Resonant tunnelling in the landscape?

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Based on work with Ed Copeland and Paul Saffin

See arXiv:0709.0261 [hep-th], 0804.3801 [hep-th]

Overview

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- The string landscape

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- Resonant tunnelling in the landscape?

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- Resonant tunnelling in optics!

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- Resonant tunnelling in optics!
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- What is required for resonant tunnelling in QFT?
- A no-go theorem
- Getting around our no-go theorem

The string theory landscape

At the last count, there were 10^{500} different vacua in string theory.

Each vacuum has its own properties depending on the compact internal space (eg: physical laws, particle content, values for fundamental constants).

With 10^{500} vacua available, seems likely that at least one could correspond to our Universe (ie. contains the Standard Model, and a positive vacuum energy with $\rho_{vac} = \Lambda/8\pi G \sim 10^{-12}(eV)^4$)

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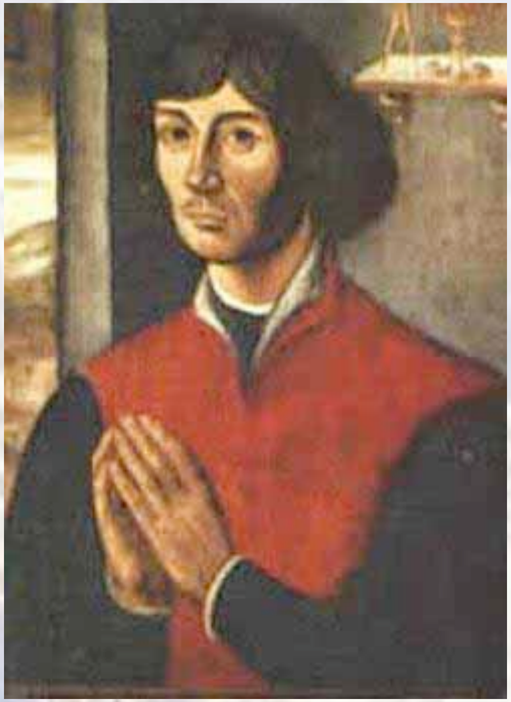
How and why was our Universe selected amongst the 10^{500} possible choices???

The Anthropic Principle



The Anthropic Principle

The Anthropic Principle

Only a Universe like our own would have the right conditions for intelligent(!) observers to evolve.

If the cosmological constant were too large an positive, Universe would expand too fast for structures to form

If the cosmological constant were too large and negative, Universe would have collapsed before structures had the chance to form.

The Anthropic Principle

This explanation might be regarded as unscientific.

It does not *predict* anything.
It is not falsifiable.

How do we measure *probability* in the landscape? Are we in a probable Universe?

An alternative to anthropic selection would be desirable…..

Resonant tunnelling in QM

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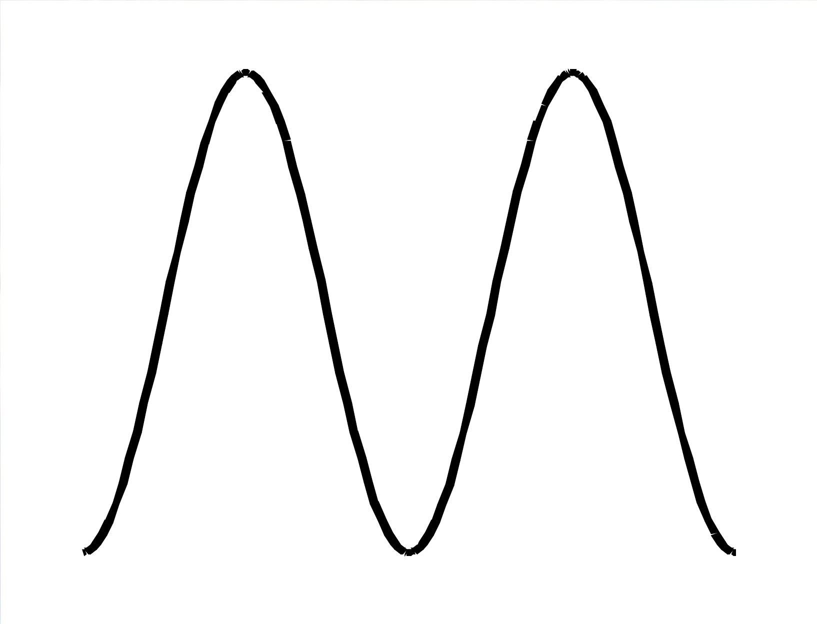
Imagine a double barrier in Quantum Mechanics



Resonant tunnelling in QM

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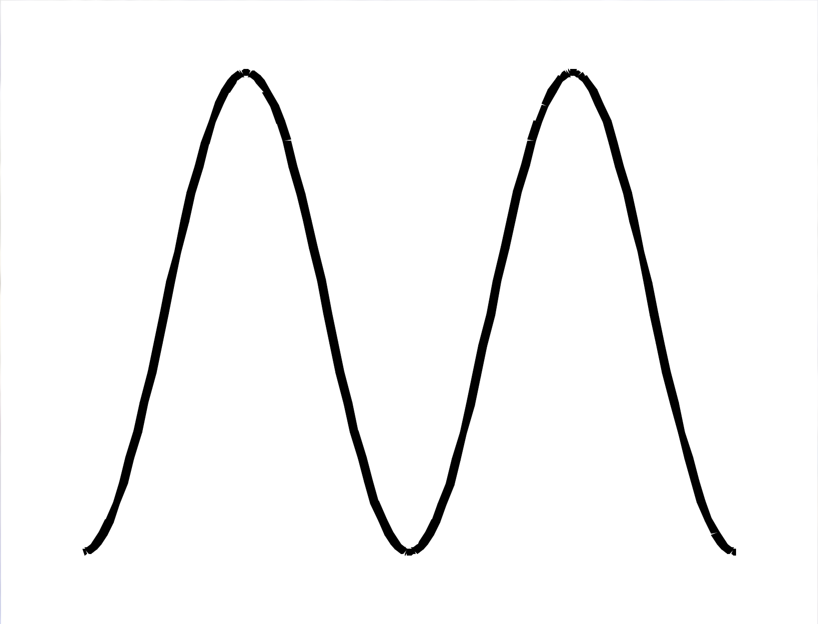

Probability of tunnelling through a single barrier is always exponentially suppressed

But, when “conditions are right”, probability of tunnelling *at once* through the *double* barrier can be order unity!!!

This is **resonant tunnelling** in QM. Has been observed experimentally (eg. in semiconductors)

What if something similar happened in the landscape?

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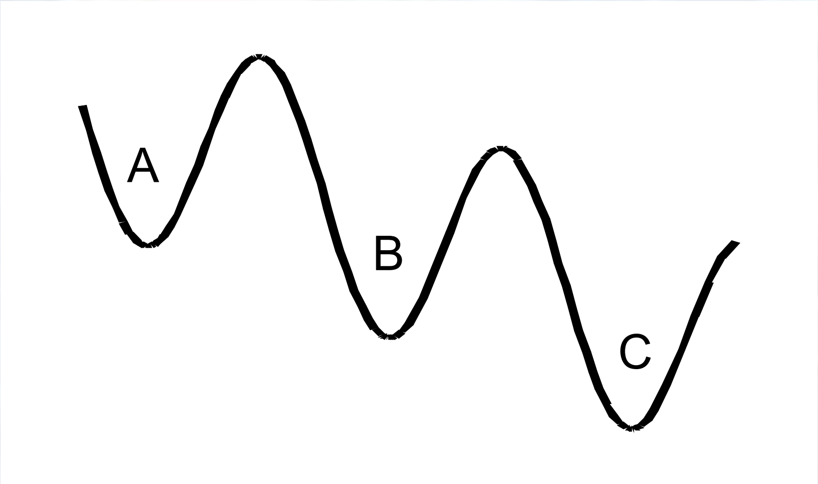
Tye, hep-th/0611148



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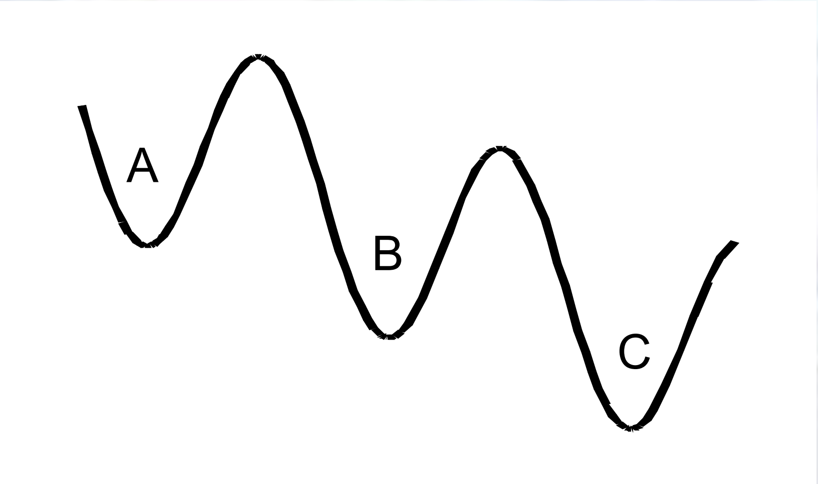

Probability of tunnelling between adjacent vacua is suppressed

$$T_{A\to B} \ll 1 \qquad T_{B\to C} \ll 1$$

Usually, probability of tunnelling from A to C is given by the product, and is also suppressed

$$T_{A\to C} = T_{A\to B} T_{B\to C} \ll 1$$

But, if conditions are right, resonance occurs and $T_{A\to C} = \mathcal{O}(1)$

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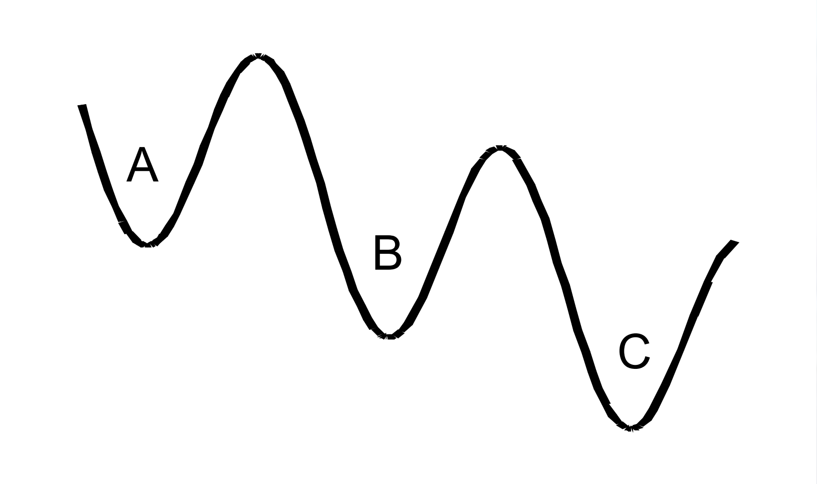

What is meant by this in the landscape?

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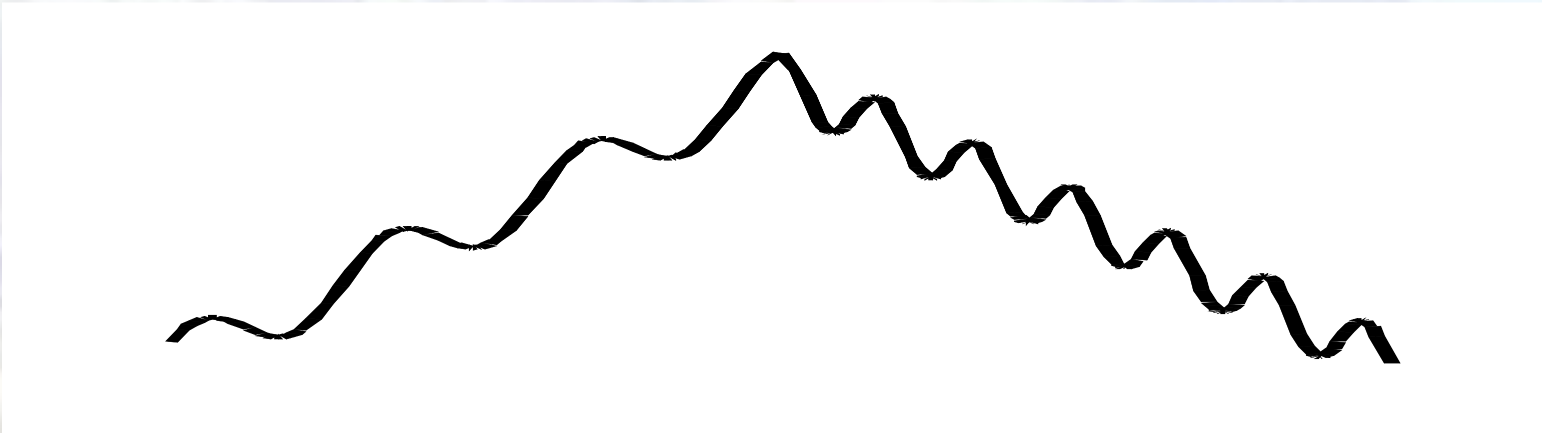

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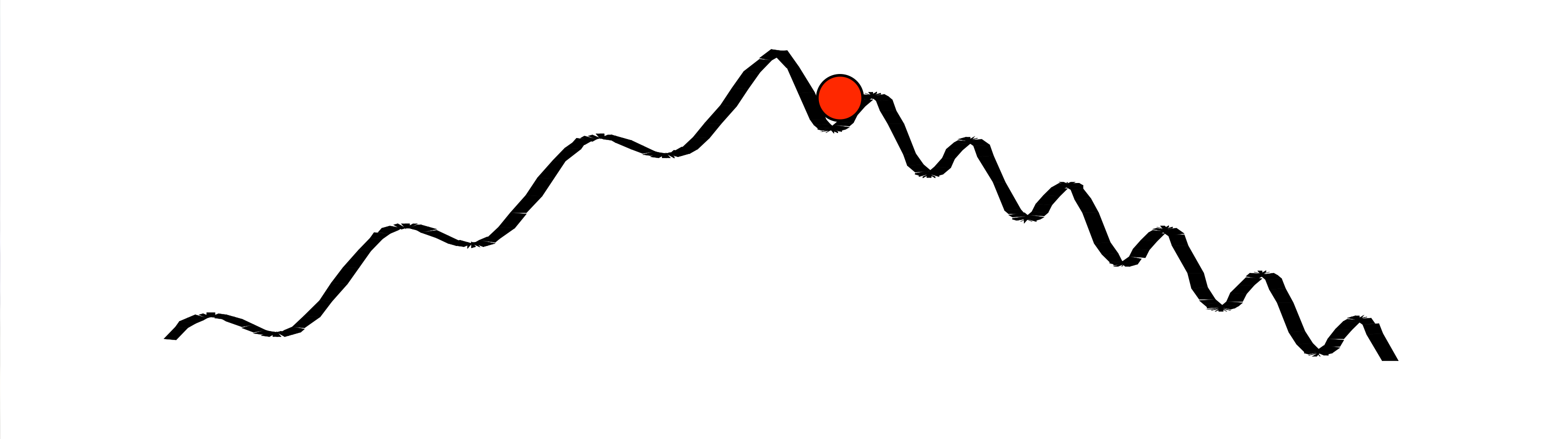
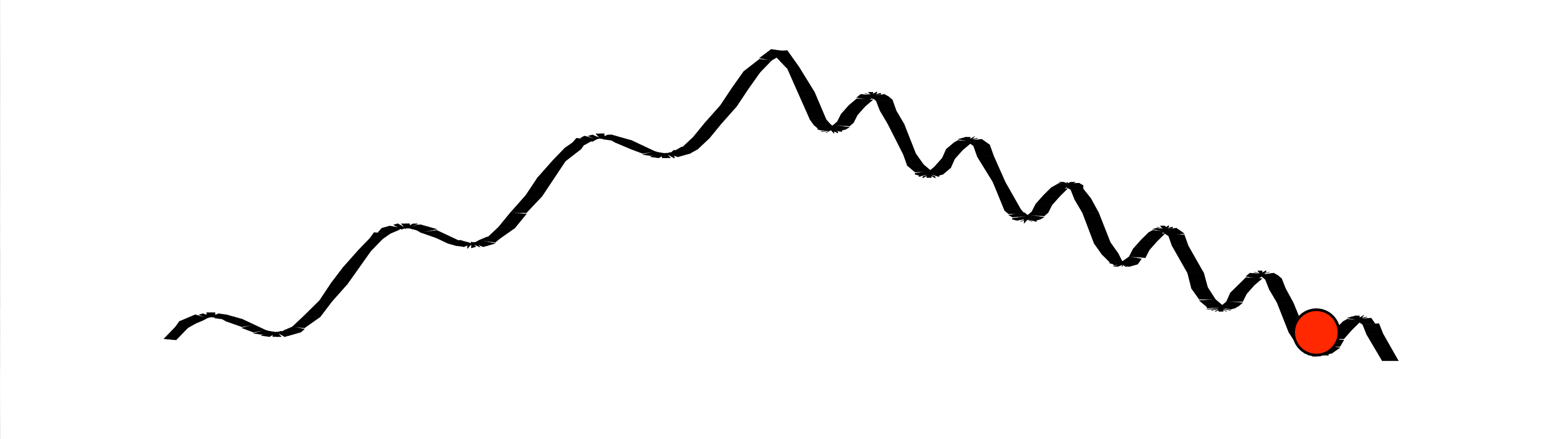


Suppose the Universe began with a very large vacuum energy, then there are lots and lots of vacua with lower energy.

