1 Introduction and review

Two central mysteries: (1) discrepancy between G_{Fermi} and G_{Newton}, (2) origin of the patterns in the quark and lepton Yukawa couplings.

Weak sale SUSY a well-motivated answer to the first question; connect to flavor?

Compelling for many reasons. If nothing else, **avoids modularity of current model-building.** Cuts down on the proliferation of scales and dynamics, and becomes much more predictive.

But also provides an answer to the flavor problem. If you have a theory of flavor related to SUSY breaking, this removes the mystery of SUSY flavor. Flavor-violating processes can be understood and parameterized.

An old endeavor, dating back to 't Hooft and others; more recently, to a spate of work in the mid- and late-nineties.

But in the last few years our understanding of SUSY breaking in gauge theories has increased dramatically. Now have examples of thoroughly calculable dynamical SUSY breaking in vectorlike gauge theories. These have all the right ingredients.

Today I want to talk about models with the following features:

1. An asymptotically free gauge group G with a large flavor symmetry, of which we will gauge a subgroup.
2. A weakly-coupled IR dual for this gauge group with mesons transforming under the flavor symmetry.
3. Calculable SUSY breaking a la ISS
4. A subset of SM fields arising in the mesons of the IR theory, as pseudo-moduli of SUSY breaking
5. The remainder of SM fields elementary
6. Some flavor physics in the UV coupling elementary and composite fields.

The picture is, loosely, $M_{GUT} \sim M_F > \Lambda \gg \mu_{\not{S}USY} \gg \tilde{m}_{SM}$. That said, I should also say what I am *not* going to do. I am not going to

1. Do anything exotic with the Higgs sector. Will not solve μ problem. It is possible to embed the Higgs fairly naturally in this setup, but will only treat this briefly.

2. Solve the usual problems of GUT model-building. Doublet-triplet splitting; proton decay; mass relations. Though the approaches I will present do not worsen these.

3. Write down a convincing theory where all SM fields are composite. Theories that avoid Landau poles and flavor problems involve only a small degree of compositeness.

4. Solve the R-symmetry breaking issues of ISS. Will use a conventional approach which still generates a hierarchy between gauginos and sfermions.

5. Incorporate SM gauge fields into gauge group of DSB; $SU(5)$ will come from weakly gauging part of the flavor symmetry of DSB.

1.1 What are we aiming for?

Perhaps the best place to begin is with a pragmatic assessment of the battlefield. By writing down a theory of SUSY breaking and flavor, what are we aiming to reproduce?

In the context of flavor, we want:

1. Weak-scale quark and lepton masses

In terms of ratios,

$$\frac{m_c}{m_u} = 300 - 800 \qquad \frac{m_t}{m_u} = 50,000 - 100,000 \tag{1.1}$$

$$\frac{m_s}{m_d} = 10 - 30 \qquad \frac{m_b}{m_d} = 700 - 1000 \tag{1.2}$$

$$\frac{m_\mu}{m_e} = 200 \qquad \frac{m_\tau}{m_e} = 3500 \tag{1.3}$$

and also

$$\frac{m_t}{m_b} \sim 35 \qquad \frac{m_t}{m_\tau} \sim 100$$

GUT-scale quark and lepton masses are similar, loosely speaking. With SUSY threshold corrections taken into account, this may clean things up a bit and make GUT predictions better.

2. Also the CKM matrix $V_{CKM} = V_L^u V_L^{d\dagger}$, which has been measured quite well

$$|V_{\rm CKM}| \simeq \begin{pmatrix} 0.97 & 0.23 & 0.004 \\ 0.23 & 0.97 & 0.04 \\ 0.009 & 0.04 & 0.99 \end{pmatrix}, \tag{1.4}$$

3. Also limits on flavor-change from FCNCs. The very strongest is from $K^0 - \bar{K}^0$ mixing. Also have $D^0 - \bar{D}^0$ mixing.

Main limit from the mass splitting between K_L and K_S: $\Delta m_K = 3.5 \times 10^{-15}$ GeV. If new processes violating flavor in first two generations enter without any alignment, scale suppressing effective SM four-fermi operators must be $\gtrsim 10^3$ TeV. Restrictive! But with some alignment, this comes down to the order 10 TeV. Important to have a theory of flavor combined with DSB!

