# a conformal field theory for eternal inflation 



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## based on:

- "A Conformal Field Theory for Eternal Inflation", MK, B. Freivogel, arXiv:0903.2048


## related work:

- MK, Freivogel, Nicolis, and Sigurdson; Bousso, Freivogel, and Yang; Garriga and Vilenkin; Guth, Garriga, and Vilenkin; Freivogel, Sekino, Susskind and Yeh; Chang, MK, and Levi; Susskind; Freivogel, Horowitz, and Shenker; Maldacena; Dyson, MK, and Susskind; Freivogel, MK, RodriguezMartinez, Susskind; Strominger; Guth and Weinberg; Hawking, Moss, and Stewart...


## de Sitter space

- dS space is central in modern cosmology
- it approximates inflation, and the universe appears to be entering a future dS phase
- if they exist in the string landscape, one expects $d S$ regions to dominate the global spacetime volume
- our observable universe would be a bubble immersed in an eternally inflating bath
- dS surrounds us in both space and time


## but...

- dS is notoriously difficult to understand
- it expands exponentially, forever fragmenting into causally disconnected pieces
- every point is surrounded by a finite area event horizon, with a quantum temperature and horizon entropy
- it has no observable S-matrix or timelike boundary on which to define a holographic dual
- field at spacelike separation fluctuates independently at late times; correlations functions are unobservable


## false vacuum eternal inflation

consider a scalar field with a potential like this classically, it will inflate (i.e. dS) forever in either minimum
quantumly, it will tunnel via bubble nucleation from either into the

(a potential like this makes a universe in a bubble, very much like the one we observe) other after finite time

I'll call this dimensionless "decay rate" per unit Hubble time per unit Hubble volume $\gamma$

## cosmology

- in the string theory landscape, our universe may be inside such a bubble
- every such bubble will eventually be struck by an infinite number of others
- these can have potentially observable effects (for example on our CMB), if $\gamma \sqrt{\Omega_{k}}\left(V_{f} / V_{i}\right)>1$
- I won't discuss these now... but can we use them to define a set of asymptotic observables?


## to begin...

...imagine starting in a region of the universe where the field is roughly homogeneous and in its false vacuum
any initial inhomgeneities or impurities will rapidly inflate away, leaving a state very close to empty de Sitter
but after about $\gamma^{-1}$ efolds, a bubble of "true" vacuum will appear and begin to expand

## a conformal diagram



past

# global slices 

late global time slice
a global time slice of $d S$ in $D$ dimensions is a d=D-I sphere

$$
d s_{D}^{2}=-d t^{2}+H^{-2} \cosh ^{2}(H t) d \Omega_{D-1}^{2}
$$

## viewed from above (fixed t)


each bubble forms part of an infinite cluster, which may or may not connect to other clusters
for decay rate less than order one, the transition (if there is one) does not percolate or complete
observers in flat bubbles at arbitrarily late times ("census takers") can see an arbitrarily large number of bubbles - all others have a cutoff

## the distribution...

...on global slices follows immediately from the dS metric
the comoving size of the bubble is just the conformal time to infinity, and $d N=\gamma d V d \eta$ the metric in conformal coordinates is

$$
d s^{2}=\frac{1}{\sin ^{2} \eta}\left(-d \eta^{2}+d \Omega_{d}^{2}\right)
$$

and so one immediately obtains

$$
d N=\gamma \sin (\psi)^{-(d+1)} d \psi d \Omega_{d}
$$

## observables?

- the full global slices are never observable
- it may be more interesting to consider the distribution on the wall an observer's bubble
- which is a census takers asymptotic sky
- but it turns out that this distribution is identical, although now d=D-2
- this is all in the approximation of noninteracting bubbles


## dS/dS?

- hence, one can define a dimension theory either from d+l dS using global slices, or d+2 dS using the census taker's sky
- finite CC/time for the census taker is a UV cutoff
- the two distributions (in this approximation) are identical, but the interpretation is very different