And therefore lots and lots of tunnelling paths to lower energy vacua.

Expect that at least one such path is a resonant path.

What if something similar happened in the landscape?



Tunnel to lower energy vacuum with probability of order unity!

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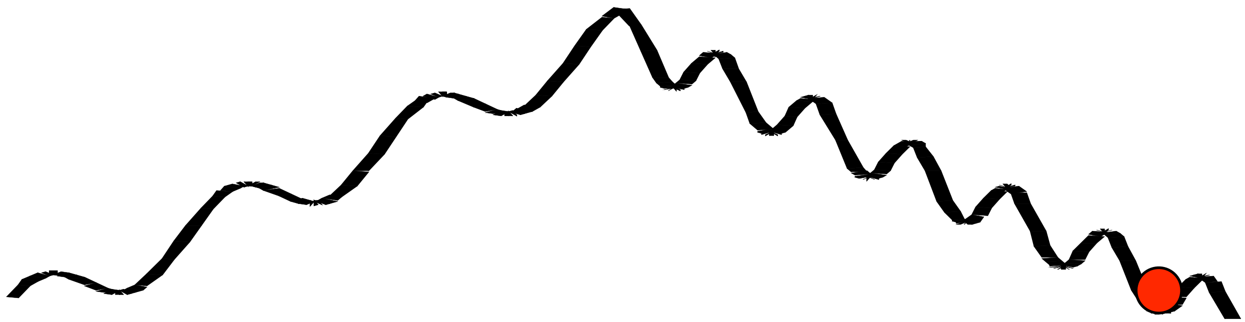

Tunnel to lower energy vacuum with probability of order unity!

Now there are far fewer vacua of lower energy (assume no AdS vacua)

And therefore far fewer tunnelling paths to lower energy vacua, and indeed, **no resonant paths**!

This explains why a small vacuum energy is much more likely!

Can such a thing happen in the landscape?

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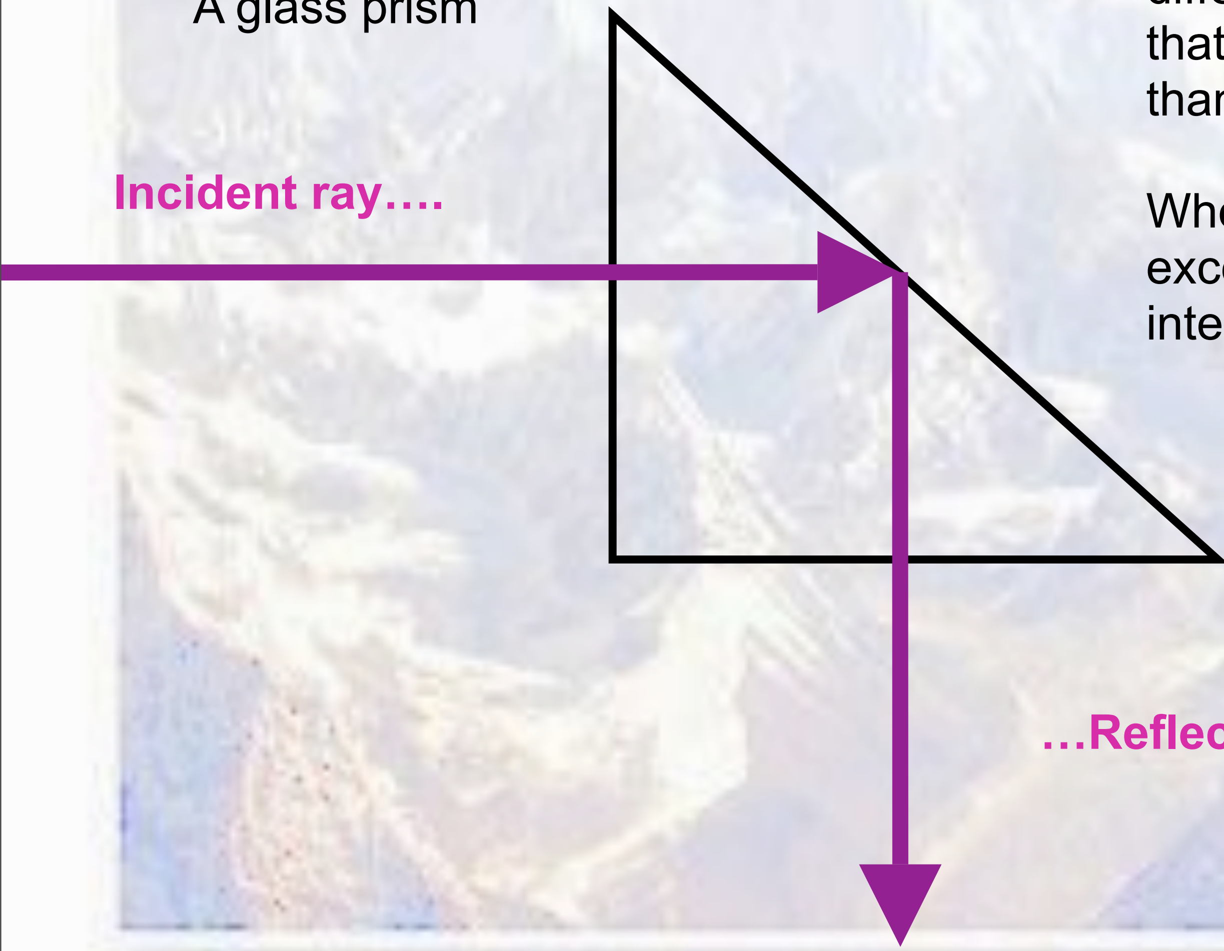
Recall some GCSE physics!!!!

Total internal reflection

A glass prism

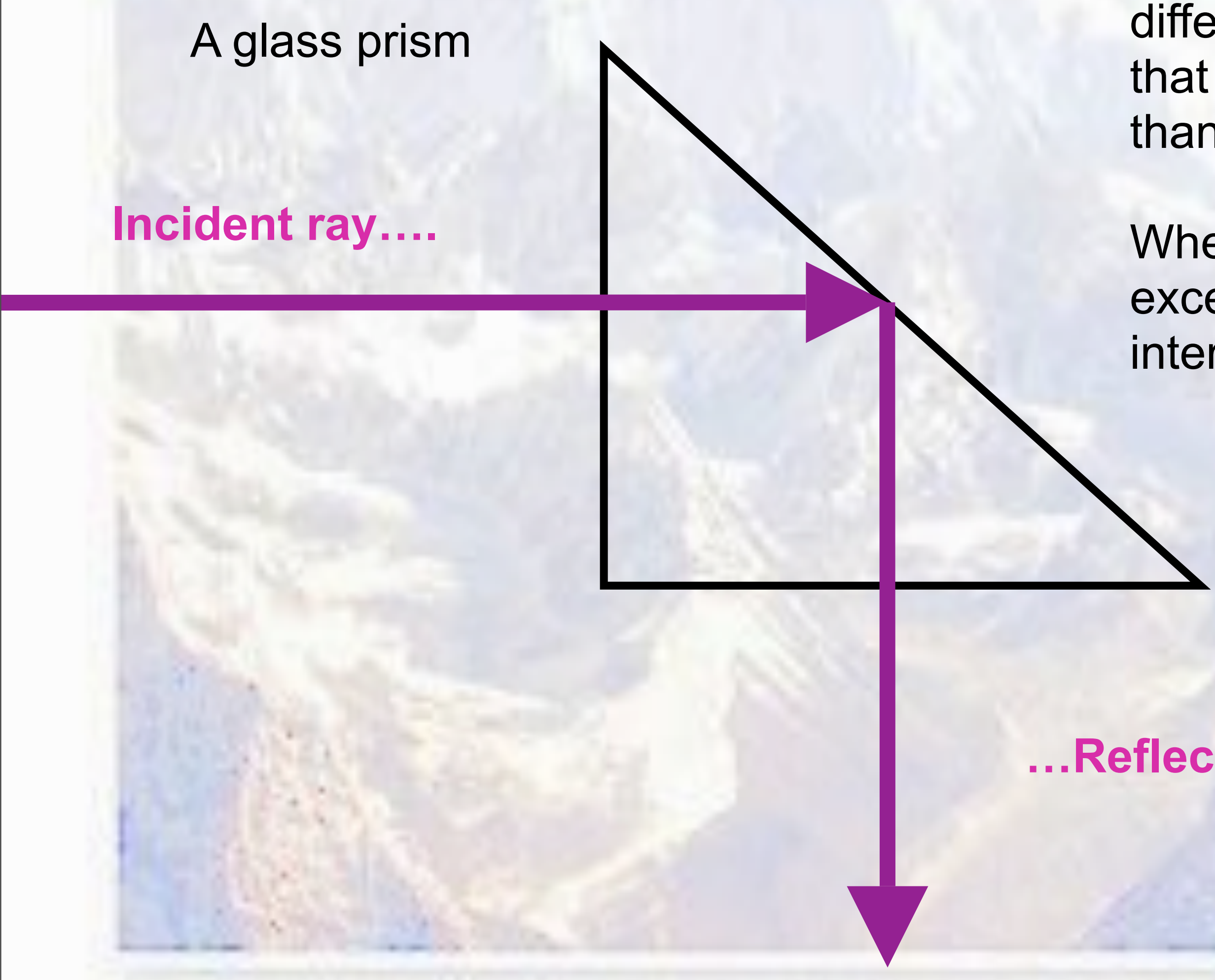
Total internal reflection



Refractive index of glass differs from that of air, so that critical angle is less than 90 degrees.

When angle of incidence exceeds critical angle, total internal reflection occurs.

Total internal reflection



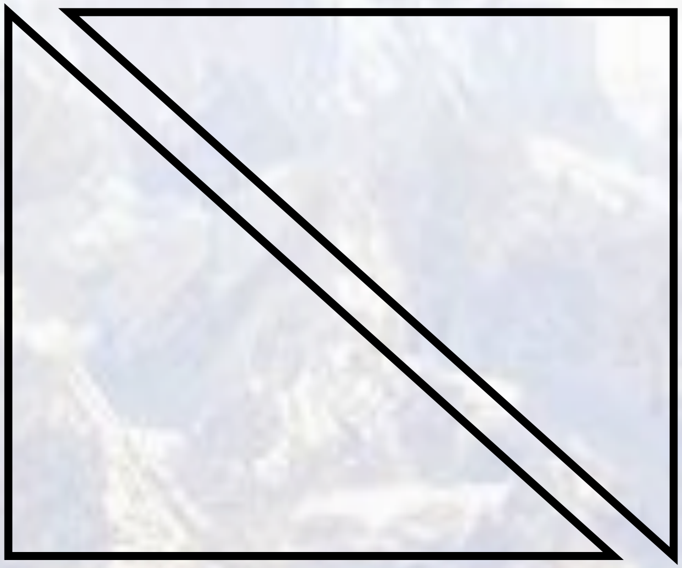
Refractive index of glass differs from that of air, so that critical angle is less than 90 degrees.

When angle of incidence exceeds critical angle, total internal reflection occurs.

This is like reflection off a potential barrier in mechanics.

Frustrated total internal reflection

Two prisms, small separation.



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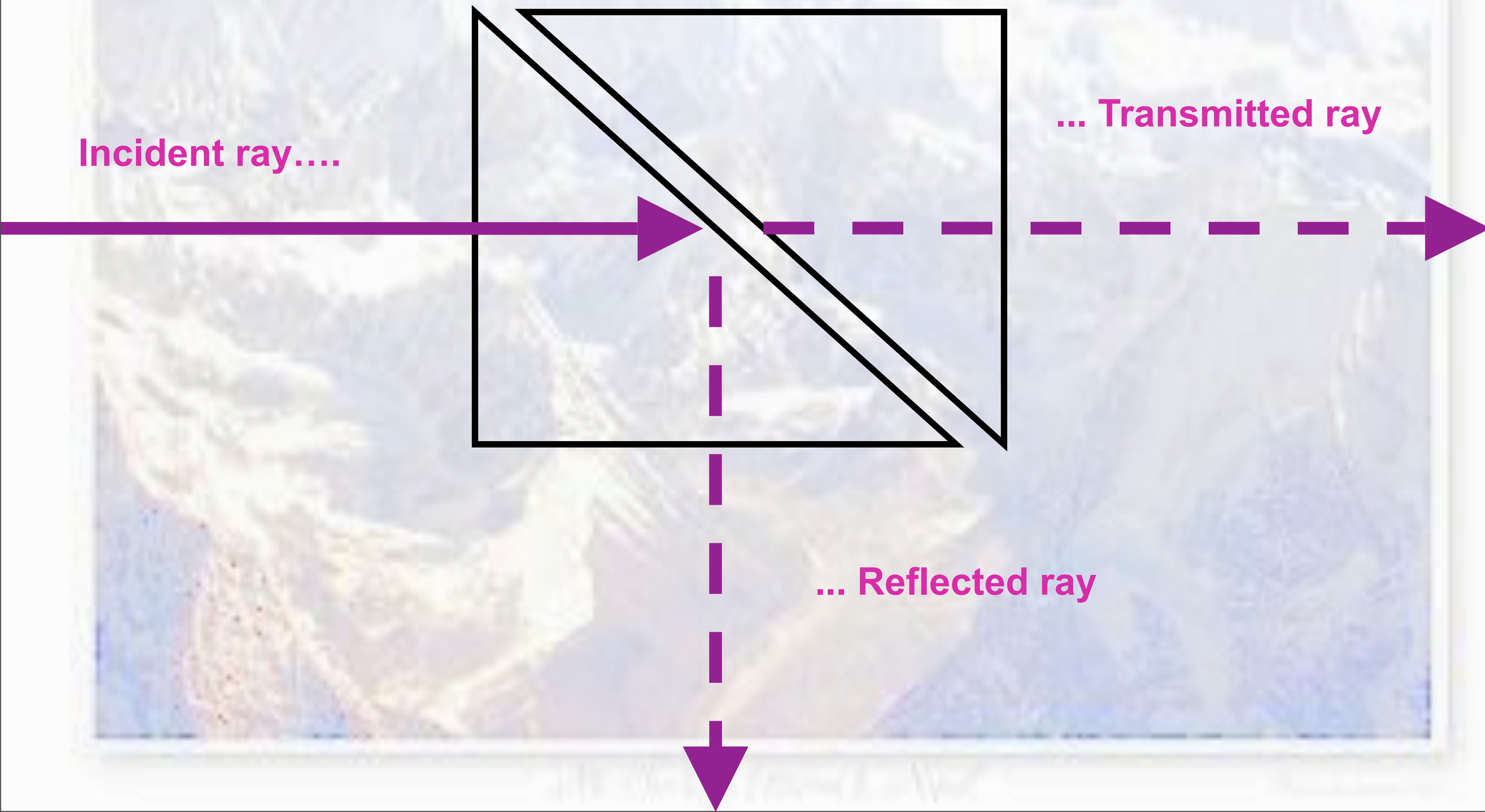

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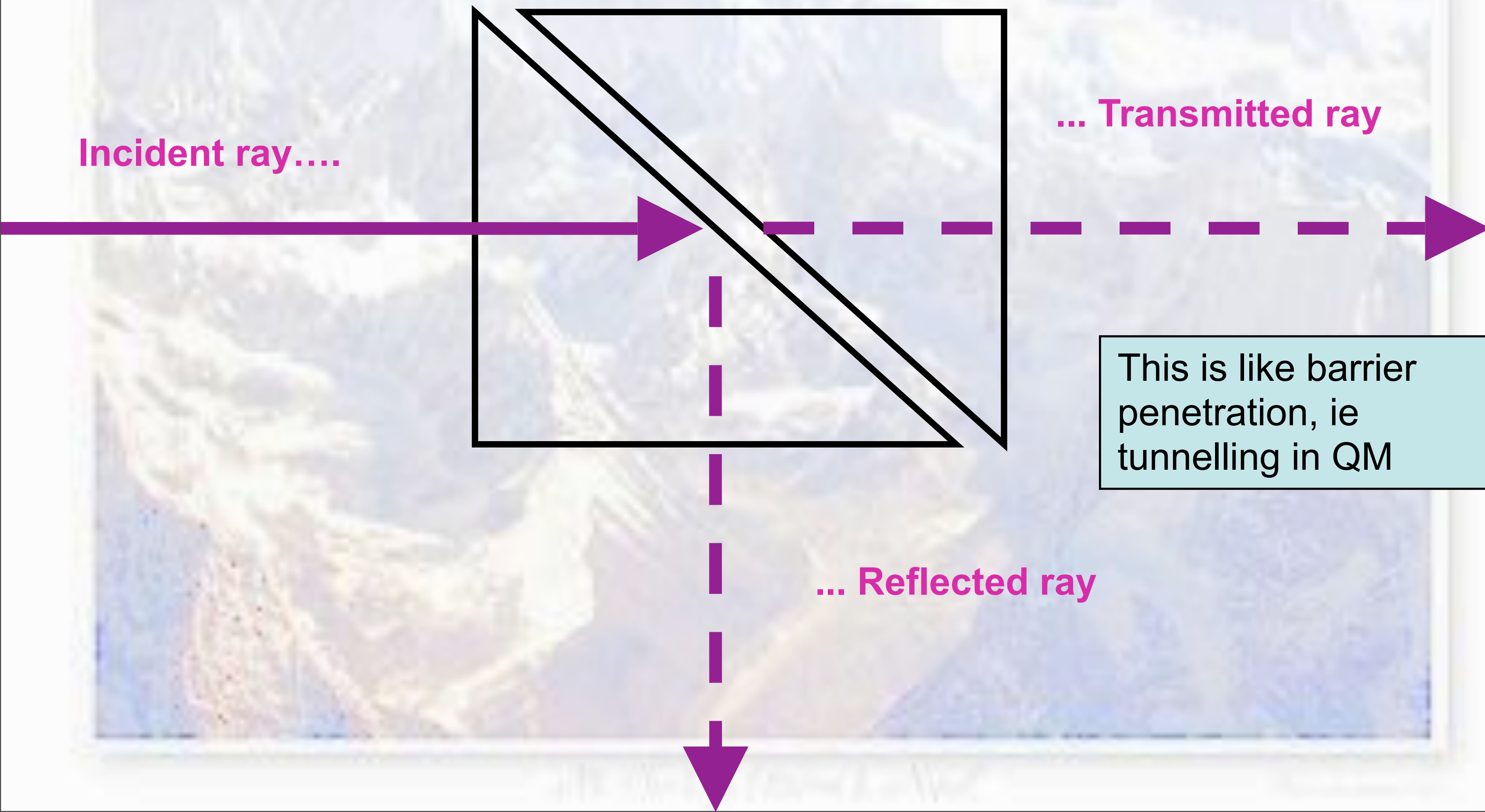