4. Also limits on the SUSY spectrum. We know where sparticles *aren't*. LEP puts limits between 100 and 200 GeV on most scalar particles.

5. Finally, if SUSY breaking is a natural explanation of the hierarchy problem, the stop cannot be too heavy. This is really the only scalar we care about for at least an order of magnitude due to top yukawa coupling.

So a good idea is to make first to generations of sfermions heavy, third generation light – solves FCNCs, naturalness (NB, can't make it too heavy).

With that in mind, let's see how we might proceed.

2 The idea

Promising idea: compositeness. First two generations composites at some intermediate scale Λ, while flavor physics is at $M_{\rm flavor} > \Lambda$, then masses and mixings of the first two generations suppressed by powers of $\epsilon \equiv \Lambda/M_{\rm flavor}$.

The third generation should be elementary because top yukawa unsuppressed (and bottom yukawa if $\tan\beta$ large). For $\epsilon \sim 10^{-1}$, this is good starting point.

1. Simplest realization: asymptotically free SQCD theory with fundamental quarks $(Q, \tilde{Q})$ and dynamical scale Λ. Will gauge an $SU(5)_{SM}$ subgroup of the flavor symmetry.

Then first two generations come from composite $Q\tilde{Q}$, which is bifundamental under flavor symmetries. Some UV physics at the scale $M_F > \Lambda$ generating Yukawa couplings schematically of the form

$$W_{Yuk} \sim \frac{1}{M_F^2}(Q\tilde{Q})H(Q\tilde{Q}) + \frac{1}{M_F}(Q\tilde{Q})H\Psi_3 + \Psi_3 H \Psi_3$$

After rescaling by Λ to get canonically normalized fields, the natural texture of the Yukawas in this basis is

$$\begin{pmatrix} \epsilon^2 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon^2 & \epsilon \\ \epsilon & \epsilon & 1 \end{pmatrix} \ . \tag{2.1}$$

For historical reasons, we call this type of theory a meson model.

Realistic? Explicit matrix has two zero eigenvalues.$\mathcal{O}(1)$ coefficients gets closer to reality. But still an approximate $SU(2)$ symmetry for the first two generations; can break this with added physics at M_F.

Simple to employ this for both y_u and y_d. However, an interesting variation is a ten-centered model. Say just the tens are composite. Then the y_u texture is as above, but the y_d texture is

$$\begin{pmatrix} \epsilon & \epsilon & 1 \\ \epsilon & \epsilon & 1 \\ \epsilon & \epsilon & 1 \end{pmatrix} \ . \tag{2.2}$$

This implements the idea that the tens mostly drive flavor hierarchies. Not a bad texture; requires fewer SM composites.

2. Another appealing notion: SQCD theory with fundamental quarks $(Q, \tilde{Q})$ and a field U in a 2-index tensor representation of the gauge group.

Each generation comes from meson of different canonical scaling dimension. Then the Yukawas look like

$$\begin{aligned} W_{Yuk} \supset\ & \frac{1}{M_F^4}(QU\tilde{Q})H(QU\tilde{Q}) + \frac{1}{M_F^3}(Q\tilde{Q})H(QU\tilde{Q}) + \\ & \frac{1}{M_F^2}(Q\tilde{Q})H(Q\tilde{Q}) + \frac{1}{M_F}(Q\tilde{Q})H\Psi_3 + \Psi_3 H\Psi_3\ . \end{aligned} \tag{2.3}$$

The corresponding Yukawa texture is

$$\begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon & 1 \end{pmatrix}\ . \tag{2.4}$$

This we call a dimensional hierarchy model, and it naturally gives three hierarchical generations.

In general, mesons $(Q\tilde{Q})$ and $(QU\tilde{Q})$ contain more matter than just the first two Standard Model generations. The idea is that some of the extra components of these fields together with the magnetic quarks yield a weakly coupled supersymmetry breaking model a la ISS.

But maybe also extra stuff that should be eliminated.