## fractal dimension

one can compute the box counting fractal dimension of the set of points that are inside zero bubbles...

$$
D_{B C} \equiv-d(\ln n(\epsilon)) / d(\ln \epsilon) \quad n(\epsilon)=V(\epsilon) \epsilon^{-d}
$$

$$
d V(\epsilon)=V r^{d} d N(r)=V \gamma(d r / r) \quad n(\epsilon)=\epsilon^{\gamma-d}
$$

$$
D_{B C}=d-\gamma
$$

Mandelbrot,Vilenkin


Numerical simulations of the model in $d=2$. Only one disk type is shown. Left pane: $\gamma=$. $1, \delta / R=.01$. Right pane: $\gamma=.5, \delta / R=.01$.

## percolation

- one expects percolation transitions in this set when fractal dimension is near integer
- but if the decay rate is very small, one is far from these transitions
- can there still be a conformal field theory description away from these special points?
- if so, quite unlike standard bond or site percolation


## correlation functions

- let's try to define some correlation functions, focussing on the case $d=2$
- for now we can forget about the bulk dS and focus only on the statistical model
- the most natural quantity is $N(z)$, the number of bubble disks that impinge on $z$
- so, try to compute $<\mathrm{N}(\mathrm{z})>$ using these distributions
- plainly this will diverge.... let's see how


## $d=2$

- in $d=(3+I)-2=(2+I)-I=2$, stereographically project the 2-sphere distribution to the plane

$$
d N=\gamma \sin (\psi)^{-(d+1)} d \psi d \Omega_{d}
$$

- SP maps disks to disks, but both the radii and center locations change
- the distribution in the plane is simply

$$
d N=\gamma\left(d r / r^{3}\right) d z d \bar{z}
$$

- this is obviously scale invariant, and the stereographic mapping proves it is fully Mobius invariant
- Mobius transformations arise in a beautiful geometric way - they are the action of the $3+1$ Lorentz group on the lightcone of the bubble's nucleation event


## I-point function of N

take the average of $N(z)$ over the ensemble of all possible configurations, with weight given by the distribution

$$
\begin{gathered}
Z_{+}=\sum_{n=0}^{\infty} \frac{\gamma^{n}}{n!} \prod_{k=1}^{n} \int_{\delta}^{R} \frac{d r_{k}}{r_{k}^{3}} \int d x_{k} d \bar{x}_{k} \\
N_{n}(z)=\sum_{k=1}^{n} \Theta\left(\left|x_{k}-z\right|-r_{k}\right) \\
\langle N(z)\rangle=Z_{+}^{-1} \sum_{n=0}^{\infty} \frac{\gamma^{n}}{n!} \prod_{k=1}^{n} \int_{\delta}^{R} N_{n}(z) \frac{d r_{k}}{r_{k}^{3}} d x_{k} d \bar{x}_{k} \\
\langle N(z)\rangle=Z_{+}^{-1} Z_{+} \gamma \int_{\delta}^{R} \frac{d r}{r^{3}} \pi r^{2}=\pi \gamma \ln (R / \delta)
\end{gathered}
$$

## disks and anti-disks

- the divergence occurs because $\mathrm{N}_{+}>0$
- but in dS, transitions will occur either way
- try allowing for "anti-bubbles" - up transitions - and count the difference

$$
N(z)=N_{+}(z)-N_{-}(z)
$$

## bulk field theory

- simplest non-trivial possibility - bubbles can nucleate inside other bubbles, always with the same rate as the parent false vacuum
- such a rule would follow from a bulk field with a periodic potential
- will be interesting to extend this to non-periodic potentials, but will probably require approximation