This is like barrier penetration, ie tunnelling in QM

Fabry-Perot interferometer

Two parallel, partially silvered mirrors

Fabry-Perot interferometer

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Light is partially reflected and partially transmitted through each mirror

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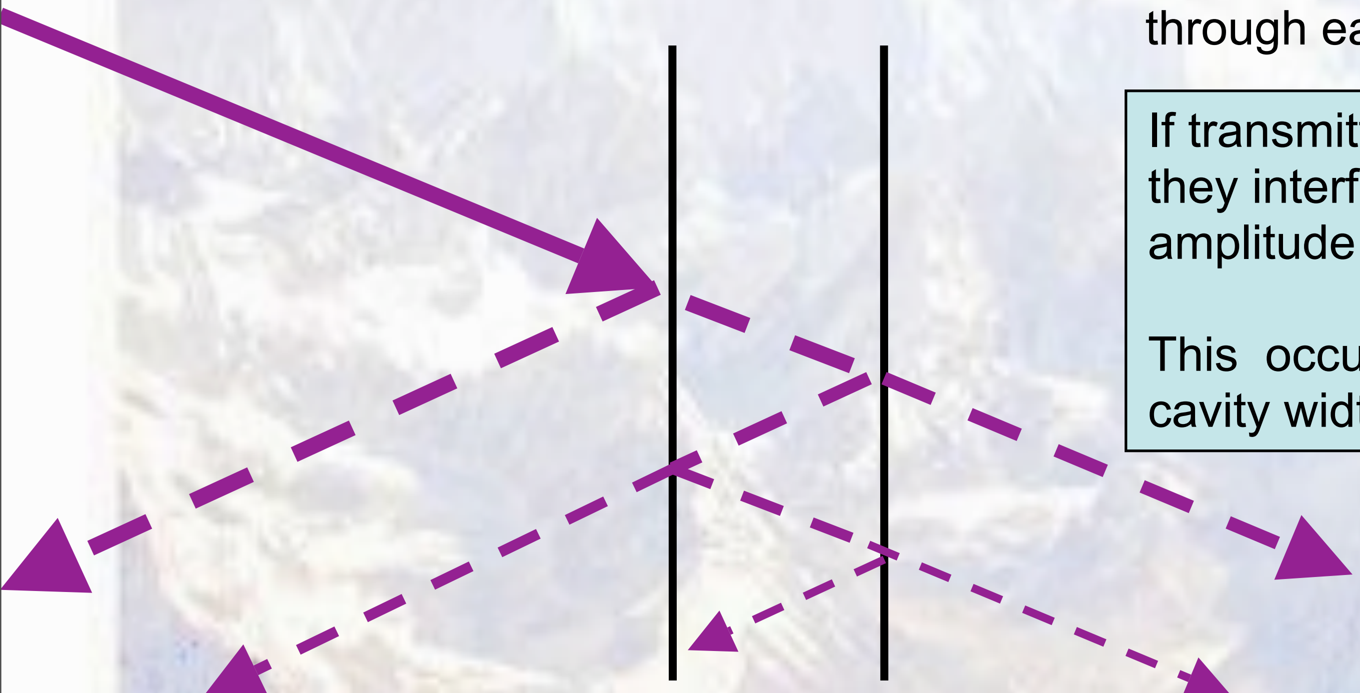

If transmitted rays are in phase, they interfere constructively and amplitude is enhanced.

This occurs when cavity width=(n+1/2)(wavelength)

Fabry-Perot interferometer

Two parallel, partially silvered mirrors

Light is partially reflected and partially transmitted through each mirror

If transmitted rays are in phase, they interfere constructively and amplitude is enhanced.

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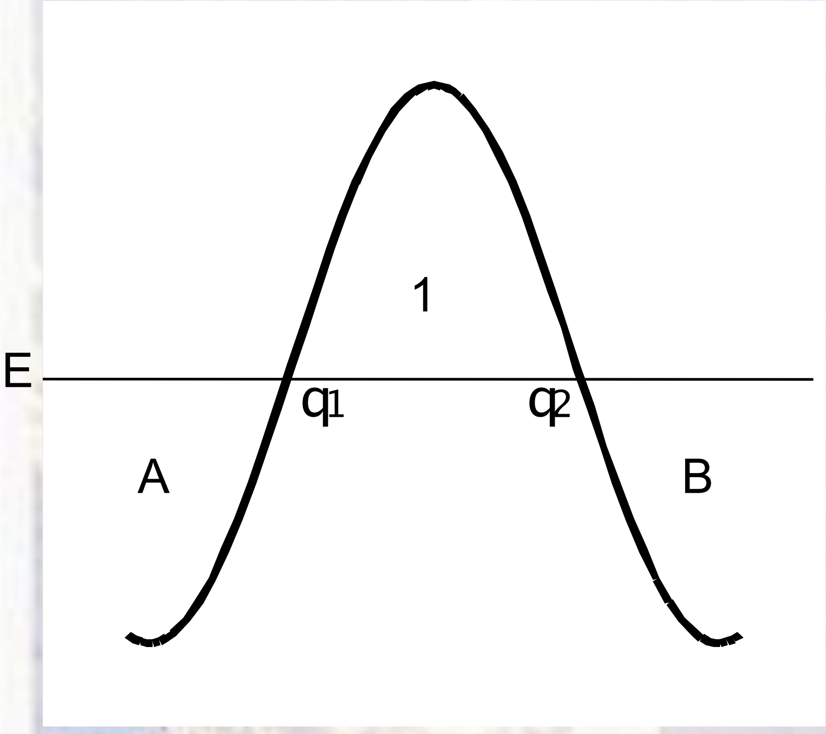
This is like resonant tunnelling in QM!

Tunnelling in QM



Consider a particle of mass, m, and energy, E.

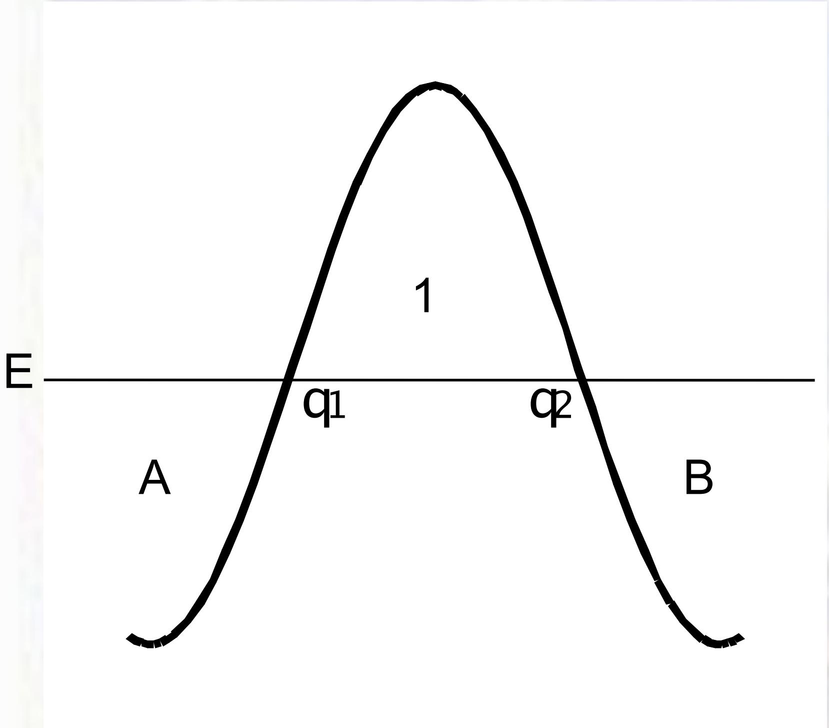
Classically, particle in region A incident on barrier 1 will be reflected at turning point q = q1.

Quantum mechanically particle is described by Schrodinger eqn:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dq^2} + V(q)\psi = E\psi$$

and can tunnel through to B

Tunnelling in QM



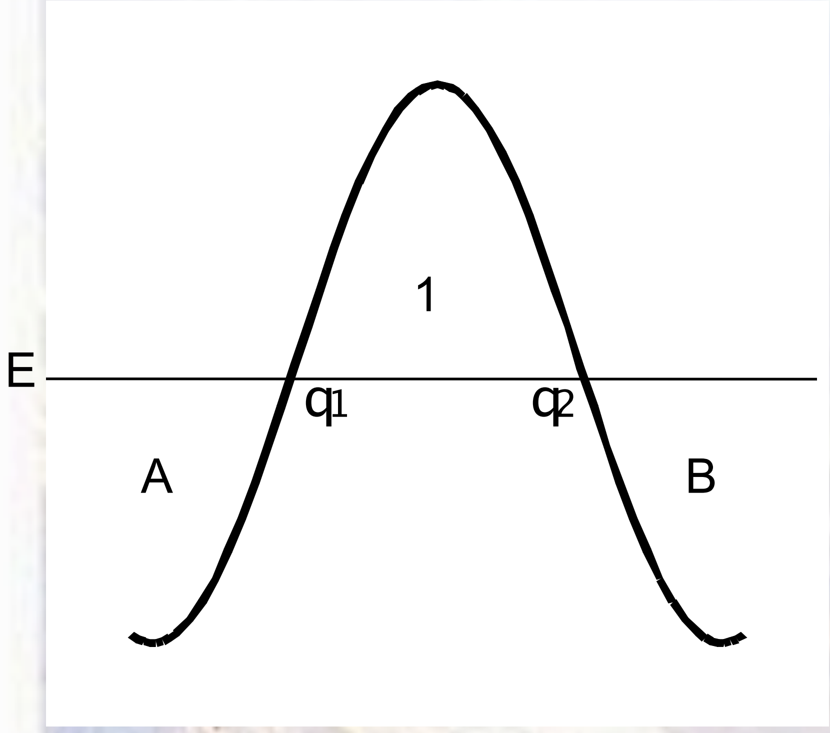
<u>WKB approximation</u>

In classically allowed regions, A or B, we have E>V(q) and

$$\psi(q) \cong \frac{\alpha_+}{\sqrt{k(q)}} \exp\left[\frac{i}{\hbar}\int^q dq' k(q')\right]$$
$$+\frac{\alpha_-}{\sqrt{k(q)}} \exp\left[-\frac{i}{\hbar}\int^q dq' k(q')\right],$$

$$k(q) = \sqrt{2m(E - V(q))},$$

Tunnelling in QM



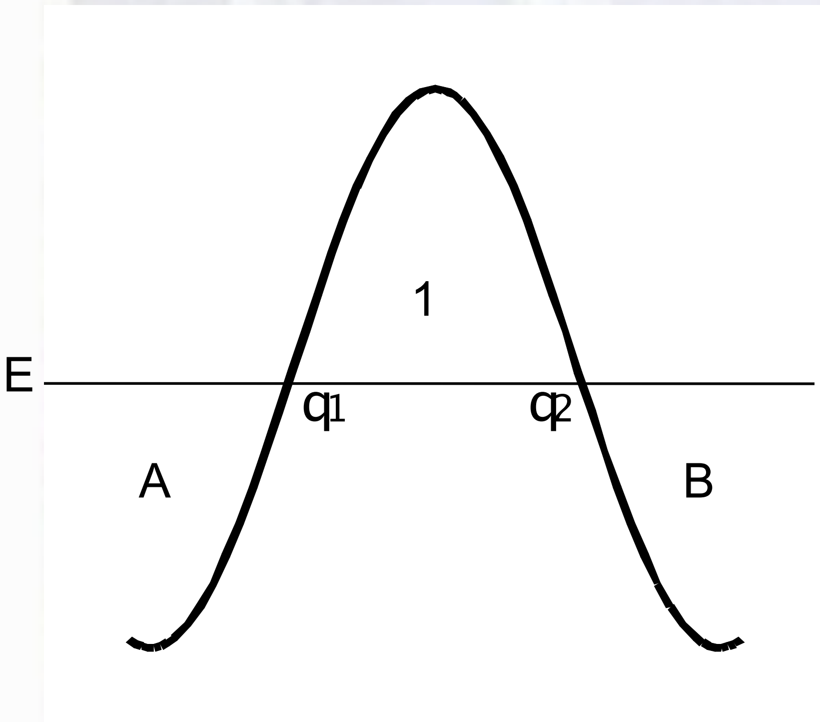
WKB approximation

In classically forbidden region, 1, we have E<V(q) and

$$\psi(q) \cong \frac{\beta_+}{\sqrt{\kappa(q)}} \exp\left[\frac{1}{\hbar}\int^q dq' \kappa(q')\right] + \frac{\beta_-}{\sqrt{\kappa(q)}} \exp\left[-\frac{1}{\hbar}\int^q dq' \kappa(q')\right],$$

$$\kappa(q) = \sqrt{2m(V(q) - E)},$$

Tunnelling in QM



WKB approximation

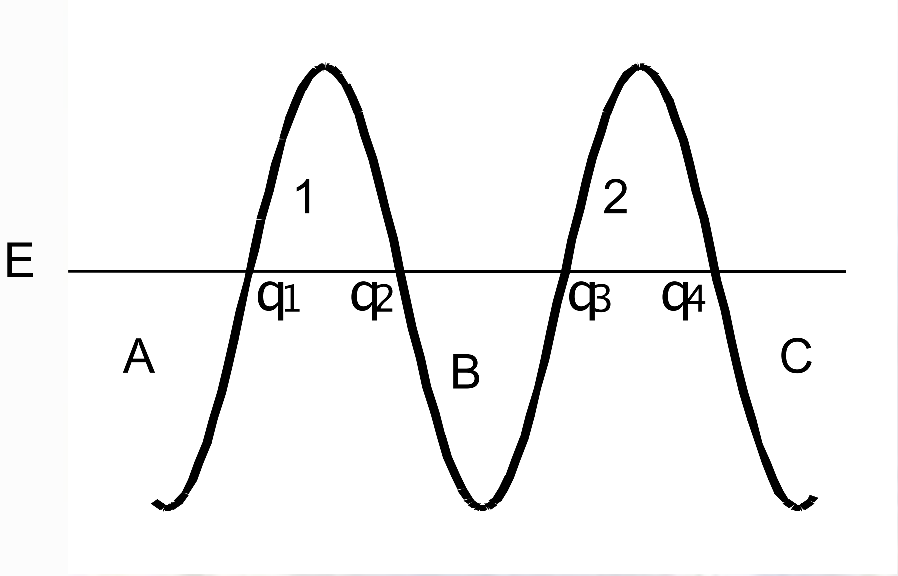
Can use WKB connection formulae to evaluate the probability of tunnelling from A to B

$$T_{A\to B} = \left|\frac{\alpha_+^B}{\alpha_+^A}\right|^2 = 4/\Theta^2,$$

where

$$\Theta = 2\exp\left[\frac{1}{\hbar}\int_{q_1}^{q_2} dq'\kappa(q')\right].$$

Resonant tunnelling in QM



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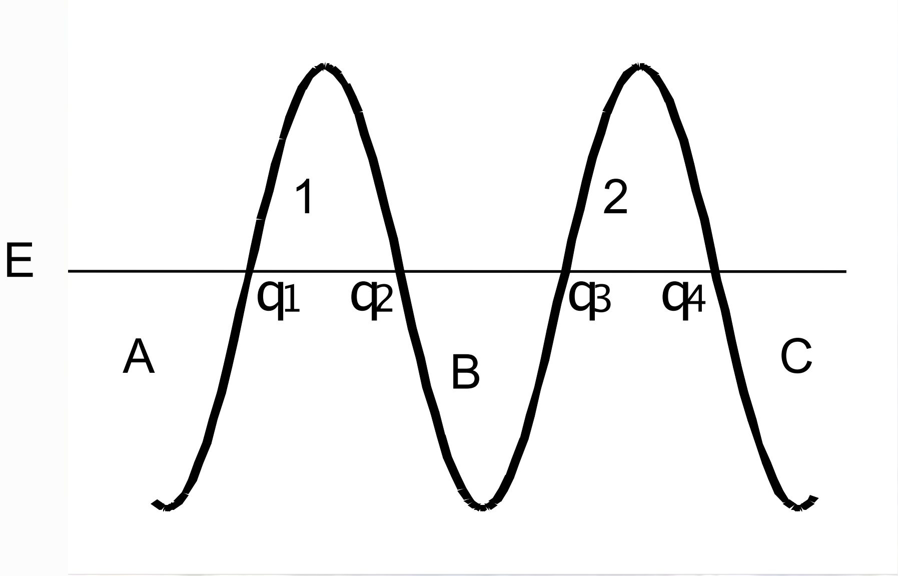