This strategy gives light fermions via compositeness. Composites will couple more strongly to SUSY breaking than elementary fields. Therefore, one is led to phenomenology of "more minimal" scenario of Cohen, Kaplan and Nelson, w/ first and second generation sfermion masses larger than third gen. **The mantra is: light fermions, heavy sfermions.**

Let's now build this explicitly. First, a review of the SUSY breaking dynamics.

2.1 ISS

Basic ISS is SQCD in the free magnetic range $N_c + 1 \leq N_f < \frac{3}{2}N_c$ with massive flavors

$$W\ =\ m\,\mathrm{Tr}\,\tilde{Q}Q. \tag{2.5}$$

IR free magnetic dual has $SU(N)$ gauge group (where $N = N_f - N_c$), N_f flavors q and $\tilde{q}$, and mesons M. From the perspective of the electric theory, the mesons are composite operators: $M_{ij} = Q_i \cdot \tilde{Q}_j$.

The superpotential for the dual theory is, for rescaled mesons with canonical dimension $\Phi = M/\Lambda_e$,

$$W \;=\; h\,\mathrm{Tr}\,q\,\Phi\,\tilde{q} \;-\; h\mu^2 \mathrm{Tr}\,\Phi \tag{2.6}$$

For $\mu \neq 0$, the superpotential breaks the global symmetry group down to

$$SU(N) \times SU(N_f) \times U(1)_B \times U(1)_R \tag{2.7}$$

with $SU(N_f)$ the diagonal subgroup in $SU(N_f)^2$. In the SUSY breaking vacuum the global symmetry will break to $SU(N) \times SU(N_c)$. So we parametrize fields as

$$q^T = \begin{pmatrix} \chi_{N\times N} \\ \rho_{N_c\times N} \end{pmatrix}, \qquad \tilde{q} = \begin{pmatrix} \tilde{\chi}_{N\times N} \\ \tilde{\rho}_{N_c\times N} \end{pmatrix} \qquad \Phi = \begin{pmatrix} Y_{N\times N} & Z^T_{N\times N_c} \\ \tilde{Z}_{N_c\times N} & X_{,N_c\times N_c} \end{pmatrix}. \tag{2.8}$$

Supersymmetry is broken at tree-level by the rank-condition. There is a classical moduli space of non-supersymmetric vacua with vacuum energy $V_0 = N_c|h^2\mu^4|$, corresponding to $\langle \chi\tilde{\chi}\rangle = \mu^2$ and X arbitrary. The pseudomoduli, namely the classically flat directions that are not Nambu-Goldstone bosons of any broken global symmetry, are stabilized at 1-loop at

$$X = 0 \qquad\qquad \chi = \tilde{\chi} = \mu\,\mathbf{1}_N \tag{2.9}$$

getting $\mathcal{O}(|h^2\mu|)$ masses. All other directions are fixed at zero at tree level, with $\mathcal{O}(|h\mu|)$ masses. The theory also has a supersymmetric vacuum, which decouples.

The stabilization comes at one loop from CW potential:

$$V_{eff} = \frac{1}{64\pi^2}\mathrm{STr}\mathcal{M}^4 \log\frac{\mathcal{M}^2}{\Lambda^2} \sim \frac{\log 4 - 1}{8\pi^2} h^4\mu^2 \mathrm{Tr} X^\dagger X + ...$$

The vev of the dual quarks in the vacuum breaks the global symmetries down to

$$SU(N)_D \times SU(N_c) \times U(1)_B \times U(1)_R \tag{2.10}$$

Fairly large global symmetry left over!

For realistic SUSY, need to break the $U(1)_R$ which is unbroken in the metastable vacuum. Such an unbroken $U(1)_R$ would forbid (Majorana) gaugino masses. The simplest way to break the $U(1)_R$ is to add a quartic superpotential perturbation in the electric theory:

$$W_4 = \frac{1}{\Lambda_0}\,\mathrm{tr}((Q\tilde{Q})^2) \to h^2\mu_\phi \mathrm{tr}(M^2) \tag{2.11}$$

w/ $\mu_\phi = \Lambda^2/\Lambda_0$ at the end of the day. Gives parametrically larger *spontaneous* breaking – the singlet $\mathrm{tr}\,X$ acquires a vacuum expectation value.