$$
Z=Z_{+} Z_{-}=\left(\sum_{n=0}^{\infty} \frac{\gamma^{n}}{n!} \prod_{k=1}^{n} \int_{\delta}^{R} \frac{d r_{k}}{r_{k}^{3}} \int d x_{k} d \bar{x}_{k}\right)\left(\sum_{n=0}^{\infty} \frac{\gamma^{n}}{n!} \prod_{k=1}^{n} \int_{\delta}^{R} \frac{d r_{k}}{r_{k}^{3}} \int d x_{k} d \bar{x}_{k}\right)
$$

$$
\langle N(z)\rangle=\left\langle N_{+}(z)-N_{-}(z)\right\rangle=\left(Z_{-} Z_{-}^{-1}\right)\left\langle N_{+}\right\rangle-\left(Z_{+} Z_{+}^{-1}\right)\left\langle N_{-}\right\rangle=0
$$

(note that the transition rates are equal)

## what about 2-point functions?

$$
\begin{array}{r}
\left\langle N\left(z_{1}\right) N\left(z_{2}\right)\right\rangle=\left\langle\left(N_{+}\left(z_{1}\right)-N_{-}\left(z_{1}\right)\right)\left(N_{+}\left(z_{2}\right)-N_{-}\left(z_{2}\right)\right)\right\rangle=2\left\langle N_{+}\left(z_{1}\right) N_{+}\left(z_{2}\right)\right\rangle-2\left\langle N_{+}\right\rangle^{2} \\
\left\langle N_{+}\left(z_{1}\right) N_{+}\left(z_{2}\right)\right\rangle=Z_{+}^{-1} \gamma Z_{+} \int_{\delta}^{R} \frac{d r}{r^{3}} \int d z d \bar{z} \Theta\left(\left|z-z_{1}\right|-r\right) \Theta\left(\left|z-z_{2}\right|-r\right) \\
=\gamma \int_{\delta}^{R} \frac{d r}{r^{3}} A_{12}\left(d_{12}, r\right)
\end{array}
$$



$$
\begin{aligned}
& A_{12}\left(d_{12}, r\right)=2 r^{2}\left(\cos ^{-1}\left(d_{12} / 2 r\right)-\left(d_{12} / 2 r\right) \sqrt{1-\left(d_{12} / 2 r\right)^{2}}\right) \Theta\left(2 r-d_{12}\right) \\
& \int_{\delta}^{R} \frac{d r}{r^{3}} A_{12}\left(d_{12}, r\right)=\pi \ln \left(R / d_{12}\right)-\pi / 2+\mathcal{O}\left(R^{-1}\right) \\
& \quad \phi(z) \equiv\left(N_{+}(z)-N_{-}(z)\right) / \sqrt{\pi \gamma}, \\
& \quad\left\langle\phi\left(z_{1}\right) \phi\left(z_{2}\right)\right\rangle=-\ln \left|z_{1}-z_{2}\right|^{2}+\ln R^{2}-1
\end{aligned}
$$

this result holds in ALL d: N is always a field with dimension 0

## massless scalars

- fields with log correlators are not particularly well-defined - and these fields have a (discrete) shift symmetry
- one usually either differentiates or exponentiates them
- let's try exponentials...

$$
V_{\beta}(z) \equiv e^{i \alpha \phi(z)}=e^{i \beta\left(N_{+}(z)-N_{-}(z)\right)}, \text { where } \beta=\alpha / \sqrt{\pi \gamma}
$$

## I-point function

$$
\begin{gathered}
\left\langle V_{\beta}\left(z_{1}\right)\right\rangle=Z_{+}^{-2}\left|\exp \left\{\gamma \int_{\delta}^{R} \frac{d r}{r^{3}} \int d x d \bar{x}\left[e^{i \beta} \Theta\left(2 r-\left|x-z_{1}\right|\right)+\Theta\left(-2 r+\left|x-z_{1}\right|\right)\right]\right\}\right|^{2} \\
\left\langle V_{\beta}\left(z_{1}\right)\right\rangle=\exp \left[-2 \gamma(1-\cos \beta) \int_{\delta}^{R} \frac{d r}{r^{3}} A_{1}(r)\right] \\
\left\langle V_{\beta}(z)\right\rangle=\left(\frac{R}{\delta}\right)^{-2 \pi \gamma(1-\cos \beta)}=\left\{\begin{array}{cc}
1 & \text { if } \beta=2 \pi n, n \in \mathcal{Z} \\
0 & \text { otherwise }
\end{array}\right\}
\end{gathered}
$$

