{10/3}φ^{2/3}
N = 50
N = 60
r
0.4
0.2
0.1
0.0
0.92
0.94
0.96
0.98
1.0
1.02
n_s
small-field inflation:
e.g. D3-D̄3, D3-D7, racetrack, Kahler moduli, ...

Gravity Waves from Monodromies

[Silverstein & AW '08]
[McAllister, Silverstein & AW '08]

- Lyth bound: $\frac{\Delta\phi}{M_P} \sim \left(\frac{r}{0.01}\right)^{1/2} \geq 1 \Leftrightarrow \begin{cases} \text{observable!} \\ \text{UV sensitive} \end{cases}$ **5 yrs!**

  (tensor/scalar)

  chaotic inflation [Linde '83]
  natural inflation [Freese et al. '90]

  $\Rightarrow$ $\Delta\phi > M_P$ protected by symmetry:

  Can this arise naturally in string theory?

- **Yes:** Monodromy $\begin{cases} \text{would-be periodic direction (brane position, axions, ...)} \\ \textbf{not} \text{ periodic in presence of } \textbf{wrapped} \text{ brane:} \end{cases}$

  $\rightarrow$ kinematically unbounded field range; potential from brane action:

$$\mathcal{L} \sim \begin{cases} u\dot{u}^2 - \sqrt{1+u^2} \;\Rightarrow\; V(\phi) \sim \phi^\alpha\,,\; \alpha = 2/3\,, & \text{Nil manifold} \\ f_a^2 \dot{a}^2 - \sqrt{\ell^4 + a^2} \;\Rightarrow\; V(\phi) \sim \phi^\alpha\,,\; \alpha = 1\,, & \text{axions in e.g. CYs} \end{cases}$$

- Systematic control:

  predictive: $n_s = 0.98 \quad r = 0.04$; $n_s = 0.975 \quad r = 0.07$

  shift symmetry weakly broken by $V(\phi)$;
  Nil manifold case: simple & explicit construction, $O(1\%)$ tuning
  Calabi-Yau case: holomorphy & exponential suppression of instantons control corrections naturally for axion monodromies