For $\mu_\phi \ll \mu$, the 1-loop effective potential still has a reliable supersymmetry-breaking minimum. The vev of $\mathrm{tr}\,X$ and the MSSM gaugino mass are $\sim \mu_\phi$.

Our goal is to construct models in which some of the MSSM fields are composite from the perspective of the UV theory. Hence, they should arise as (part of) the magnetic mesons Φ, and in particular should live in X, the part that transforms under unbroken $SU(N_c)$

3 Some in-principle models

Now we have seen how SUSY breaking works, with a large pseudomodulus transforming under the flavor symmetries. Build models!

3.1 Meson model

To produce a model with 2 composite generations, start w/ SQCD with $N_c = 16$ and $N_f = 17$. The $SU(17)$ global symmetry is higgsed down to $SU(16)$. MSSM gauge group is a weakly gauged $SU(5)$ subgroup of $SU(16)$; embedded so electric quarks transform as

$$\begin{aligned} Q &= (\mathbf{5} + \mathbf{5} + \overline{\mathbf{5}} + \mathbf{1}) + \mathbf{1} \\ \tilde{Q} &= (\overline{\mathbf{5}} + \overline{\mathbf{5}} + \mathbf{5} + \mathbf{1}) + \mathbf{1} \end{aligned} \tag{3.1}$$

where the parenthesis separate the $\mathbf{16}$ and $\overline{\mathbf{16}}$ of $SU(16)$ from the singlet. Magnetic meson is

$$\Phi = \begin{pmatrix} Y_{1\times 1} & Z^T_{1\times 16} \\ \tilde{Z}_{16\times 1} & X_{16\times 16} \end{pmatrix} \tag{3.2}$$

with $Y, Z, \tilde{Z}$ and X transforming in the $\mathbf{1}$, $\overline{\mathbf{16}}$, $\mathbf{16}$ and $(\mathbf{Adj}+\mathbf{1})$ of $SU(16)$. X decomposes into $SU(5)$ representations as:

$$X = 2\times(\mathbf{10}+\bar{\mathbf{5}}) + \left[5\times\mathbf{24} + 2\times\mathbf{15} + 2\times\overline{\mathbf{15}} + 2\times\overline{\mathbf{10}} + 3\times\mathbf{5} + \bar{\mathbf{5}} + 6\times\mathbf{1}\right] \tag{3.3}$$

In addition, we need to add an "elementary" third MSSM generation $\Psi^{(3)}$ in the $(\mathbf{10}+\bar{\mathbf{5}})$ of $SU(5)$, plus the Higgs fields.

Note the extra stuff! This we can lift by adding spectators $S_{\mathbf{R}}$ to the SQCD dynamics which transform under $SU(5)$, and we pair them up via UV superpotential terms

$$S_{\mathbf{R}}(Q\tilde{Q})_{\bar{\mathbf{R}}} \to S_{\mathbf{R}}\Phi_{\bar{\mathbf{R}}}$$

These become mass terms for the IR degrees of freedom, and we may integrate them out around the scale Λ.

Should worry that adding these spectators significantly alters the vacuum structure. Adding spectators does not destroy the ISS vacuum, but can also induce vacua of lower energy that break SM gauge symmetry and are not parametrically far away in field space (basically because this violates the rank condition). A simple fix is to create a hierarchy in the UV quark masses, with one big mass and the rest smaller; then ISS vacuum can have lower vacuum energy.

So can decouple extra matter. This leaves us with the Standard Model generations plus extra fields from the magnetic quarks and $Z, \tilde{Z}$; these are $6\times(\mathbf{5}+\bar{\mathbf{5}})$ "messengers" coming from the (ρ, Z) sector. Communicate SUSY breaking to elementary fields.