Again, we can use the WKB connection formulae to show that, probability of tunnelling from A to C is

$$T_{A\to C} = \left| \frac{\alpha_+^C}{\alpha_+^A} \right|^2 = 4/(\Theta\Phi\cos W)^2,$$

where

$$W = \frac{1}{\hbar}\int_{q_2}^{q_3} dq' \; k(q'),$$

$$\Phi = 2\exp\left[\frac{1}{\hbar}\int_{q_3}^{q_4} dq' \kappa(q')\right].$$

$$k(q) = \sqrt{2m(E - V(q))}$$

$$\kappa(q) = \sqrt{(2m(V(q) - E)}$$

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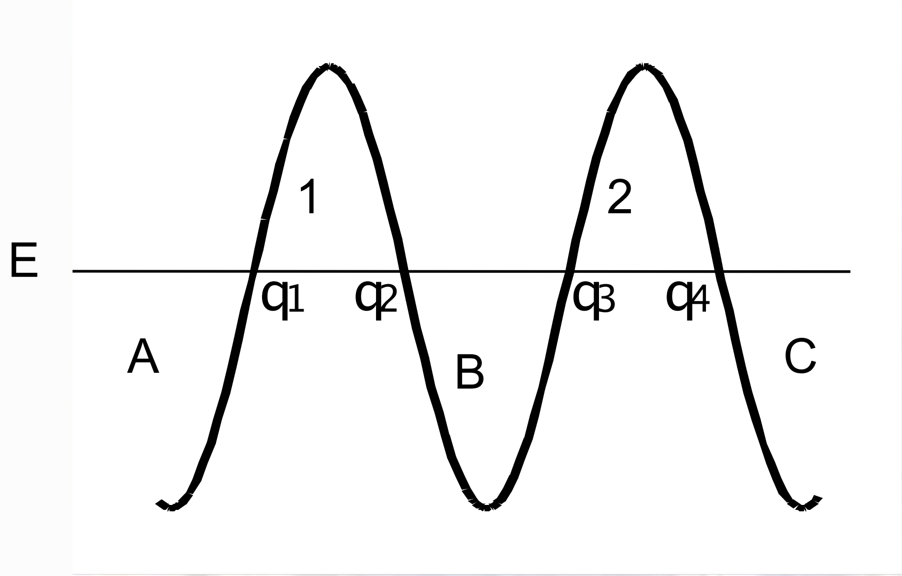

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Resonant tunnelling when
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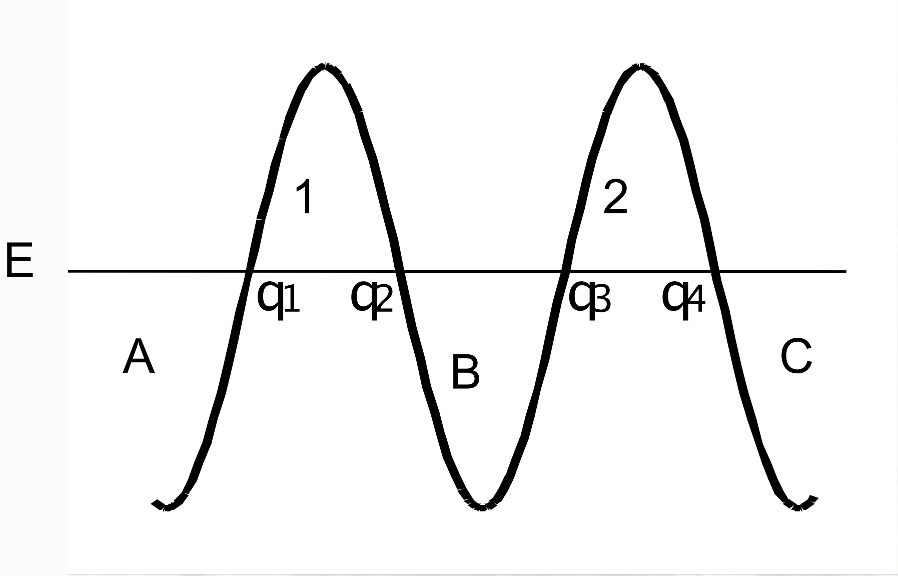
Resonant tunnelling in QM



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Bohr-Sommerfeld quantization condition for existence of a bound state in B

Width of B=(n+1/2)(de Broglie wavelengths)

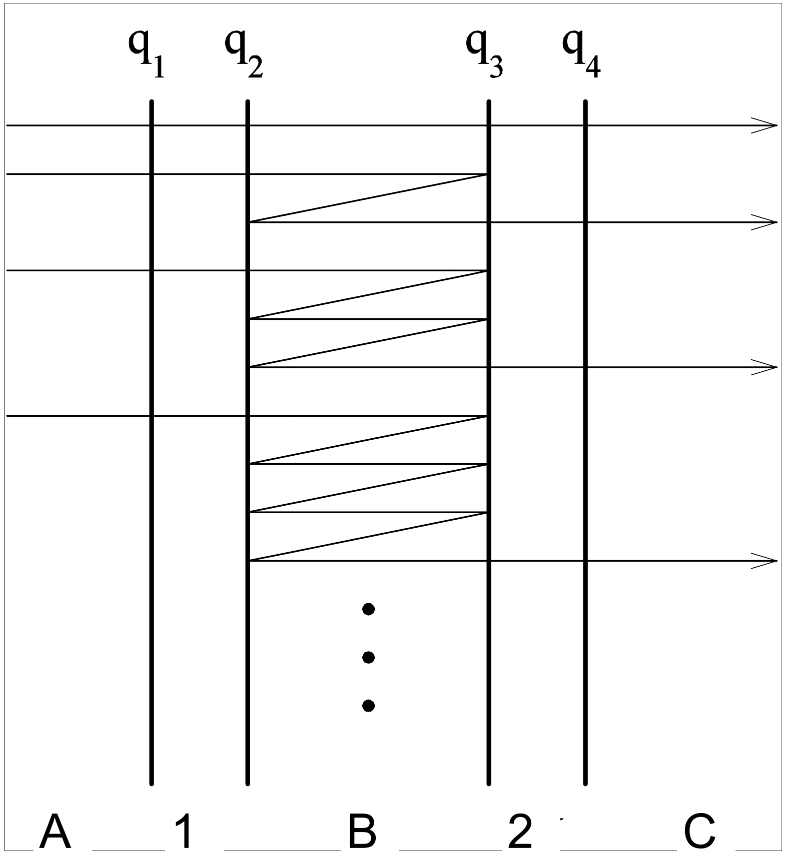
Resonant tunnelling when $\cos W = 0 \implies W = (n+1/2)\pi$

Resonant tunnelling in QM

Bound state corresponds to a particle oscillating between turning points in the central classically allowed region, B.

As it oscillates it picks up a quantum phase.

If this phase is $(n+1/2)\pi$ then resonant tunnelling occurs



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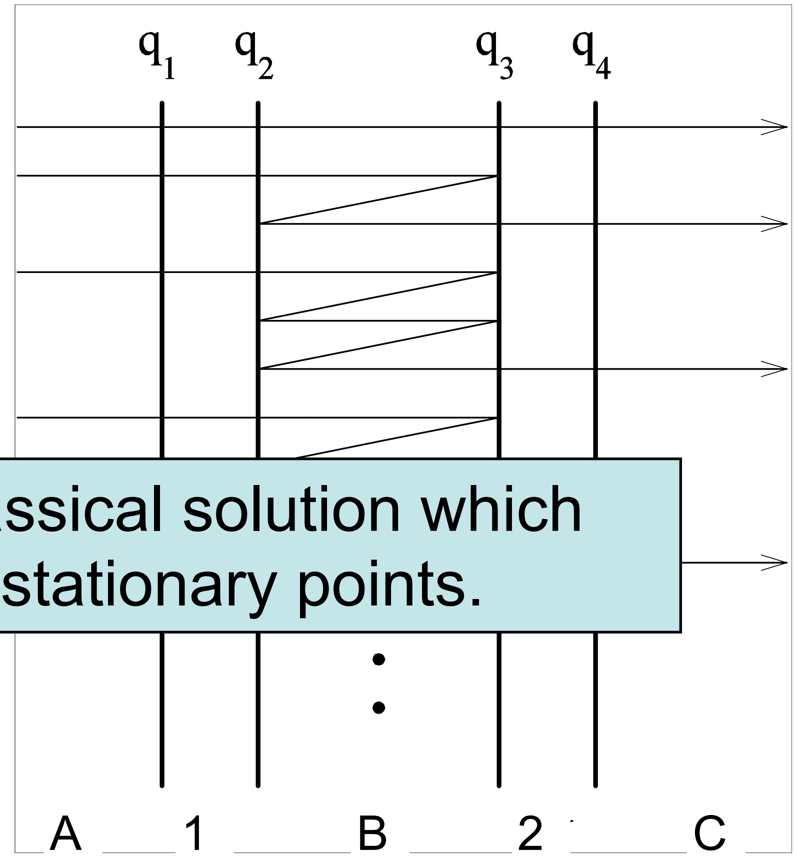

• The existence of a classical solution which oscillates between two stationary points.

Resonant tunnelling in QM

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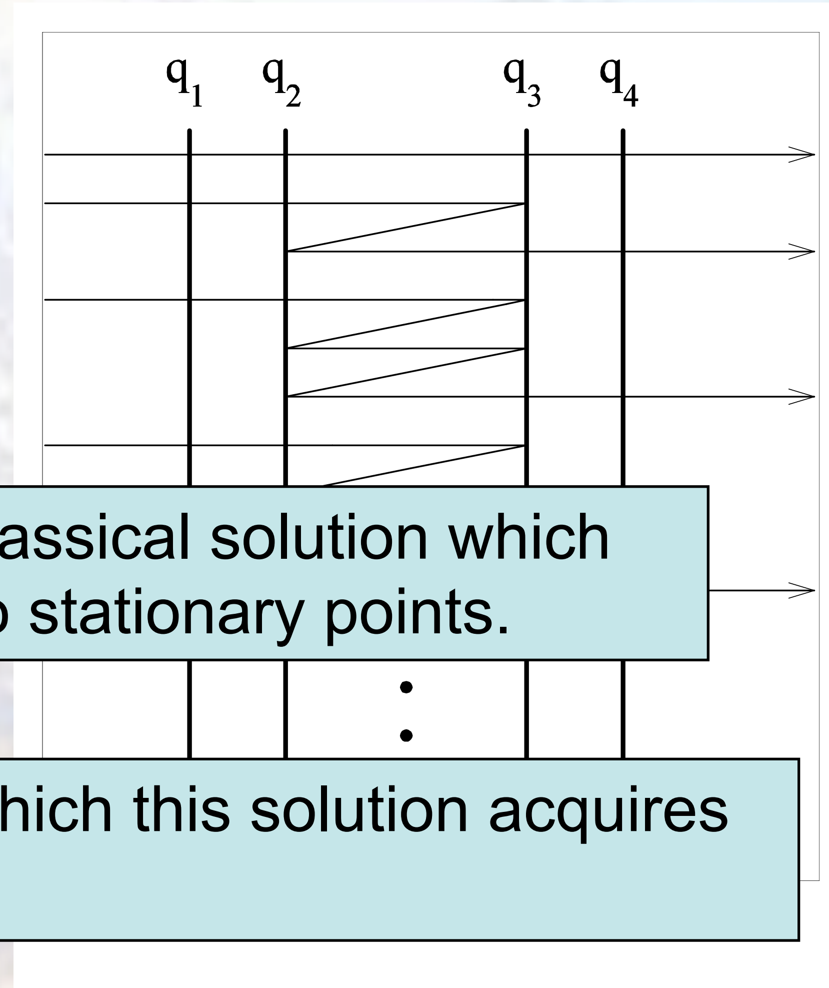

- The existence of a classical solution which oscillates between two stationary points.

- The quantum phase which this solution acquires is $(n+1/2)\pi$.

Tunnelling in N dimensional QM

We can think of QFT as $N \to \infty$ limit of N dimensional QM

Tunnelling in N dimensional QM

We can think of QFT as $N \to \infty$ limit of N dimensional QM

Consider the mechanics of a particle of unit mass in N-dimensions. The classical path of the particle is given by $\vec{q}(t) = (q1(t), \ldots qN(t))$, and is found by extremizing the action,

$$S = \int dt \, \left[\frac{1}{2} \dot{\vec{q}} \cdot \dot{\vec{q}} - V(\vec{q}) \right]$$

Quantum mechanically, a particle, of energy E is described by the wavefunction $\psi(\vec{q})$ satisfying the time independent Schrodinger equation,

$$\left[-\frac{\hbar^2}{2} \vec{\nabla}^2 + V(\vec{q}) \right] \psi = E\psi.$$

Tunnelling in N dimensional QM

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extremi

Direct application of the WKB approximation run into diificulties owing to ambiguity in direction of gradient $\vec{\nabla}$

This problem was resolved by Banks, Bender and Wu (PRD 8 (1973) 3346) by reducing the problem to one-dimensional QM along classical paths and MPEPs

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Classical Paths and MPEPs

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N dimensional space is split into classically allowed regions with E>V, and classically forbidden regions with E<V

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N dimensional space is split into classically allowed regions with E>V, and classically forbidden regions with E<V

In classically allowed region, wavefunction is peaked along classical paths (well known)

In classically forbidden region, wavefunction is peaked along "Most probable escape paths" (MPEPs)

Classical Paths and MPEPs

Consider a curve, $\vec{Q}(\lambda) \in \mathbb{R}^N$, parametrized by λ.
The curve has tangent vector $\vec{v}_{\|}(\lambda) = \partial\vec{Q}/\partial\lambda$, and $N-1$ orthogonal normal vectors $\vec{v}_{\perp}^{\,i}(\lambda),\ i = 1, \ldots, N-1$, satisfying

$$\vec{v}_{\|} \cdot \vec{v}_{\perp}^{\,i} = 0, \qquad \vec{v}_{\perp}^{\,i} \cdot \vec{v}_{\perp}^{\,j} \propto \delta^{ij}.$$

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Curve is a classical path or MPEP if wavefunction is peaked there, ie

$$\vec{v}_{\perp}^{\,i} \cdot \vec{\nabla}\sigma|_{\vec{q}=\vec{Q}} = 0, \qquad i = 1, \ldots, N-1.$$

Classical Paths and MPEPs

CP satisfies

$$\frac{d^2\vec{Q}}{d\lambda^2} + V = 0$$

where λ plays the role of real time and is related to the proper distance along the curve:

$$ds = \sqrt{2(E-V)}d\lambda$$

In semi-classical approximation, wavefunction is dominated by its value close to the CP

$$\psi(\vec{q}) \cong \frac{1}{[2(E-V(\vec{q}))]^{\frac{1}{4}}} \left[\alpha_+ e^{\frac{i}{\hbar}\int^s ds\ \sqrt{2(E-V(\vec{q}))}} + \alpha_- e^{-\frac{i}{\hbar}\int^s ds\ \sqrt{2(E-V(\vec{q}))}}\right],$$

Classical Paths and MPEPs

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MPEP satisfies

$$\frac{d^2\vec{Q}}{d\lambda^2} + V = 0$$

where λ plays the role of imaginary time and is related to the proper distance along the curve:

$$ds = \sqrt{2(V-E)}d\lambda$$

In semi-classical approximation, wavefunction is dominated by its value close to the CP

$$\psi(\vec{q}) \cong \frac{1}{[2(V(\vec{q})-E)]^{\frac{1}{4}}} \left[\beta_+ e^{\frac{1}{\hbar}\int^s ds\ \sqrt{2(V(\vec{q})-E)}} + \beta_- e^{-\frac{1}{\hbar}\int^s ds\ \sqrt{2(V(\vec{q})-E)}}\right],$$