periodic conservation of charge

## 2-point functions

$$
\begin{gathered}
\left\langle V_{\beta_{1}}\left(z_{1}\right) V_{\beta_{2}}\left(z_{2}\right)\right\rangle=\left|\left\langle\exp \left[i \beta_{1} N\left(z_{1}\right)+i \beta_{2} N\left(z_{2}\right)\right]\right\rangle\right|^{2}= \\
\exp \left\{\gamma \int \frac{d r}{r^{3}} \int d x d x\left(-2+2 \cos \left[\beta_{1} \Theta\left(r-\left|x-z_{1}\right|\right)+\beta_{2} \Theta\left(r-\left|x-z_{2}\right|\right)\right]\right)\right\}
\end{gathered}
$$

$$
\begin{gathered}
\left\langle V_{\beta_{1}}\left(z_{1}\right) V_{\beta_{2}}\left(z_{2}\right)\right\rangle=\exp \left\{-2 \gamma \int \frac{d r}{r^{3}}\left[\left(1-\cos \beta_{1}\right) A_{1}^{0}+\left(1-\cos \beta_{2}\right) A_{2}^{0}+\left(1-\cos \left(\beta_{1}+\beta_{2}\right)\right) A_{12}^{0}\right]\right\} \\
=\exp \left\{-2 \pi \gamma\left[\left(1-\cos \beta_{1}\right) I_{1}^{0}+\left(1-\cos \beta_{1}\right) I_{1}^{0}+\left(1-\cos \beta_{2}\right) I_{2}^{0}+\left(1-\cos \left(\beta_{1}+\beta_{2}\right)\right) I_{12}^{0}\right]\right\}
\end{gathered}
$$

$$
\begin{aligned}
& I_{1}^{0}\left(z_{1}, z_{2}\right) \equiv \frac{1}{\pi} \int \frac{d r}{r^{3}} A_{1}^{0}\left(r, z_{1}, z_{2}\right) \\
& \begin{array}{c}
I_{1}^{0}=I_{1}-I_{12} \\
I_{2}^{0}=I_{2}-I_{12} \\
I_{12}^{0}=I_{12}
\end{array}
\end{aligned}
$$

$$
\begin{aligned}
I_{1}^{0}=I_{2}^{0} & =\ln \frac{\left|z_{12}\right|}{\delta} \\
I_{12}^{0} & =\ln \frac{R}{\left|z_{12}\right|}
\end{aligned}
$$

Need the coeff of the overlap area $=0$, i.e. $\beta_{1}+\beta_{2}=2 \pi n$

$$
\left\langle V_{\beta_{1}}\left(z_{1}\right) V_{\beta_{2}}\left(z_{2}\right)\right\rangle=\left(\frac{\delta}{z_{1}-z_{2}}\right)^{2 \pi \gamma\left(1-\cos \beta_{1}\right)}\left(\frac{\delta}{\overline{z_{1}}-\bar{z}_{2}}\right)^{2 \pi \gamma\left(1-\cos \beta_{1}\right)}
$$

$$
\Delta(\beta)=\bar{\Delta}(\beta)=\pi \gamma(1-\cos \beta)
$$

This is the correct behavior for a conformal field!

And reminiscent of bulk dispersion relation for a periodic potential...