Several features should stand out. What is the spectrum? First two generations get soft masses from one loop CW potential,

$$m^2_{CW} \sim \frac{h^2}{16\pi^2} h^2\mu^2$$

All elementary fields get gauge mediation masses, since there are SM messengers connected to SUSY breaking:

$$m_{GM}^2 \sim \left(\frac{\alpha}{4\pi}\right)^2 h^2\mu^2$$

Finally, gaugino masses come from R-symmetry breaking and go as

$$m_\lambda \sim g_{SM}^2 \mu_\phi$$

To make this all viable, need $h\mu \sim 100 - 200$ TeV and $\mu_\phi \sim 1$ TeV. Puts sfermions from composites around 10 TeV, elementary fields around 1 TeV.

3.2 Dimensional hierarchy

That was a meson model. Can generalize this idea simply to a dimensional hierarchy model. I will only review this schematically.

3.2.1 SUSY Breaking

This is SQCD plus an adjoint, and adjoint has a general renormalizable superpotential

$$W_{el} = \frac{g_U}{3}\,\mathrm{Tr}U^3 + \frac{m_U}{2}\,\mathrm{Tr}U^2 \tag{3.4}$$

This superpotential will not have any metastable SUSY breaking vacua, which requires additional perturbations.

Generically the gauge group is broken to

$$SU(N_c) \to SU(r_1) \times SU(r_2) \times U(1)\,. \tag{3.5}$$

The low energy theory splits into two decoupled SQCD sectors with only fundamental matter (as long as $m_U \neq 0$). Stable vacuum means $N_f \geq N_c/2$. IR freedom means $N_f < \frac{2}{3}N_c$, so we want

$$\frac{N_c}{2} < N_f < \frac{2}{3}N_c \tag{3.6}$$

Interested in the two mesons

$$(M_1)_{ij} = \tilde{Q}_i Q_j \quad , \quad (M_2)_{ij} = \tilde{Q}_i U Q_j\,, \tag{3.7}$$

where the gauge indices are contracted and suppressed.

The magnetic dual theory consists of SQCD, with gauge group $SU(\tilde{N}_c = 2N_f - N_c)$ and strong coupling scale $\tilde{\Lambda}$, N_f quarks $(q, \tilde{q})$, one magnetic adjoint field $\tilde{U}$, and two gauge singlet fields corresponding to the mesons (**??**). And superpotential for canonical fields $\Phi \sim M_1/\Lambda, \Phi_U \sim M_2/\Lambda^2$:

$$\begin{aligned} W_{mag} = & \frac{\tilde{g}_U}{3} \mathrm{Tr}\tilde{U}^3 + \frac{\tilde{m}_U}{2} \mathrm{Tr}\tilde{U}^2 + \tilde{\lambda}' \, \mathrm{Tr}\tilde{U} + \\ & + \frac{h}{\Lambda} \left[c_1 \tilde{m}_U \, \mathrm{tr}(\Phi q \tilde{q}) + c_2 \, \mathrm{tr}(\Phi q \tilde{U} \tilde{q}) \right] + h \, \mathrm{tr}(\Phi_U q \tilde{q}) \, . \end{aligned} \tag{3.8}$$

The $(\Phi_U, q, \tilde{q})$ sector is very similar to the magnetic theory of ISS except, e.g., Φ_U is of dimension 3 in the UV, while the ISS meson has scaling dimension 2.

For simplicity focus on vacua with $\langle \mathrm{Tr} \, \tilde{U}^2 \rangle = 0$, corresponding to $r_1 = N_f$, $r_2 = N_c - N_f$ and unbroken magnetic gauge group. Simplest choice is $\tilde{N}_c = 1$ (for this choice the magnetic gauge group is trivial); no magnetic adjoint, and the magnetic superpotential simplifies to

$$W_{mag} = c_1 h \frac{\tilde{m}_U}{\Lambda} \, \mathrm{tr}(\Phi q \tilde{q}) + h \, \mathrm{tr}(\Phi_U q \tilde{q}) \, . \tag{3.9}$$

The limit $\tilde{m}_U \ll \Lambda$ simplifies the analysis considerably.