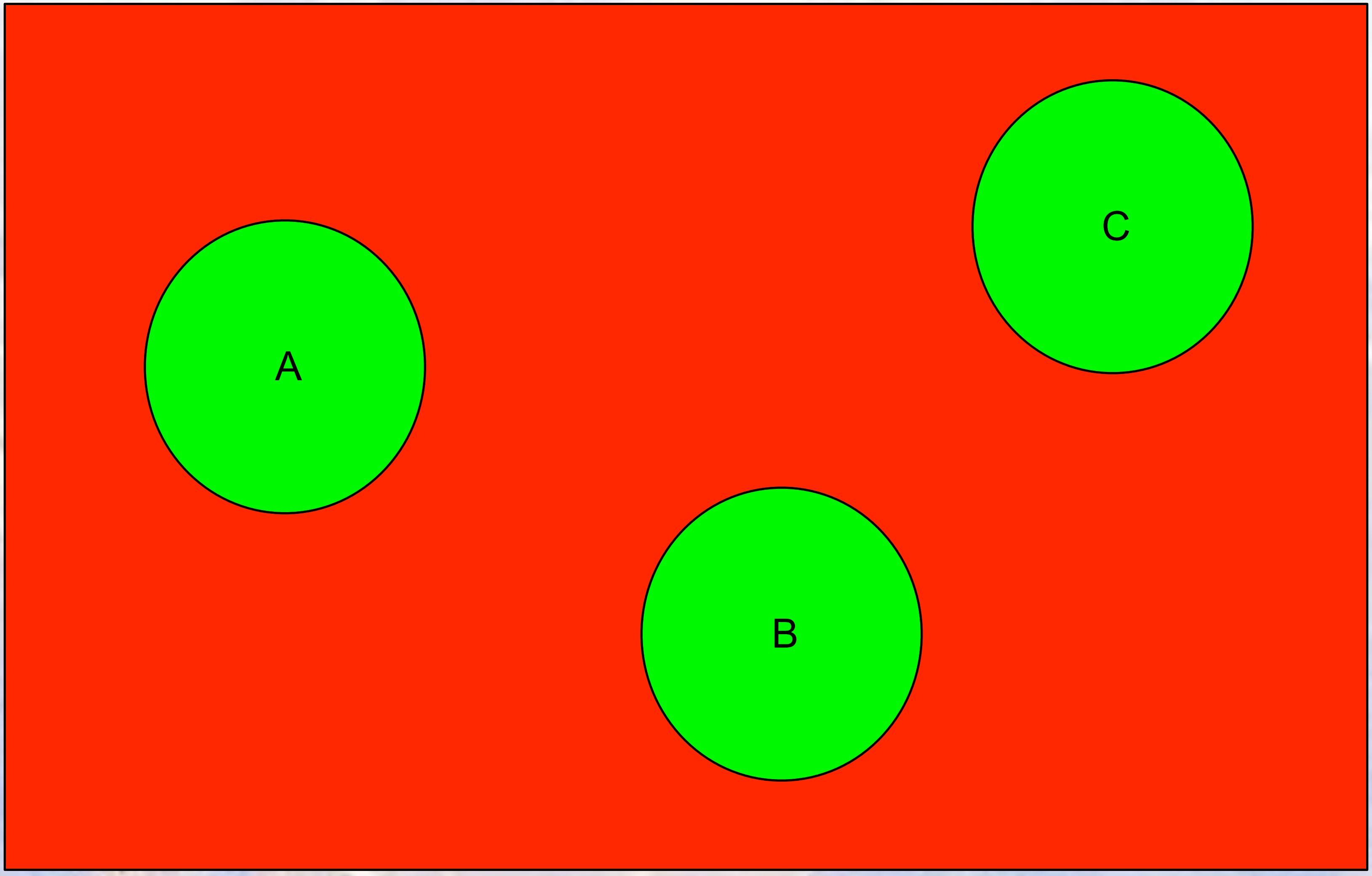
The tunnelling highway

•N-dimensional QM has been reduced to one-dimensional QM along a tunnelling highway

•The tunnelling highway is the combination of classical paths and MPEPs involved in the tunnelling process.

•Classical paths and MPEPs must be connected to one another at classical turning points (E=V)

The tunnelling highway



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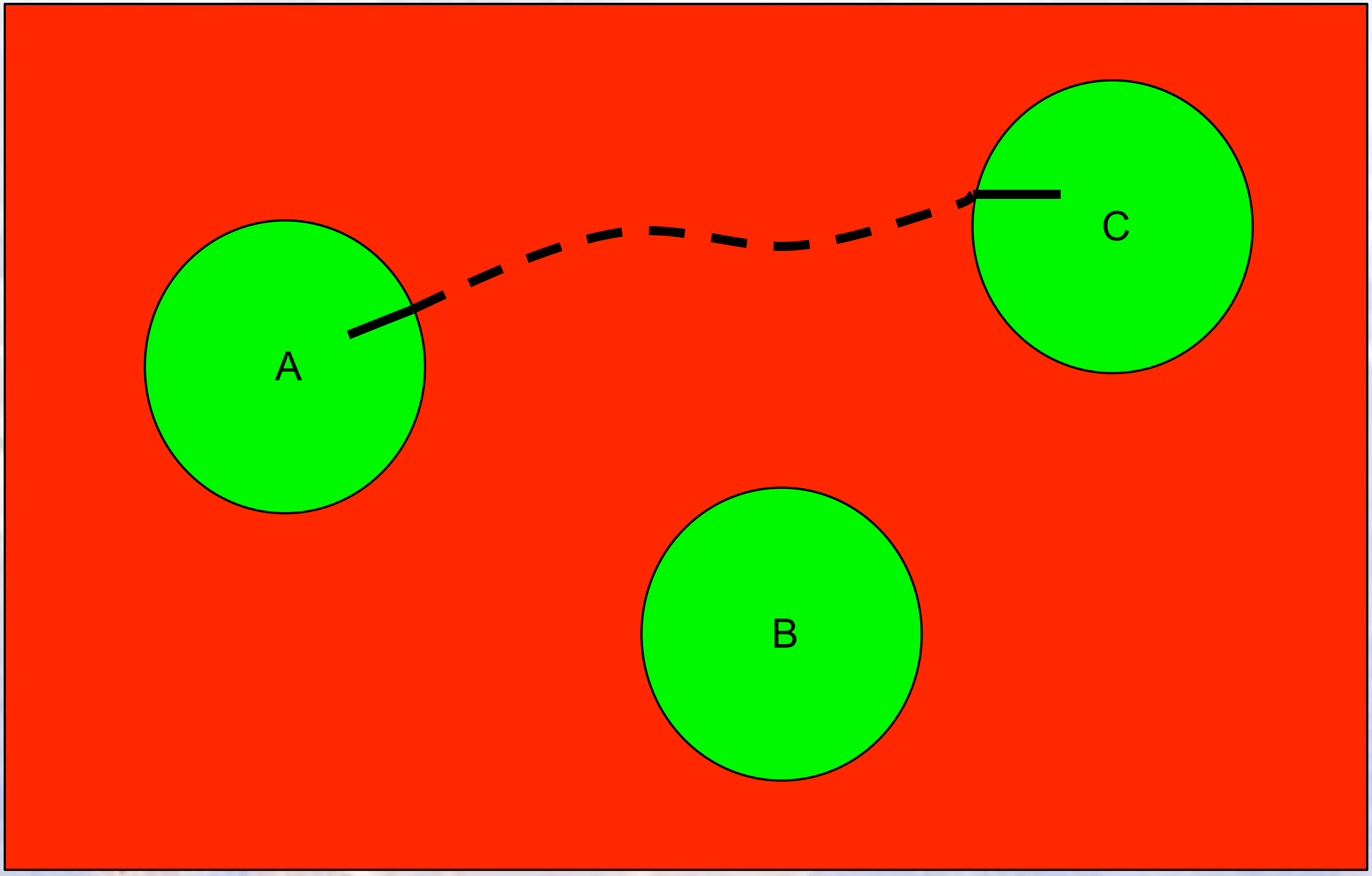

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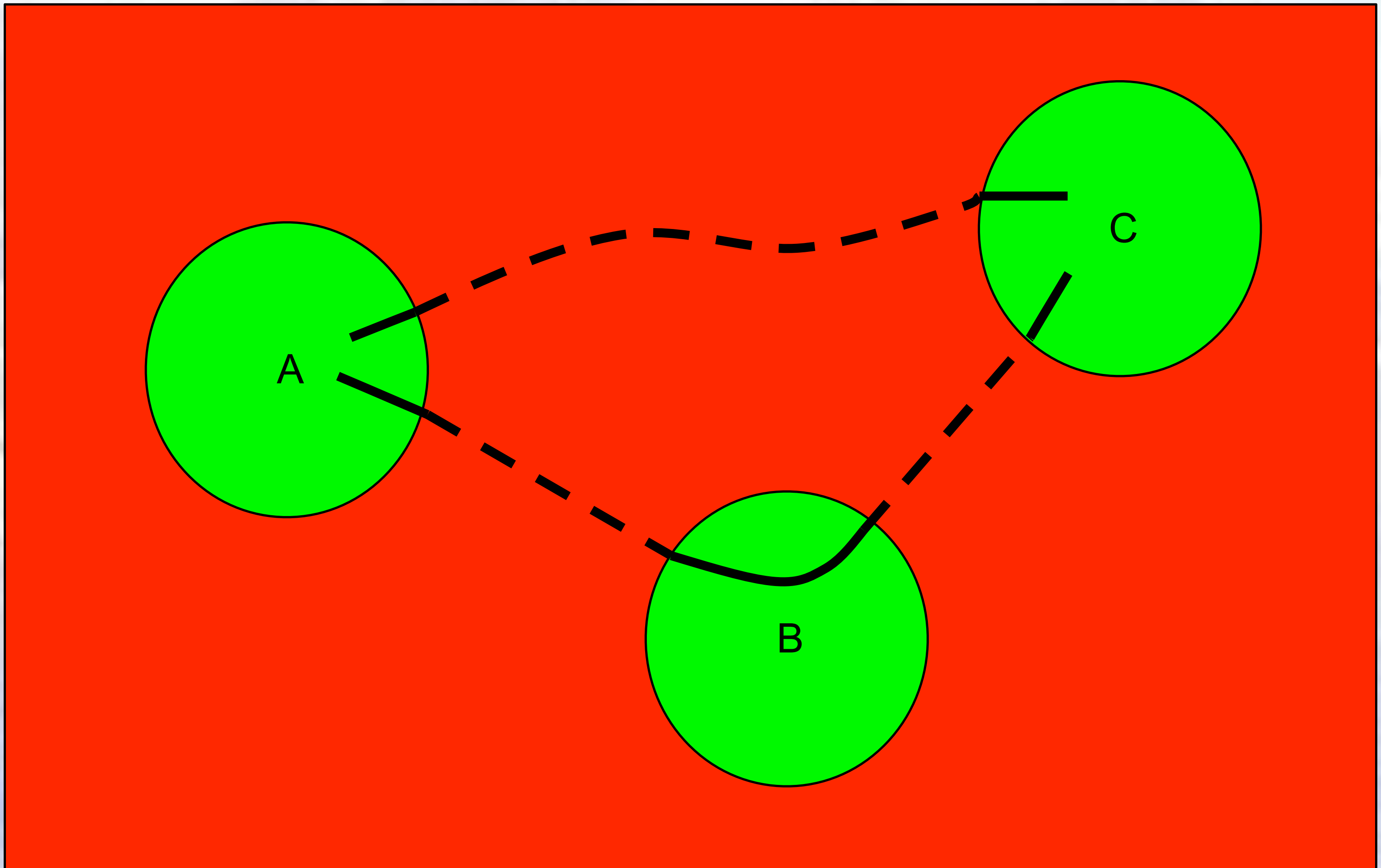

Tunnelling in QFT

Consider the standard theory of a scalar field, $\phi(t;x)$, evolving in time through a spatial volume,$\mathcal{V}$, under the infuence of a potential, $V(\phi)$. This is described by the action

$$S = \int dt \int_{\mathcal{V}} dx \; \left[\frac{1}{2}\dot{\phi}^2 - \frac{1}{2}\phi'^2 - V(\phi)\right]$$

We can think of the field $\phi(t;x)$ as describing a quantum mechanical system in infinite-dimensional space, like so

$$\{\phi(t,x), x \in \mathcal{V}\} = \{\phi(t,x_1), \phi(t,x_2), \ldots\}.$$

Wavefunction, $\psi[\phi]$ is a functional acting on an appropriately chosen "configuration space".

The "configuration space" is taken to be the space of real valued functions on $\mathcal{V}$, satisfying some boundary condition on $\partial\mathcal{V}$.

The norm, $|\psi[\phi]|^2$, therefore measures the probability density for a given configuration ϕ

by the action

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The "configuration space" is taken to be the space of real valued functions on $\mathcal{V}$, satisfying some boundary condition on $\partial\mathcal{V}$.

The norm,$|\psi[\phi]|^2$, therefore measures the probability density for a given configuration ϕ

by the action

$$S =$$

We can think of the fiel

infinite-dimensional spac

$\{\phi(t,$

Generalised Schrodinger equation

$$\left[-\frac{\hbar^2}{2}\int_{\mathcal{V}} dx\ \frac{\delta^2}{\delta\phi(x)^2} + U[\phi]\right]\psi[\phi] = E\psi[\phi]$$

Generalised potential

$$U[\phi] = \int_{\mathcal{V}} dx\ \left[\frac{1}{2}\phi'^2 + V(\phi)\right]$$

Classical Paths and MPEPs

“Configuration space” is split into classically allowed regions with E>U, and classically forbidden regions with E<U

- In classically allowed region, wavefunction is peaked along classical paths (well known)
- In classically forbidden region, wavefunction is peaked along “Most probable escape paths” (MPEPs)

Classical Paths and MPEPs

A CP $\Phi(\lambda; x)$ is defined at each point $x \in \mathcal{V}$, and is parametrised in configuration space by λ. It satisfies

$$\frac{d^2\Phi}{d\lambda^2} - \frac{d^2\Phi}{dx^2} + V'(\Phi) = 0,$$

where λ plays the role of real time, and is related to the proper distance along the curve

$$ds = \sqrt{2(E - U[\phi])}d\lambda$$

In semi classical approx, wavefunction is dominated by its value close to the MPEP

$$\psi[\phi] \cong \frac{1}{[2(E - U[\phi])]^{\frac{1}{4}}} \left[\alpha_+ e^{\frac{i}{\hbar}\int^s ds\ \sqrt{2(E-U[\phi])}} + \alpha_- e^{-\frac{i}{\hbar}\int^s ds\ \sqrt{2(E-U[\phi])}}\right]$$

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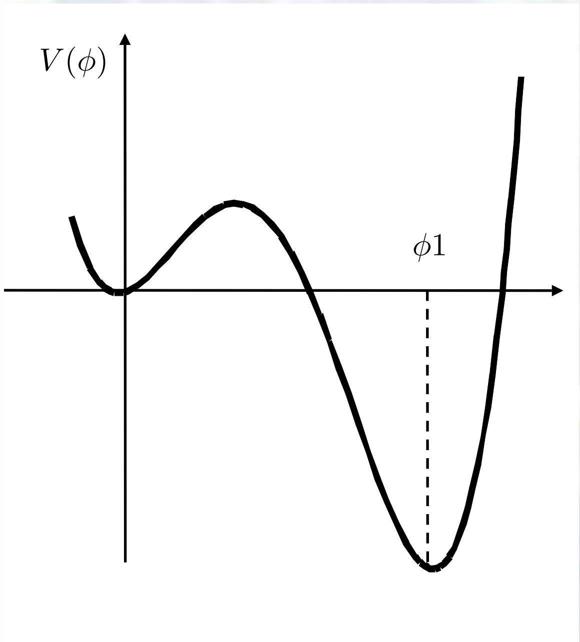
What is required for resonant tunnelling in QFT?