## 3-point functions

$$
\begin{aligned}
& \left\langle V_{\beta_{1}}\left(z_{1}\right) V_{\beta_{2}}\left(z_{2}\right) V_{\beta_{3}}\left(z_{3}\right)\right\rangle=\exp \left\{-2 \pi \gamma\left[\left(1-\cos \beta_{1}\right) I_{1}^{0}+\left(1-\cos \beta_{2}\right) I_{2}^{0}+\left(1-\cos \beta_{3}\right) I_{3}^{0}+\right.\right. \\
& \left.\left.\left(1-\cos \left(\beta_{1}+\beta_{2}\right)\right) I_{12}^{0}+\left(1-\cos \left(\beta_{1}+\beta_{3}\right)\right) I_{13}^{0}+\left(1-\cos \left(\beta_{2}+\beta_{3}\right)\right) I_{23}^{0}+\left(1-\cos \left(\beta_{1}+\beta_{2}+\beta_{3}\right)\right) I_{123}^{0}\right]\right\} \\
& \beta_{1}+\beta_{2}+\beta_{3}=2 \pi n \\
& \left\langle V_{\beta_{1}}\left(z_{1}\right) V_{\beta_{2}}\left(z_{2}\right) V_{\beta_{3}}\left(z_{3}\right)\right\rangle= \\
& \exp \left\{-2 \pi \gamma\left[\left(1-\cos \beta_{1}\right)\left(I_{1}^{0}+I_{23}^{0}\right)+\left(1-\cos \beta_{2}\right)\left(I_{2}^{0}+I_{13}^{0}\right)+\left(1-\cos \beta_{3}\right)\left(I_{3}^{0}+I_{12}^{0}\right)\right]\right\} \\
& I_{1}^{0}=I_{1}-I_{12}-I_{13}+I_{123} \\
& I_{23}^{0}=I_{23}-I_{123}
\end{aligned}
$$

## and the result is...


$\left\langle V_{\beta_{1}}\left(z_{1}\right) V_{\beta_{2}}\left(z_{2}\right) V_{\beta_{3}}\left(z_{3}\right)\right\rangle=\left|\left(\frac{\delta}{z_{1}-z_{2}}\right)^{\Delta_{1}+\Delta_{2}-\Delta_{3}}\left(\frac{\delta}{z_{1}-z_{3}}\right)^{\Delta_{1}+\Delta_{3}-\Delta_{2}}\left(\frac{\delta}{z_{2}-z_{3}}\right)^{\Delta_{2}+\Delta_{3}-\Delta_{1}}\right|^{2}$
this is the correct conformally covariant 3-point function for a set of conformal primaries with dimensions as above
these quantities are IR finite and have consistent and positive dimensions, which is non-trivial

Mobius invariance fixes this form for operators of definite weight, because the lowest Mobius invariant is the cross ratio of four points

## forging ahead...



$$
\begin{aligned}
& \left\langle V_{\beta_{1}}\left(z_{1}\right) V_{\beta_{2}}\left(z_{2}\right) V_{\beta_{3}}\left(z_{3}\right) V_{\beta_{4}}\left(z_{4}\right)\right\rangle=\exp \left\{-2 \pi \gamma\left[\sum_{i}\left(1-\cos \beta_{i}\right) I_{i}^{0}+\sum_{i<j}\left(1-\cos \left(\beta_{i}+\beta_{j}\right)\right) I_{i j}^{0}+\right.\right. \\
& \left.\left.\sum_{i<j<k}\left(1-\cos \left(\beta_{i}+\beta_{j}+\beta_{k}\right)\right) I_{i j k}^{0}+\left(1-\cos \sum_{i} \beta_{i}\right) I_{1234}^{0}\right]\right\} \\
& =\left|\exp \left\{-\Delta_{1}\left(I_{1}^{0}+I_{234}^{0}\right)-\Delta_{2}\left(I_{2}^{0}+I_{341}^{0}\right)-\Delta_{3}\left(I_{3}^{0}+I_{412}^{0}\right)-\Delta_{4}\left(I_{4}^{0}+I_{123}^{0}\right)-\sum_{i<j} \Delta_{i j} I_{i j}^{0}\right\}\right|^{2} \\
& I_{1}^{0}=I_{1}-I_{12}-I_{13}-I_{14}+I_{123}+I_{124}+I_{134}-I_{1234} \\
& I_{12}^{0}=I_{12}-I_{123}-I_{124}+I_{1234} \\
& I_{123}^{0}=I_{123}-I_{1234} \\
& I_{1234}^{0}=I_{1234}
\end{aligned}
$$