Add deformations to break SUSY:

$$W_{mag} = c_1 h \frac{\tilde{m}_U}{\Lambda} \, \mathrm{tr}(\Phi q \tilde{q}) + \frac{1}{2} m_\Phi \, \mathrm{tr} \, \Phi^2 + \left[-h\mu^2 \, \mathrm{tr} \, \Phi_U + h \, \mathrm{tr}(\Phi_U q \tilde{q}) + \frac{1}{2} h^2 \mu_\phi \, \mathrm{tr}(\Phi_U^2) \right] . \tag{3.10}$$

Off-diagonal components of Φ_U and Φ will be identified with the first and second Standard Model generations. Of course, such components cannot have large vector-like supersymmetric masses via superpotential terms that couple them to conjugate fields. The Standard Model composite generations will be made massless by introducing heavy spectator fields coupled to the unwanted conjugate fields.

In the limit $\mu_\phi \to 0$ supersymmetry is broken at tree level by the rank condition, and Φ_U is stabilized at the origin due to one-loop effects. For finite $\mu_\phi \ll \mu$, the $U(1)'_R$ is explicitly broken and supersymmetric vacua appear at a distance μ^2/μ_ϕ from the origin. The SUSY breaking vacuum is displaced slightly from the origin and is still parametrically long-lived.

Pattern of supersymmetry breaking: parameterize the fields as

$$\Phi_U = \begin{pmatrix} Y_{U,\,\tilde{N}_c\times\tilde{N}_c} & Z^T_{U,\,\tilde{N}_c\times(N_f-\tilde{N}_c)} \\ \tilde{Z}_{U,\,(N_f-\tilde{N}_c)\times\tilde{N}_c} & X_{U,\,(N_f-\tilde{N}_c)\times(N_f-\tilde{N}_c)} \end{pmatrix} \;,\; \Phi = \begin{pmatrix} Y_{\tilde{N}_c\times\tilde{N}_c} & Z^T_{\tilde{N}_c\times(N_f-\tilde{N}_c)} \\ \tilde{Z}_{(N_f-\tilde{N}_c)\times\tilde{N}_c} & X_{(N_f-\tilde{N}_c)\times(N_f-\tilde{N}_c)} \end{pmatrix} \;, \tag{3.11}$$

$$q^T = \begin{pmatrix} \chi_{\tilde{N}_c\times\tilde{N}_c} \\ \rho_{(N_f-\tilde{N}_c)\times\tilde{N}_c} \end{pmatrix} \;,\; \tilde{q} = \begin{pmatrix} \tilde{\chi}_{\tilde{N}_c\times\tilde{N}_c} \\ \tilde{\rho}_{(N_f-\tilde{N}_c)\times\tilde{N}_c} \end{pmatrix} \;. \tag{3.12}$$

The vacuum lies at

$$\langle hX_U\rangle \approx 16\pi^2\mu_\phi \;\;,\;\; \langle\chi\tilde{\chi}\rangle \approx \mu^2 \tag{3.13}$$

and

$$|W_{X_U}| \approx |h\mu^2|\,. \tag{3.14}$$

The field Φ is stabilized supersymmetrically,

$$W_\Phi = 0\;,\; \langle X\rangle = 0\;,\; \langle Y\rangle \approx -c_1\frac{\tilde{m}_U}{\Lambda}\frac{h\mu^2}{m_\Phi}\,, \tag{3.15}$$

And also

$$\langle Y_U\rangle = -c_1\frac{\tilde{m}_U}{\Lambda}\langle Y\rangle\,. \tag{3.16}$$

The rest of the fields are stabilized at the origin.

3.2.2 Explicit model

Now weakly gauge and identify a subgroup of $SU(N_f-\tilde{N}_c)$ with the Standard Model gauge group. We can now identify part of X_U and X with the first and second generation Standard Model fermions.

The minimal choice for the number of flavors and colors of the electric theory corresponds to

$$N_f = 12, N_c = 23$$

The $SU(N_f = 12)$ global symmetry is broken to $SU(N_f - \tilde{N}_c = 11)$ by the vacuum expectation value $\chi\tilde{\chi} = \mu^2$.

The Standard Model GUT group is a weakly gauged $SU(5)$ subgroup of $SU(11)$, with the following embedding of $SU(5)$ into $SU(12)$:

$$Q \sim (\mathbf{5} + \bar{\mathbf{5}} + \mathbf{1}) + \mathbf{1}\,, \tilde{Q} \sim (\bar{\mathbf{5}} + \mathbf{5} + \mathbf{1}) + \mathbf{1}, \tag{3.17}$$

where the representations in round brackets denote the embedding into $SU(11)$.