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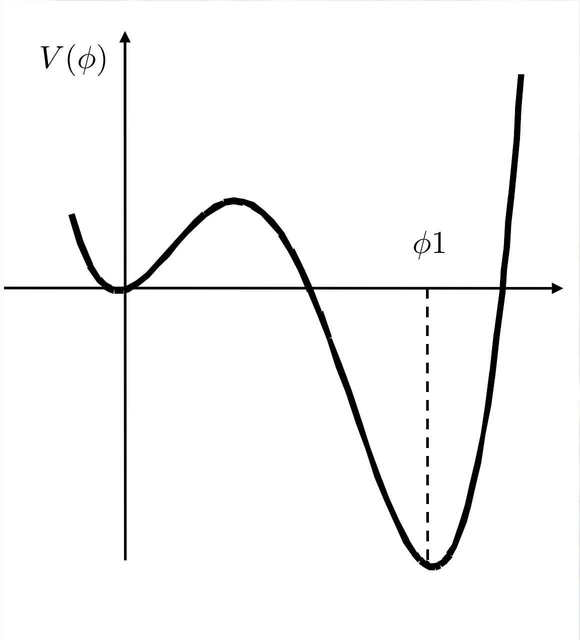

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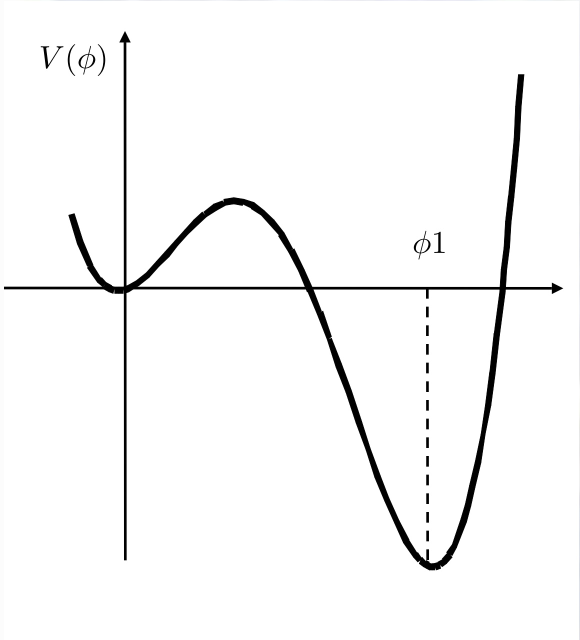

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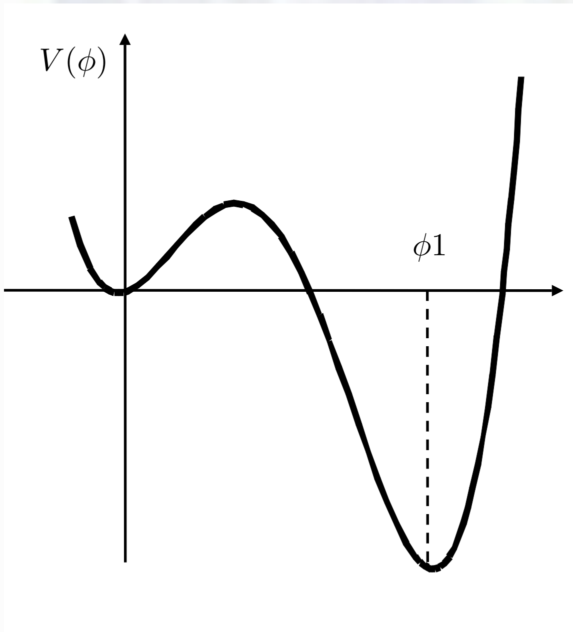

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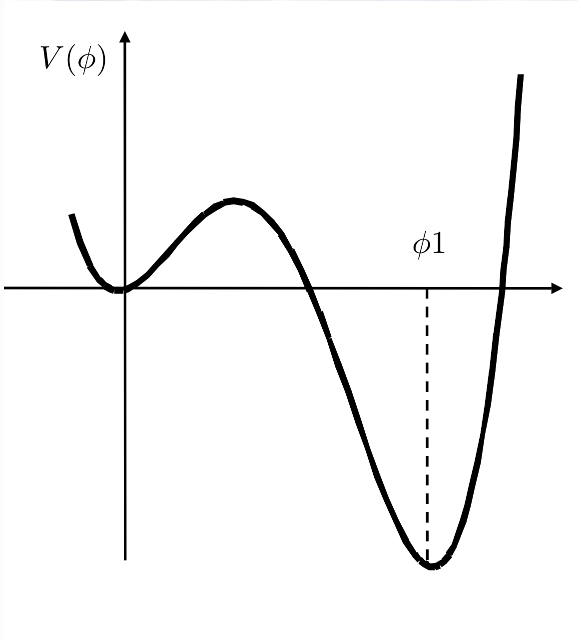

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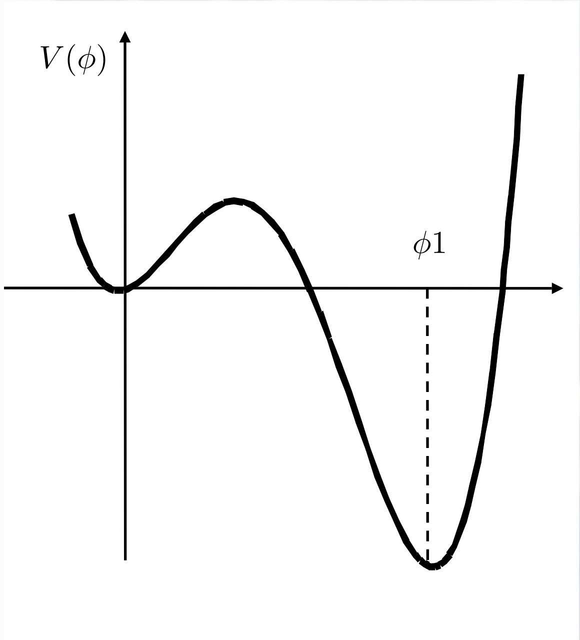

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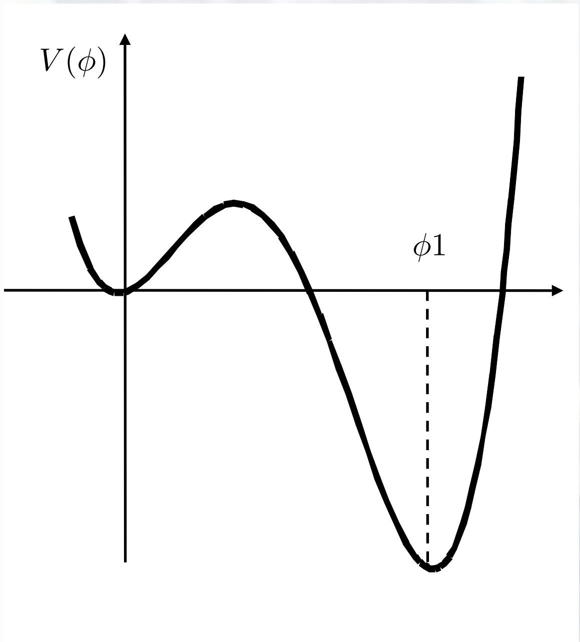
- Classical path in central well must correspond to a “bound state” solution oscillating between two stationary points, and picking up a quantum phase of (n+1/2)π

Decay of the homogeneous false vacuum



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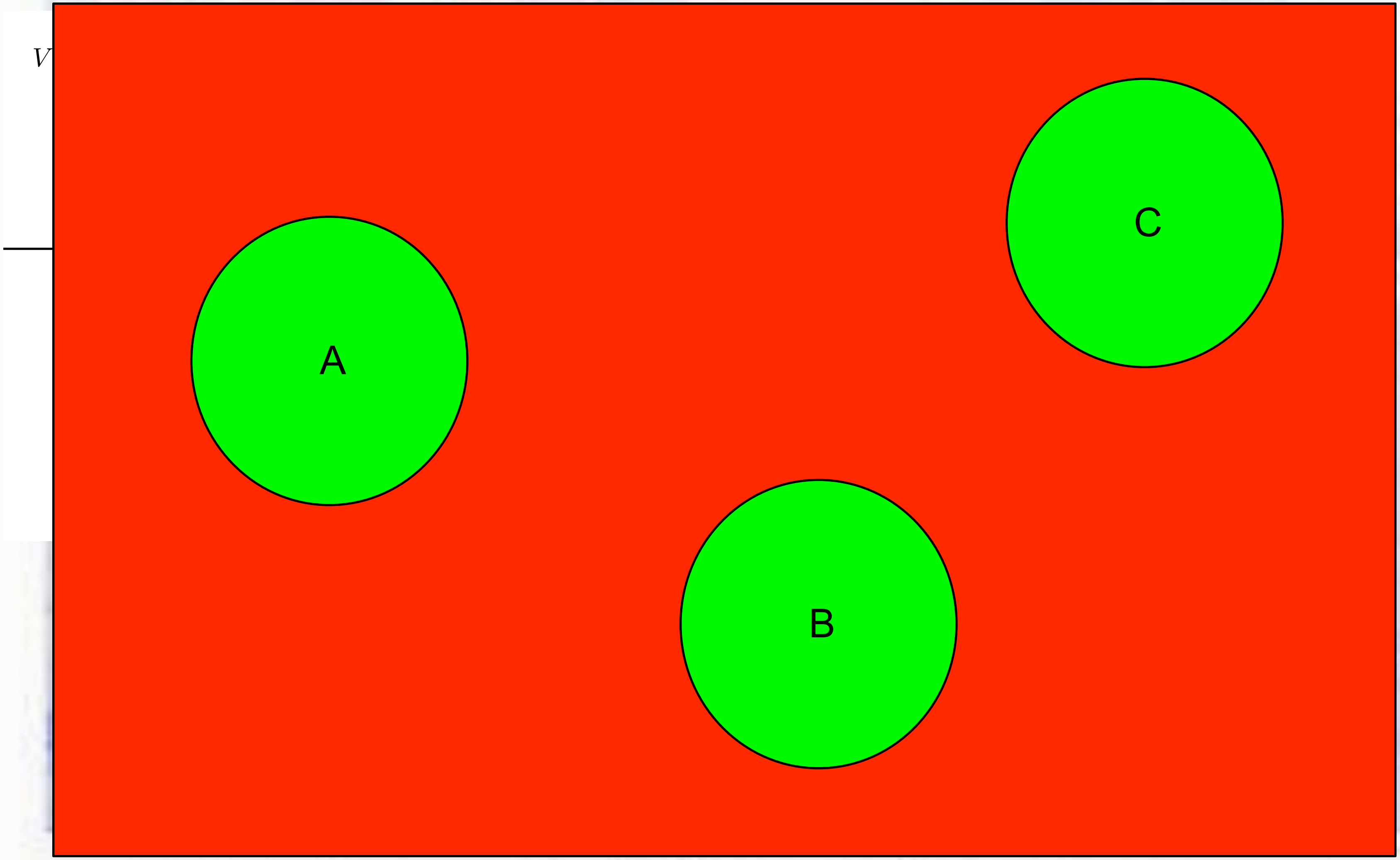

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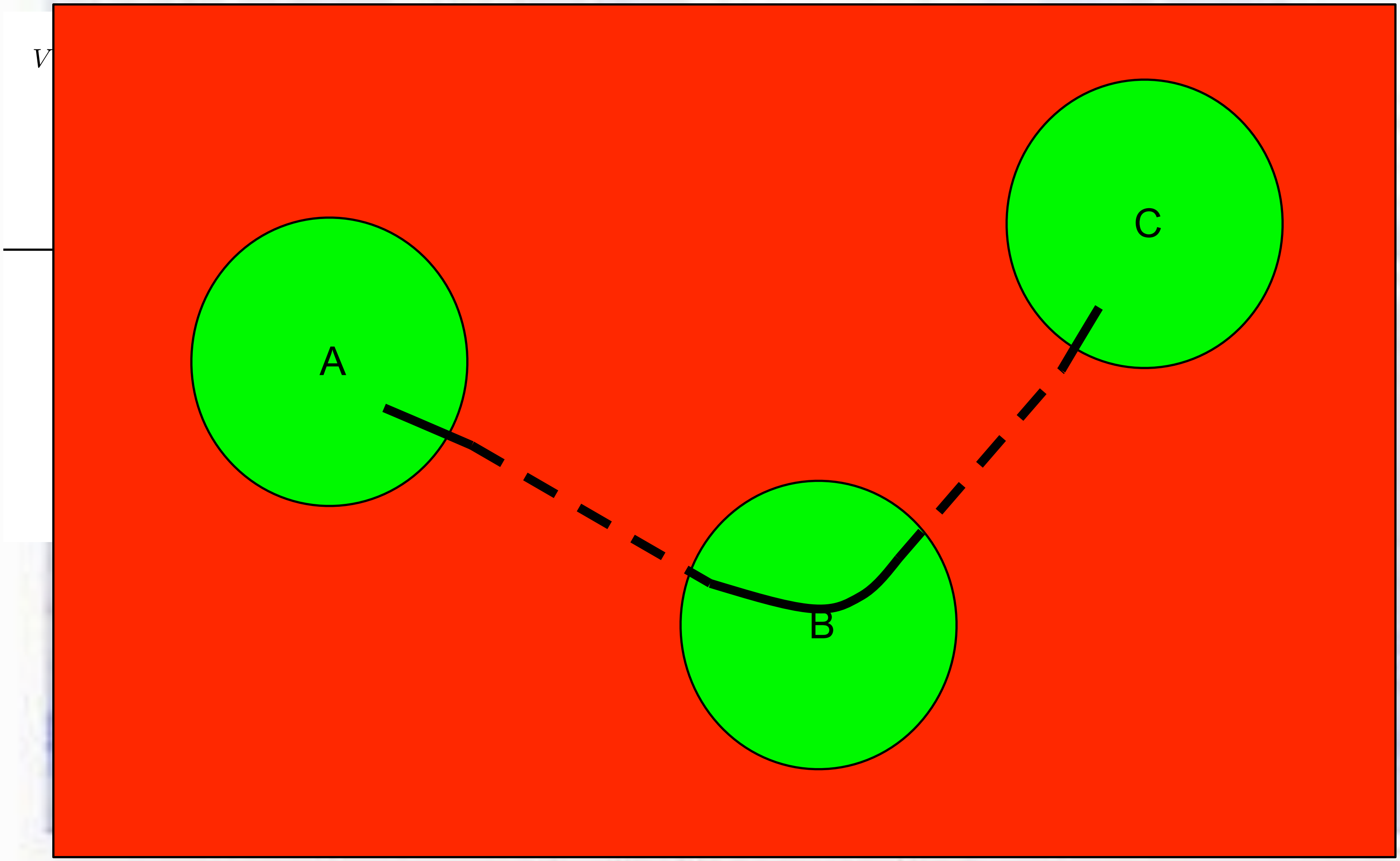

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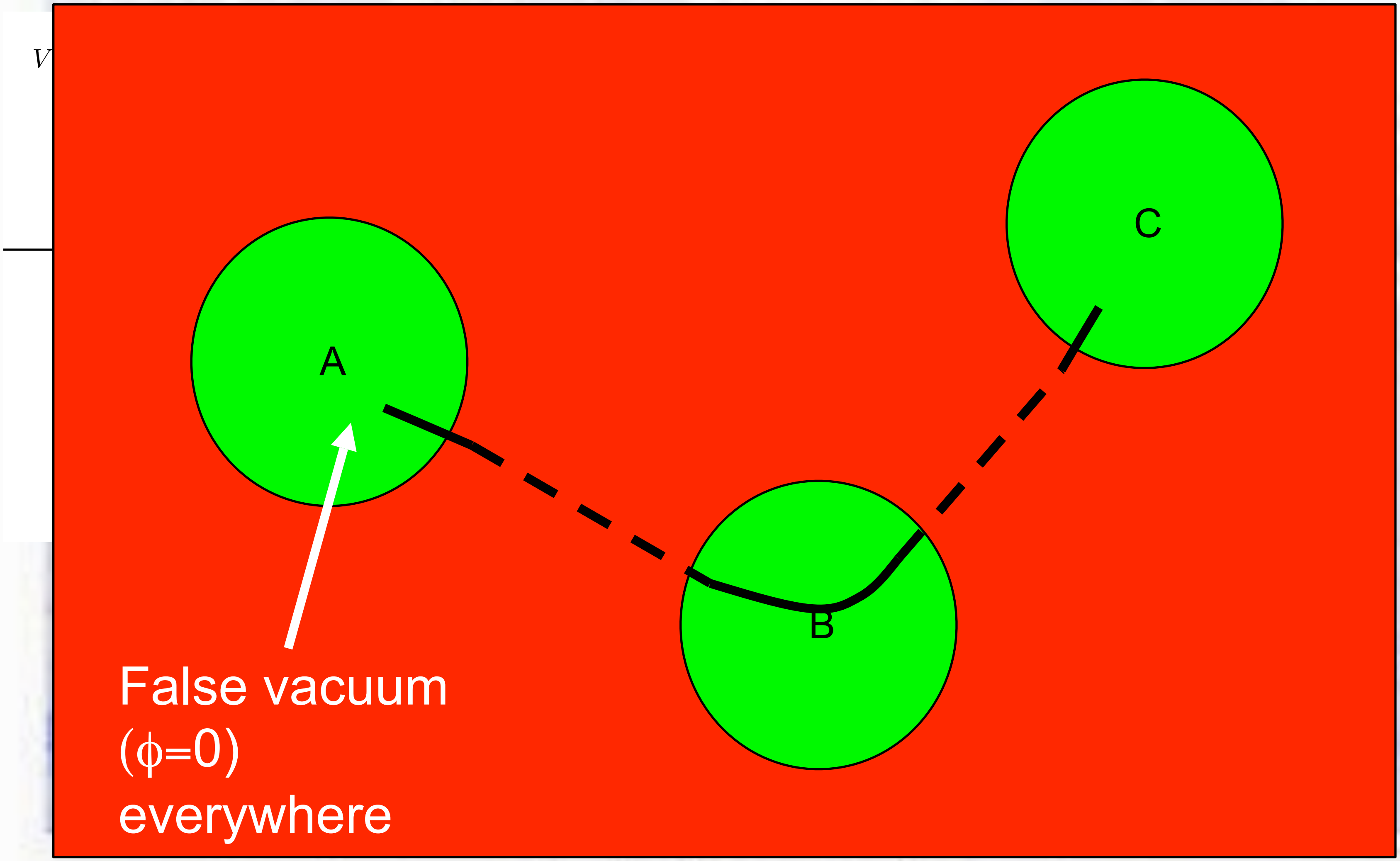