## using Mobius, map

$$
\begin{aligned}
z_{1} & =z \\
z_{2} & =0 \\
z_{3} & =1 \\
z_{4} & =\infty
\end{aligned}
$$

$\left\langle V_{\beta_{1}}(z) V_{\beta_{2}}(0) V_{\beta_{3}}(1) V_{\beta_{4}}(\infty)\right\rangle=C\left|z^{\Delta_{1}+\Delta_{2}-\Delta_{12}}(1-z)^{\Delta_{1}+\Delta_{3}-\Delta_{13}} \exp \left\{-\left(\sum_{i} \Delta_{i}-\frac{1}{2} \sum_{i<j} \Delta_{i j}\right) I_{123}\right\}\right|^{2}$
reduces 4 -point function to 3 points, but now without 3-charge conservation
the cross ratio is $\mathbf{z}$

## 4-point function

$$
\pi I_{123}=-\frac{\pi}{2}+\pi \ln R-\operatorname{sgn}(\Im(z))\left\{\Im[\ln z \ln (1-\bar{z})]+\mathrm{D}_{2}(z)\right\}
$$

## which gives

$$
\begin{aligned}
& \left\langle V_{\beta_{1}}(z) V_{\beta_{2}}(0) V_{\beta_{3}}(1) V_{\beta_{4}}(\infty)\right\rangle=C\left|z^{-\Delta_{1}-\Delta_{2}+\Delta_{12}}(1-z)^{-\Delta_{1}-\Delta_{3}+\Delta_{13}}\right|^{2} \times \\
& \exp \left\{\left(\frac{2}{\pi} \sum_{i} \Delta_{i}-\frac{1}{\pi} \sum_{i<j} \Delta_{i j}\right) \operatorname{sgn}(\Im(z))\left\{\Im[\ln z \ln (1-\bar{z})]+\mathrm{D}_{2}(z)\right\}\right\}
\end{aligned}
$$

Here $D_{2}(z)$ is the Bloch-Wigner function

$$
\mathrm{D}_{2}(z)=\Im\left[\operatorname{Li}_{2}(z)\right]+\arg (1-z) \ln |z|
$$

This function is invariant under crossing symmetry:

$$
z \rightarrow 1-\frac{1}{z} \rightarrow \frac{1}{1-z} \rightarrow \frac{1}{z} \rightarrow 1-z \rightarrow \frac{-z}{1-z}
$$

For example, using
$\operatorname{sgn}\left(\Im\left(z^{-1}\right)\right) \Im\left[\ln z^{-1} \ln \left(1-\bar{z}^{-1}\right)\right]=\operatorname{sgn}(\Im(z)) \Im[\ln z \ln (1-\bar{z})]-\pi \ln |z|$
one can check that

$$
\left\langle V_{\beta_{1}}(z) V_{\beta_{2}}(0) V_{\beta_{3}}(1) V_{\beta_{4}}(\infty)\right\rangle=z^{-2 \Delta_{1} \bar{z}^{-2 \Delta_{1}}}\left\langle V_{\beta_{1}}(1 / z) V_{\beta_{4}}(0) V_{\beta_{3}}(1) V_{\beta_{2}}(\infty)\right\rangle
$$

When $z$ is real (i.e. when all four points are on some circle), it reduces to a simple product of powers

## However...

$$
\begin{aligned}
& \left\langle V_{\beta_{1}}(z) V_{\beta_{2}}(0) V_{\beta_{3}}(1) V_{\beta_{4}}(\infty)\right\rangle=C\left|z^{-\Delta_{1}-\Delta_{2}+\Delta_{12}}(1-z)^{-\Delta_{1}-\Delta_{3}+\Delta_{13}}\right|^{2} \times \\
& \exp \left\{\left(\frac{2}{\pi} \sum_{i} \Delta_{i}-\frac{1}{\pi} \sum_{i<j} \Delta_{i j}\right) \operatorname{sgn}(\Im(z))\left\{\Im[\ln z \ln (1-\bar{z})]+\mathrm{D}_{2}(z)\right\}\right\}
\end{aligned}
$$

there is a problem. Without the sgn, the functions in the exponent are real and analytic for real z between 0 and I. Therefore, the exponent is non-analytic (its fifth derivative diverges as $z$ crosses the real axis).

This indicates a problem - this is not yet a true CFT

## Arbitrary Dimension

- One can generalize this theory to any $d$ in the obvious way
- the N -point functions can be computed using the same techniques.
- The number operator always has logarithmic correlators
- The 2- and 3-point functions are as before, but with

$$
\Delta=\bar{\Delta}=\frac{C_{d}}{2} \gamma(1-\cos \beta)
$$

- We have not computed the 4-pt function


## free field limit

$$
\begin{array}{r}
\left\langle e^{i \beta_{1} N\left(z_{1}\right)} e^{i \beta_{2} N\left(z_{2}\right)} \ldots e^{i \beta_{n} N\left(z_{n}\right)}\right\rangle=\exp \left(-2 C_{d} \gamma\left[\sum_{i=1}^{n}\left(1-\cos \beta_{i}\right) I_{i}^{0}+\sum_{i<j}\left(1-\cos \left(\beta_{i}+\beta_{j}\right)\right) I_{i j}^{0}+\right.\right. \\
\left.\left.\sum_{i<j<k}\left(1-\cos \left(\beta_{i}+\beta_{j}+\beta_{k}\right)\right) I_{i j k}^{0}+\ldots\right]\right)
\end{array}
$$

- Consider taking $\gamma \rightarrow \infty$ with $\alpha$ fixed
- One can re-write the exponent as

$$
\sum_{k=1}^{n} \frac{\Delta_{i_{1} \ldots i_{k}}}{k!} \sum_{l=0}^{n} \frac{(-1)^{l}}{l!} \sum_{j_{1}, j_{2}, \ldots, j_{l}} I_{i_{1} i_{2} \ldots i_{k} j_{1} \ldots j_{l}}
$$

- using $\Delta_{i_{1} \ldots i_{k}}=\pi \gamma\left(1-\cos \left(\beta_{i_{1}}+\ldots+\beta_{i_{k}}\right)\right)=\left(\alpha_{i_{1}}+\ldots+\alpha_{i_{k}}\right)^{2}+\mathcal{O}\left(\alpha^{4} / \gamma\right)$
- one can write the correlator as

$$
\left\langle e^{i \beta_{1} N\left(z_{1}\right)} e^{i \beta_{2} N\left(z_{2}\right)} \ldots e^{i \beta_{n} N\left(z_{n}\right)}\right\rangle=\left|\exp \left\{-\sum_{i, j} \alpha_{i} \alpha_{j}\left(\ln \frac{R}{\delta}+\ln \frac{\delta}{\left|z_{i j}\right|}\left(1-\delta_{i j}\right)\right)\right\}\right|^{2}
$$

## free fields

- the correlator is zero unless charge conservation holds:

$$
\sum_{i} \alpha_{i}=\sum_{i, j} \alpha_{i} \alpha_{j}=0
$$

- If it does, the correlator becomes simply

$$
\left\langle e^{i \sqrt{2} \alpha_{1} \phi\left(z_{1}\right)} e^{i \sqrt{2} \alpha_{2} \phi\left(z_{2}\right)} \ldots e^{i \sqrt{2} \alpha_{n} \phi\left(z_{n}\right)}\right\rangle=\prod_{i<j}\left|\frac{z_{i}-z_{j}}{\delta}\right|^{4 \alpha_{i} \alpha_{j}}
$$