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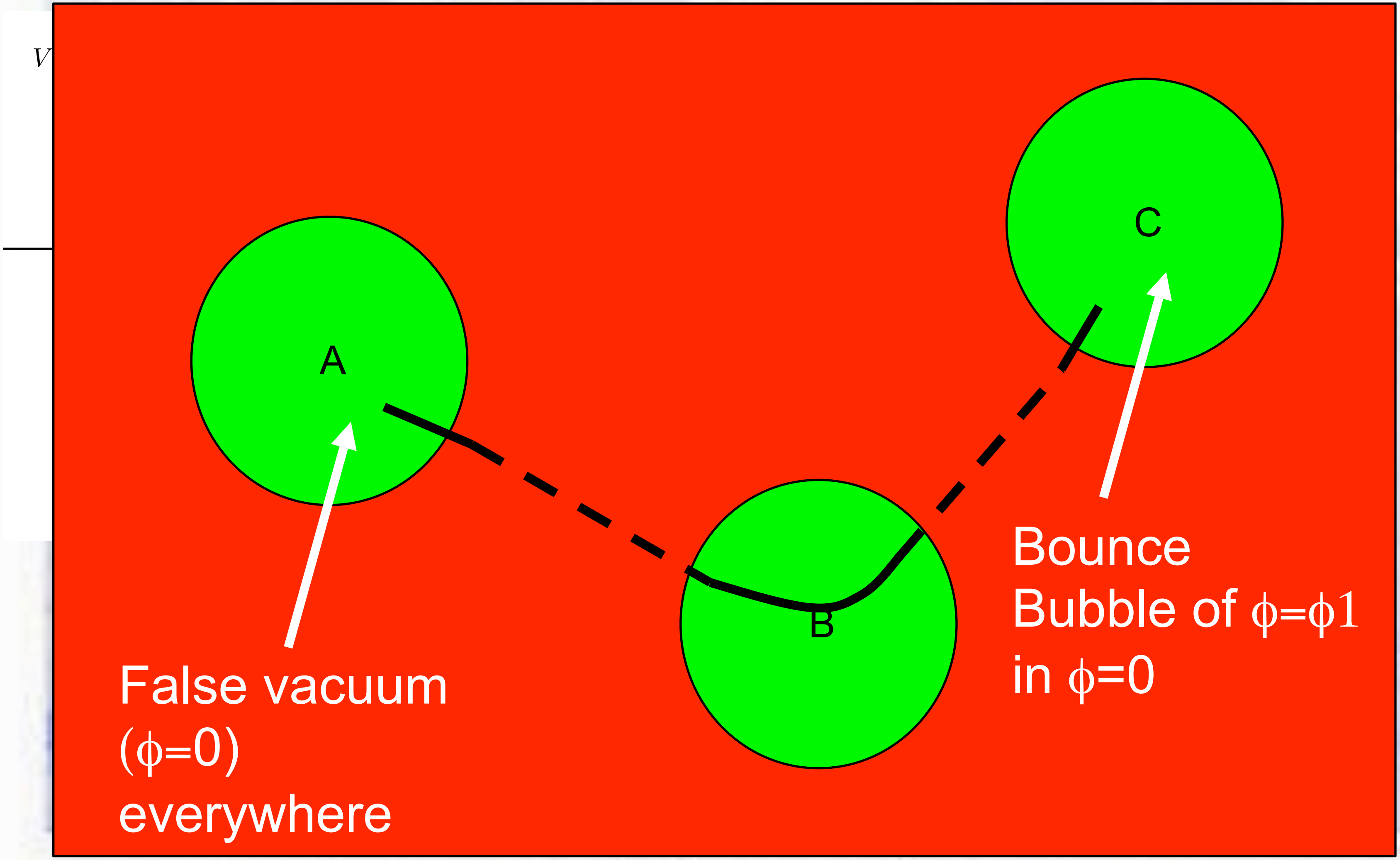

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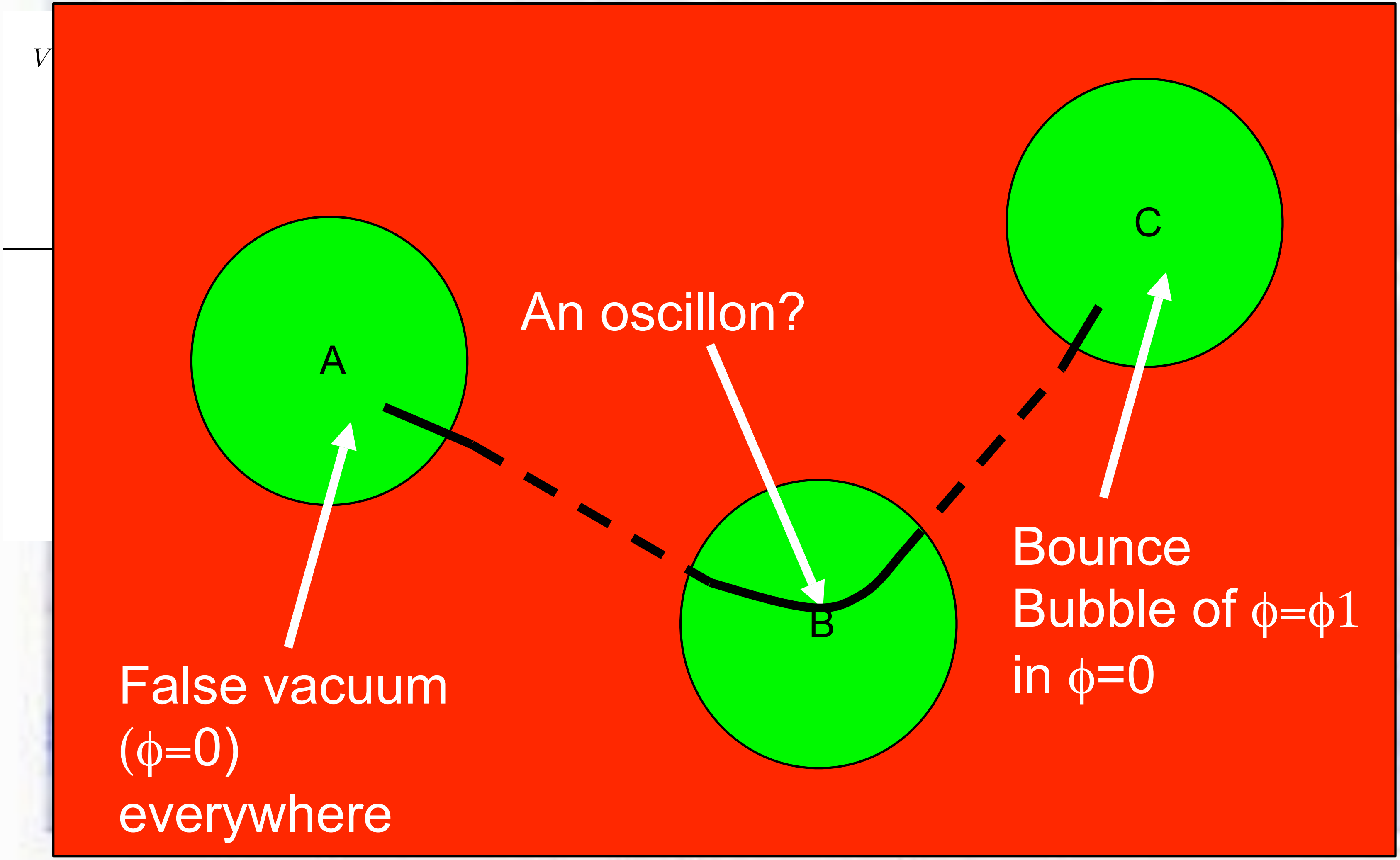
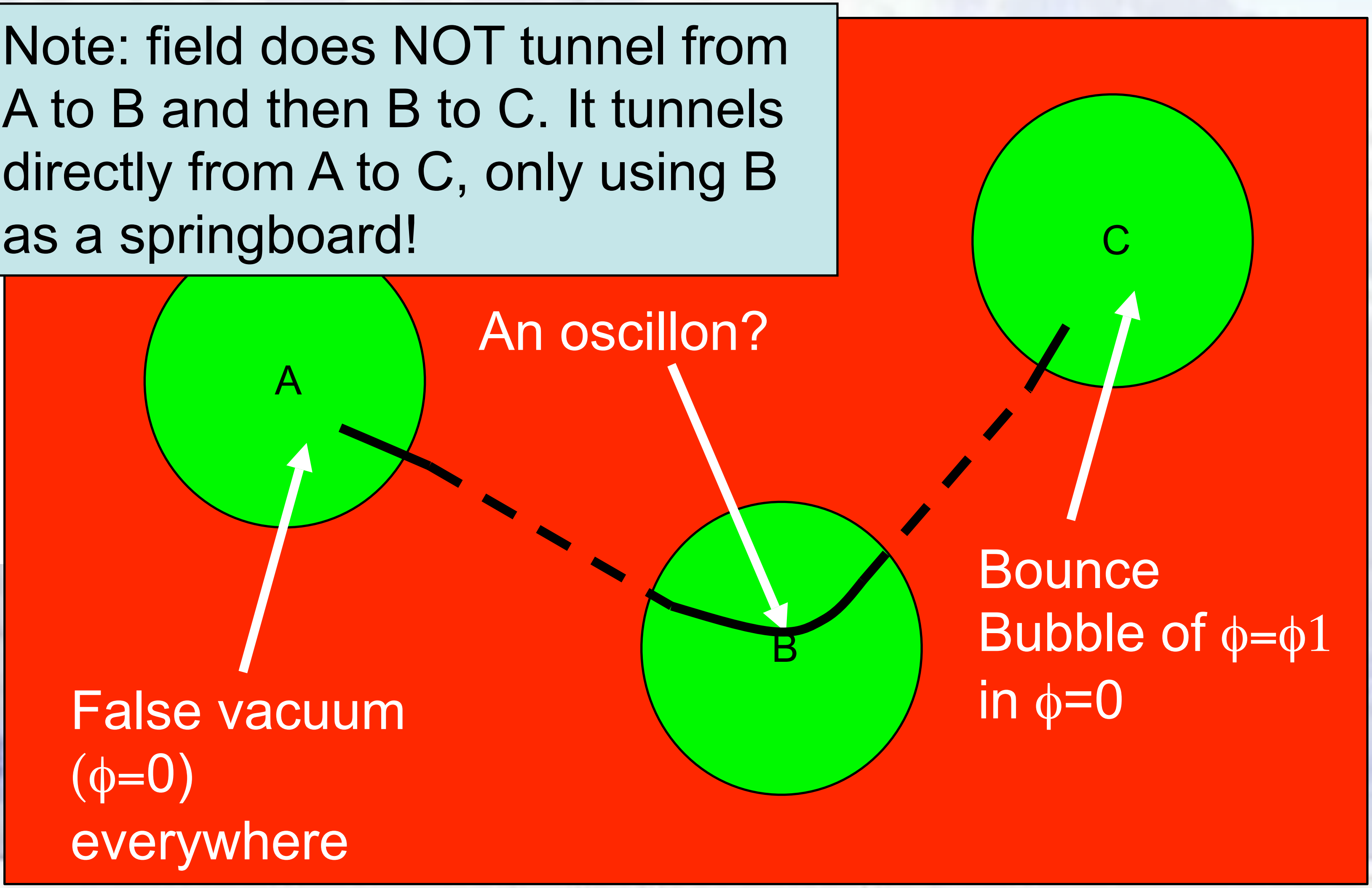


Decay of the homogeneous false vacuum

Note: field does NOT tunnel from A to B and then B to C. It tunnels directly from A to C, only using B as a springboard!



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Evaluate it on the boundary, $\partial \mathcal{V}$, and use condition 3 to show that $I = 0$.

$$\int_{t_1}^{t_2} dt\ E - \int_{\mathcal{V}} dx\ I = 0$$

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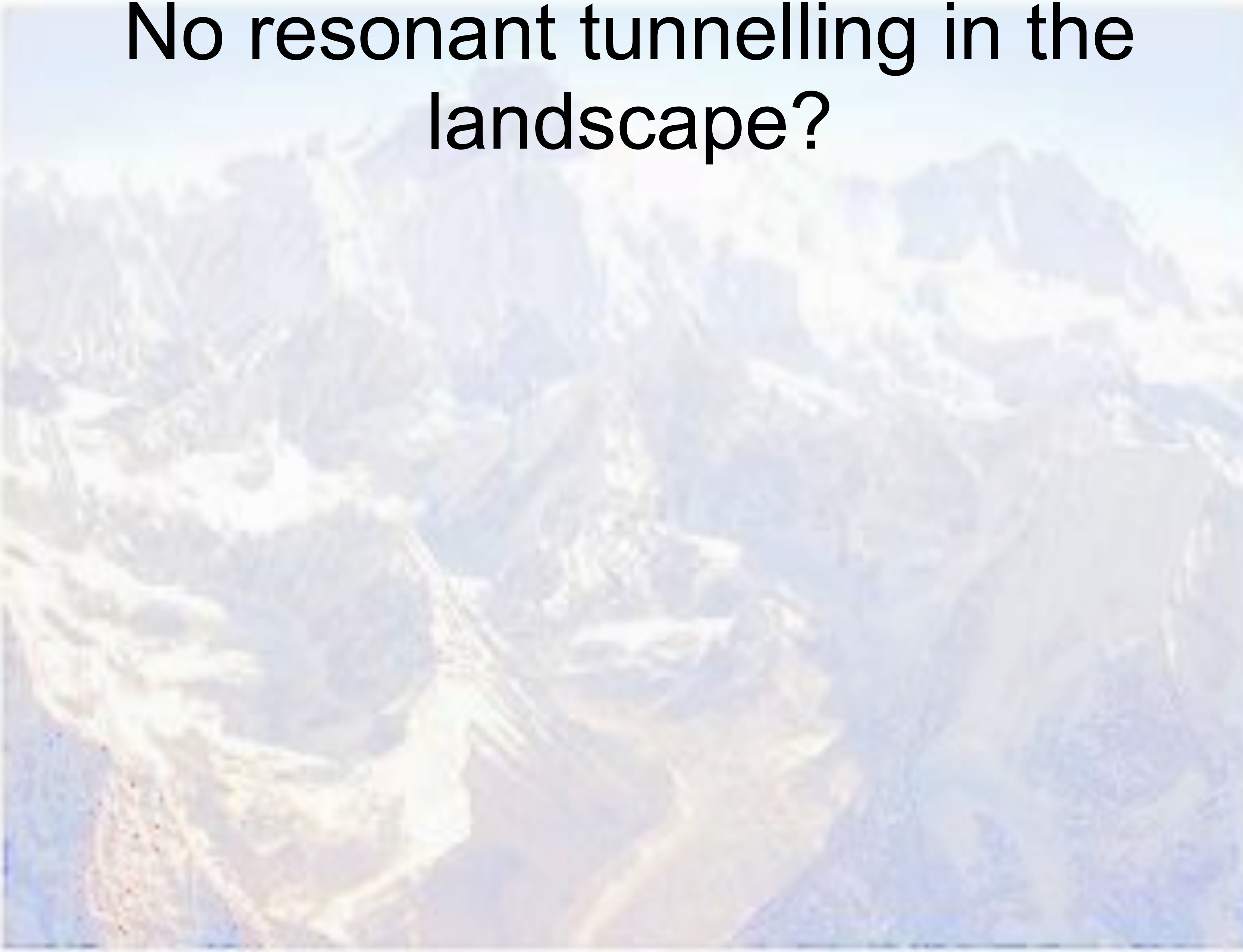
The homogeneous false vacuum cannot decay via resonant tunnelling in standard scalar QFT

No resonant tunnelling in the landscape?