This is the correlator of vertex operators of a free field with kinetic term $\phi \square^{d / 2} \phi$

## central charge

given the partition function $Z(R)$ of a 2-sphere of radius $R$, one can compute the central charge:

$$
\begin{gathered}
Z(R)=R^{c / 3} Z_{0} \\
Z=\sum_{n=1}^{\infty} \frac{\gamma^{n}}{n!} \int d \Omega_{2}\left(\int_{\epsilon}^{\pi-\epsilon} \frac{d \psi}{\sin ^{3} \psi}-\Lambda\right) \\
\ln Z=4 \pi \gamma\left(\frac{R^{2}}{\epsilon^{2}}-\ln \frac{\epsilon}{R}+\ln 2-\frac{1}{6}-\Lambda R^{2}+\mathcal{O}\left(\epsilon^{2}\right)\right) \\
c=12 \pi \gamma
\end{gathered}
$$

## eternal inflation?



- recall that the inside of a bubble is an open universe, and slices of constant density are hyperbolic 3 -space
- Lorentz transformations around the point of nucleation are isometries, and an infinite boost moves a point to the boundary (which contains all the volume)
- It is often argued that inflation cannot be past eternal. Let's see what the effects of an initial condition surface are here


## infinite boost

all initial conditions look the same after an infinite boost:
viewed from the side...

viewed from above...

initial condition surface

## preferred point

- in either sky or global slicing, the effect of an infinitely boosted initial condition surface is to pick a special point on the sphere which no disk can cover - the "persistence of memory"
- choosing this point to be Riemann infinity gives the distribution on the plane with no "inside out" disks
- no initial condition would mean there can be disks which cover infinity - but this has almost no effect on IR safe quantities such as these correlators
- therefore, there does not seem to be a problem with past eternal inflation, and one can define a true dS invariant (but fractal) equilibrium distribution


## our world, today?

- can we actually see these collisions?
- first, there must be at least one collision in our past lightcone
- second, the collision's lightcone should bisect the part of the reheating/last scattering surface accessible to us to be easier to detect
- turns out the number of such collisions is

$$
N \sim \gamma \sqrt{\Omega_{k}(t)}\left(H_{f}^{2} / H_{i}^{2}\right)
$$

## effects of one collision

- Observer C is oblivious to the collision
- Observer B will detect anisotropic redshifts in the CMB
- Observer A will detect the anisotropic redshifts and "see" the Timelike trajectory domain wall directly
- Might see radiation from collision itself, or from the other bubble.

S. Chang, MK, T. Levi


## Asymmetric redshifts



- Photons from different directions travel through different metrics, and originate from a perturbed reheating surface
- This leads to a differential redshift as a function of angle. Photons that last scatter inside the lightcone versus those outside divides the sky into two discs
- There is a preferred direction and dependence only on the angle towards it
- One can learn something about the physics of the other bubble from the sign and radial dependence of the temperature shift


## Some interesting anomalies...



WCM 3- year, real space


WCM 1-year, scale $R_{9}$


From The non-gaussian cold spot in the 3-year wmap data.
M. Cruz, L. Cayon , E. Martinez-Gonzalez, P. Vielva, J. Jin


From A measurement of large-scale peculiar velocities of clusters of galaxies: results and cosmological implications.
A. Kashlinsky, F. Atrio-Barandela, D. Kocevski, H. Ebeling

## end

- many of the conceptual difficulties with dS seem to be ameliorated by this formalism, which...
- defines (globally) conformally invariant correlation functions with positive definite conformal weights
- but the 4 -pt function is bad - fixed by integrating over shape, interactions, gravity?
- has a Gaussian limit when the bulk decay rate is large
- central charge is proportional to the decay rate
- standard FT UV cutoff behavior, dual to bulk
- observable at least to census taker, maybe to us
- applications to percolation/condensed matter?