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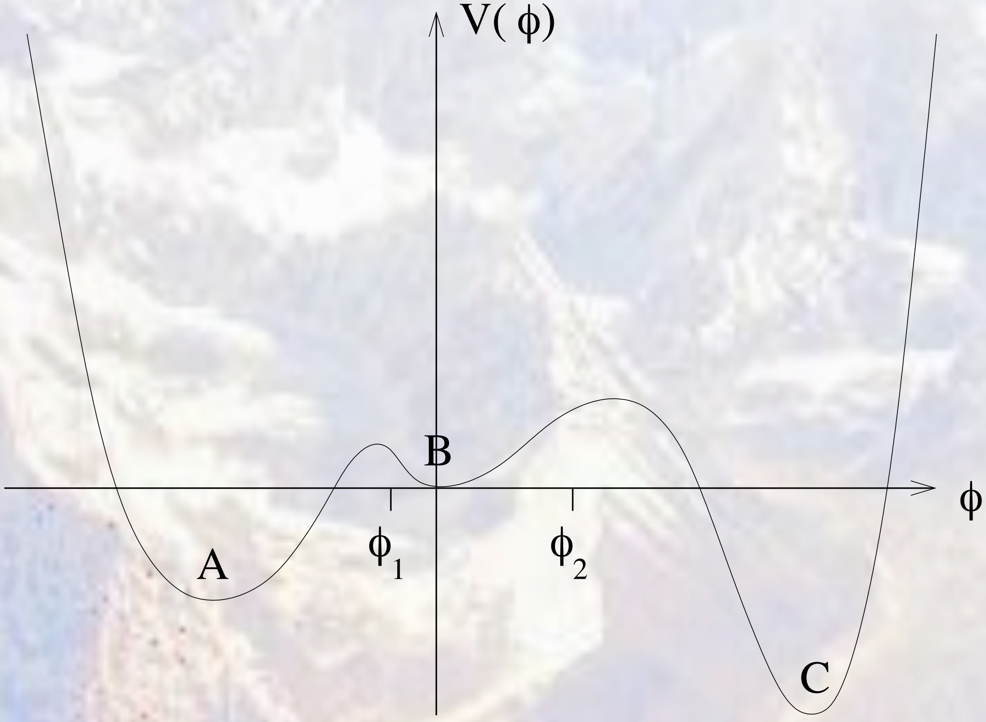

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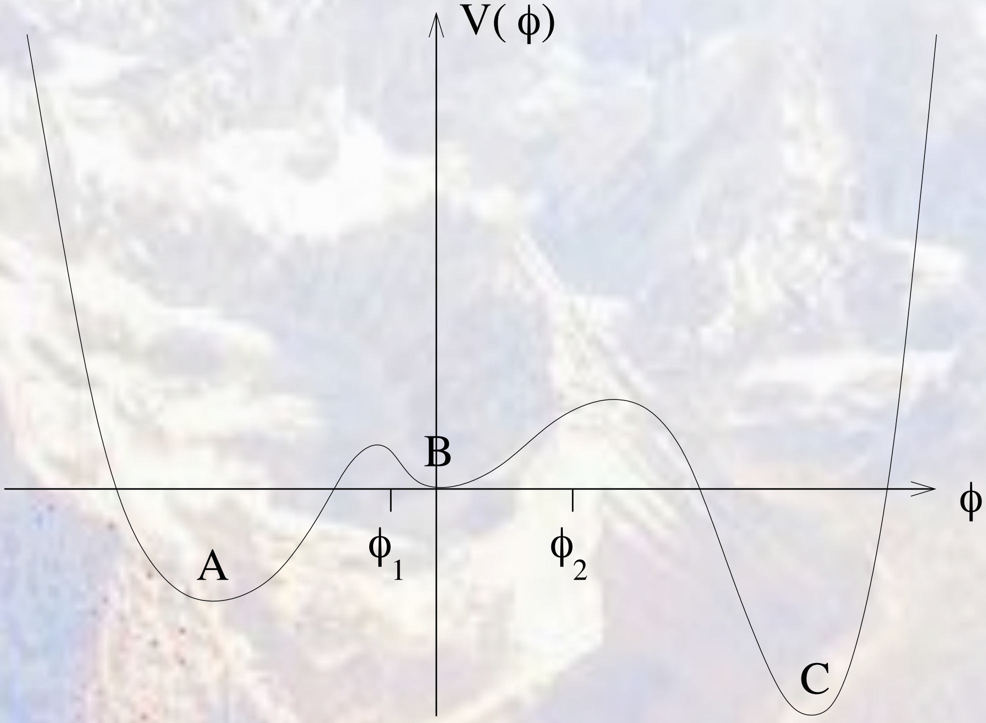

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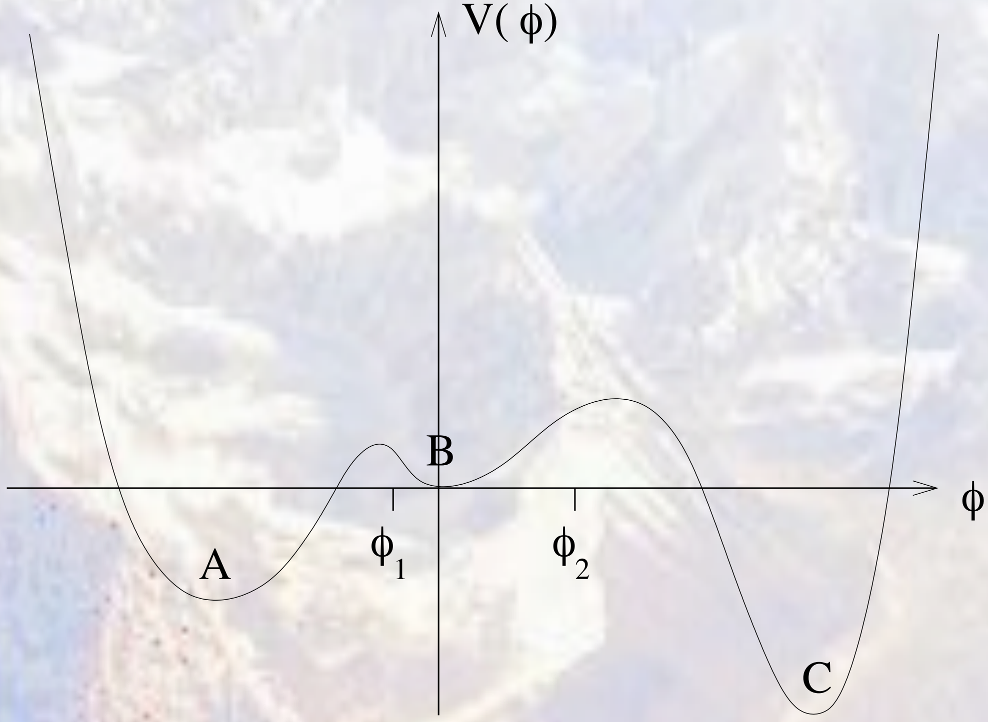

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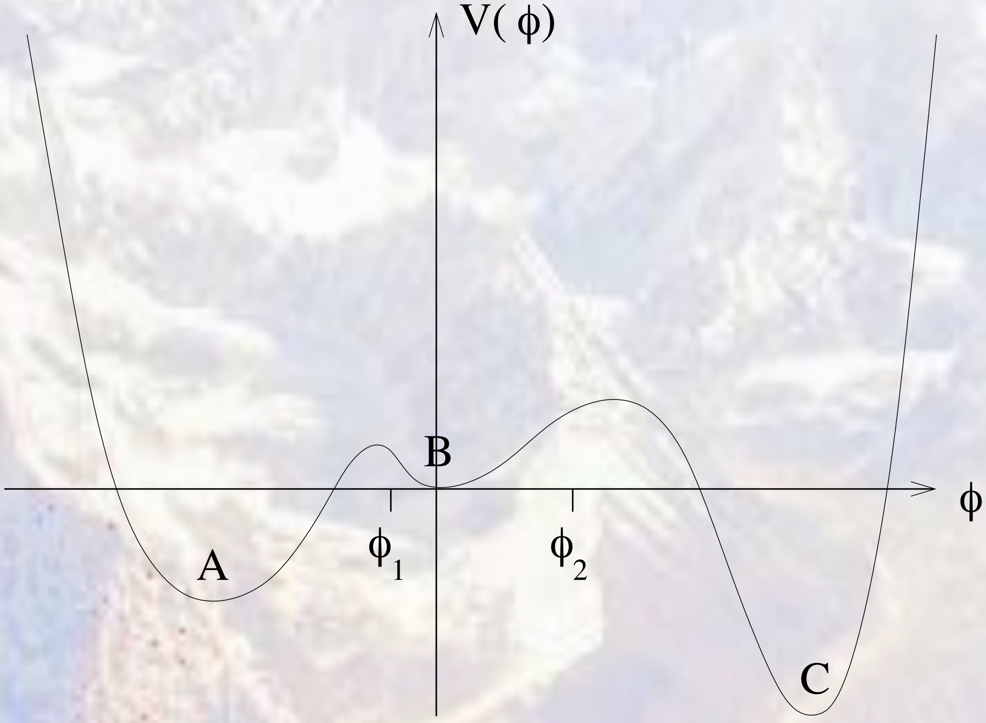

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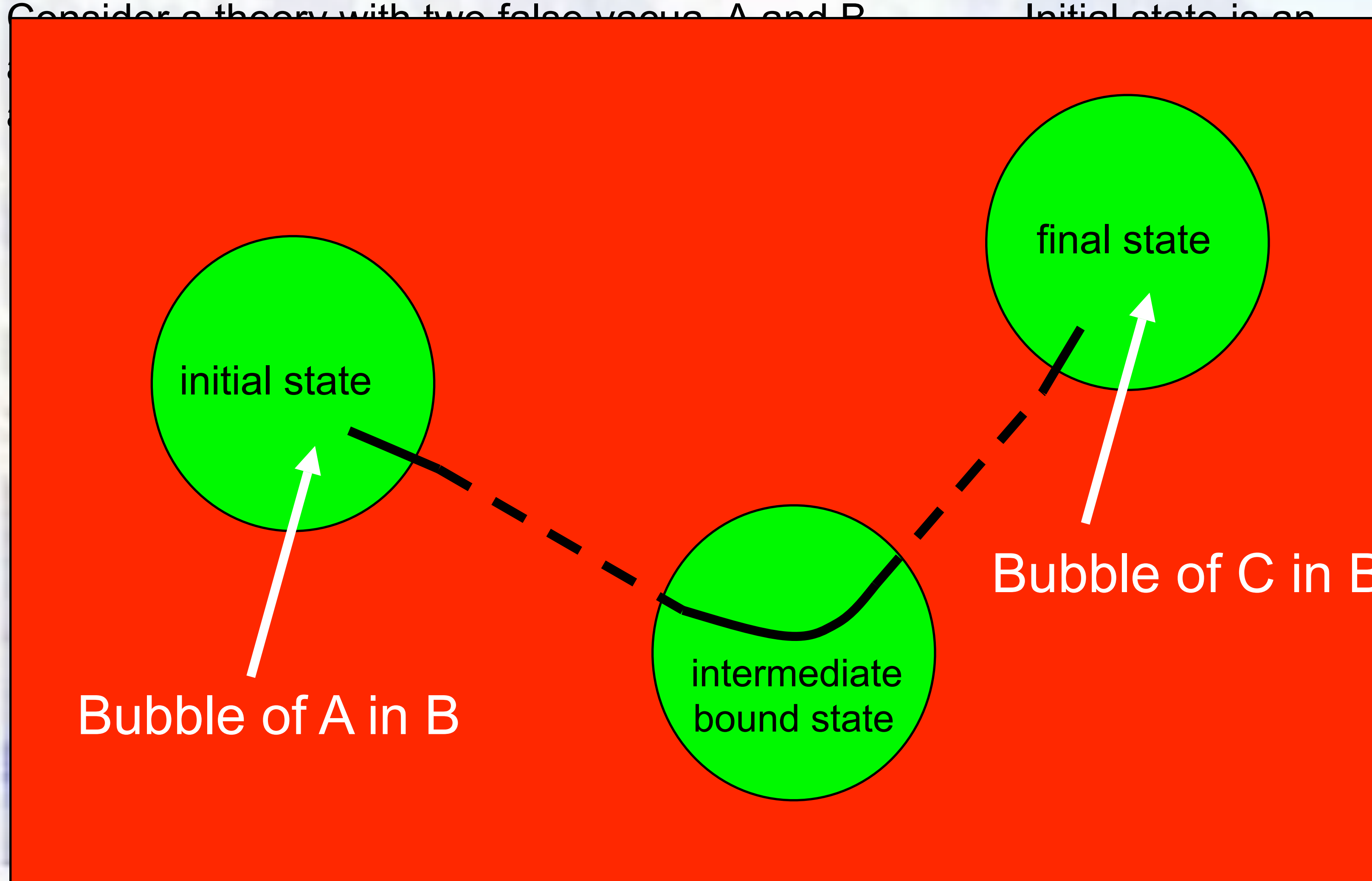

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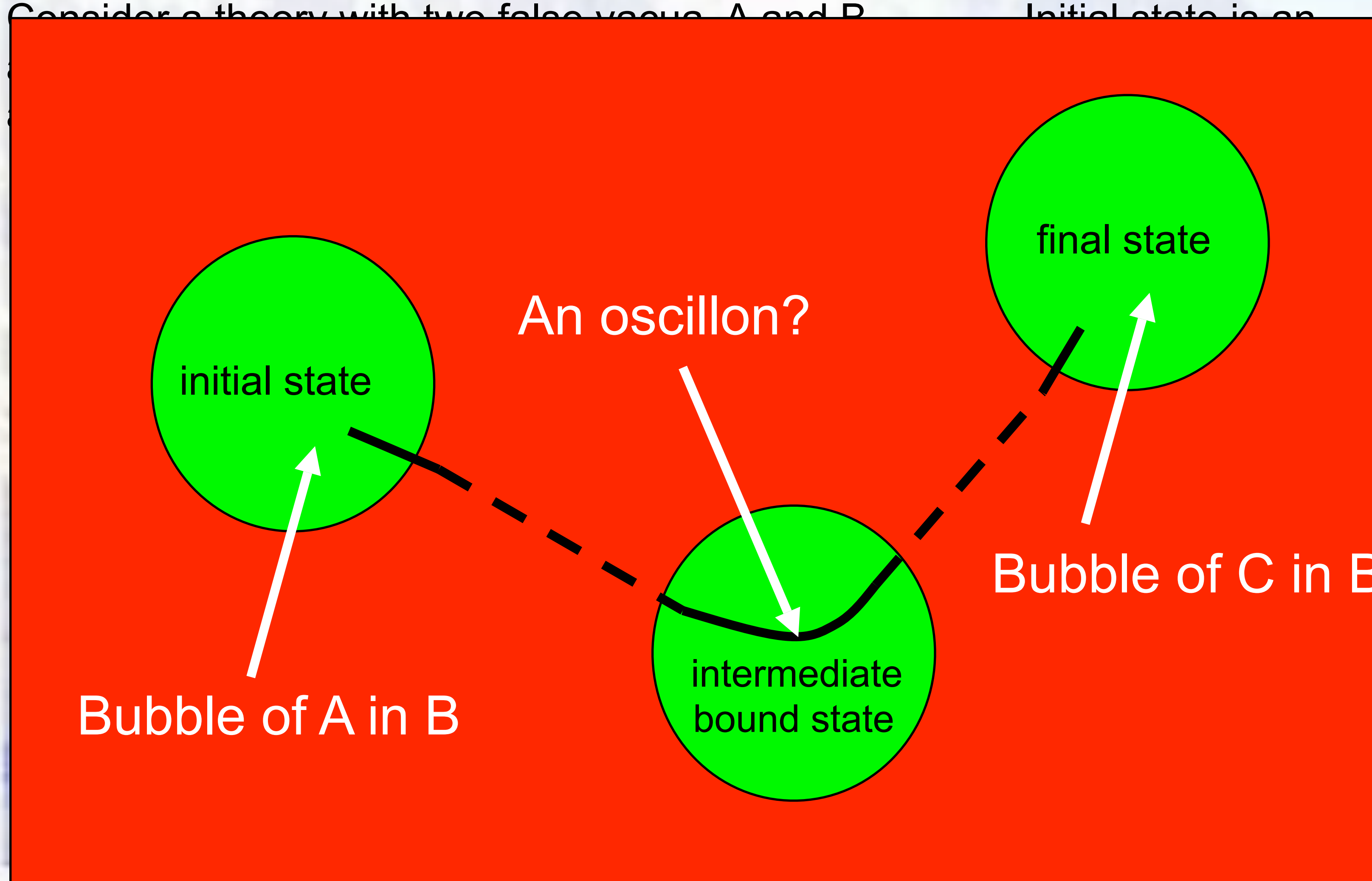

Five conditions for resonant decay

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No go theorem no longer applies, and suitable bound states can be found !

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Exact solutions exist for following action

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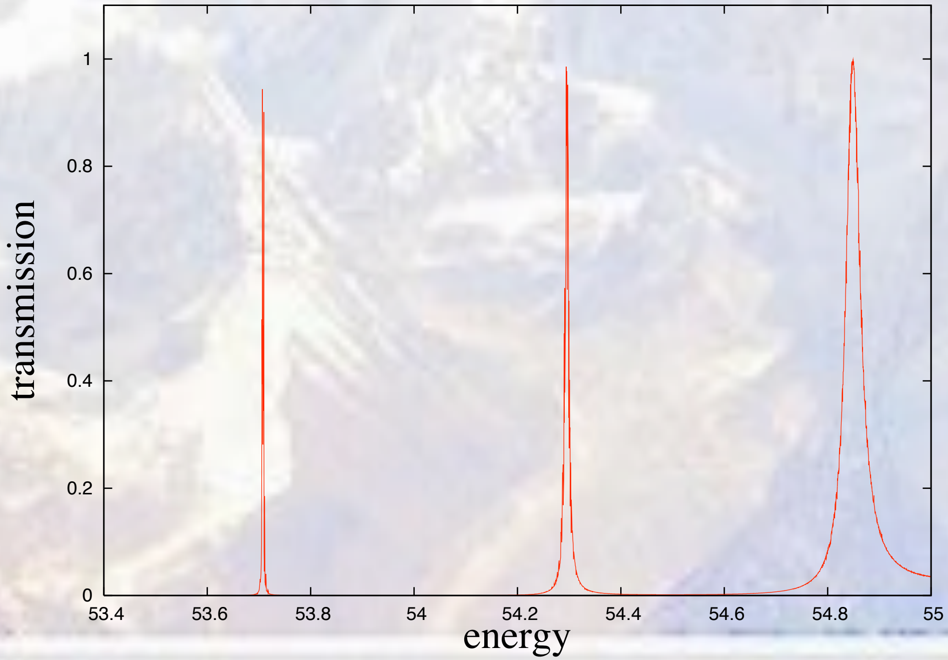

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- In the spirit of the landscape, even the most contrived set-up may be realised. Have we really gained anything?


Thanks!