The mesons of the magnetic theory decompose as (see (**??**))

$$\Phi_U = \begin{pmatrix} Y_{U,\,1\times1} & Z^T_{U,\,1\times11} \\ \tilde{Z}_{U,\,11\times1} & X_{U,\,11\times11} \end{pmatrix}\ ,\ \Phi = \begin{pmatrix} Y_{1\times1} & Z^T_{1\times11} \\ \tilde{Z}_{11\times1} & X_{11\times11} \end{pmatrix}\ , \tag{3.18}$$

The fields $(Y_i, \chi, \tilde{\chi})$ fields are all singlets under the Standard Model gauge group, while X_U and X decompose as

$$(\mathbf{10} + \bar{\mathbf{5}}) + \left[2\times\mathbf{24} + \mathbf{15} + \overline{\mathbf{15}} + \overline{\mathbf{10}} + 2\times\mathbf{5} + \bar{\mathbf{5}} + 3\times\mathbf{1}\right], \tag{3.19}$$

where the representations in round brackets will form the desired Standard Model fermions and the matter in square brackets represents additional matter that we will want to remove.

There is actually a hidden technical issue here, namely that the one-loop scalar potential couples to a particular linear combination of $\Phi, \Phi_U - \frac{N_c - N_f}{g_U}\frac{m_U}{\Lambda}\Phi + \Phi_U$ couples to $q, \tilde{q}$ and gets a mass at one loop. The orthogonal combination is stabilized by gauge mediation and two-loop contributions, but these do not give a satisfactory soft spectrum; a heavy first generation of sfermions and light second generation is prohibited by FCNCs.

This problem may be solved by essentially doubling matter, so that first and second generations $(\mathbf{10} + \bar{\mathbf{5}})$ come from different matrix elements and both come from the heavy linear combination of Φs. This adds extra light matter to lift.

3.3 An aside on the Higgs

So far I have avoided doing anything interesting with the Higgses. However, you could extend this setup readily to incorporate them in a fairly natural fashion. The simplest way to do this is to make H_u elementary and H_d a composite. This naturally gives us a good value of $\tan\beta$.

Couplings to DSB are of the form

$$\lambda_u H_u \mathcal{O}_u + \lambda_d H_d \mathcal{O}_d$$

Of course for H_d, λ_d comes straight from superpotential and is unsuppressed.

μ is generated by integrating out couplings of the Higgses to messengers. This turns out to be of order

$$\mu \sim \frac{\lambda_u \lambda_d}{16\pi^2} \langle hX \rangle \sim \lambda_u \lambda_d \mu_\phi$$

and $B\mu \gg \mu^2$. Because H_d now couples more strongly to SUSY breaking, it also obtains a larger soft mass. Ultimately there is a hierarchy

$$|m_{H_u}^2| \sim \mu^2 \ll B\mu \ll m_{H_d}^2$$

which suffices for EWSB.

4 Viable models

So far you are probably of the impression that this is a terrible idea. We have a surfeit of extra matter, the Standard Model is input in the UV through arbitrary choice of spectators, and in many cases there is enough matter to raise the spectre of Landau poles.

We would like to construct models with

- A minimum of extra matter, and perhaps no need for spectators.
- The chiral index of the Standard Model a natural ingredient.
- No Landau pole before the GUT scale.

Not impossible despite the challenges of $SU(N)$ model-building. The answer is to try $Sp(N)$. SP is advantageous because there are only quarks, and so it is possible to embed $SU(5)_{SM}$ fields in chiral ways, subject to anomaly cancellation. Moreover the matter content in mesons is much smaller because M is an antisymmetric $N_f \times N_f$ tensor.

Setup is $Sp(2N_c)$ gauge theory with $2N_f$ fundamental quarks Q_i, $i = 1, \ldots, 2N_f$ (N_f flavors); for $N_f > N_c - 2$ leads to dual w

$$Sp\left(2\tilde{N}_c \equiv 2(N_f - N_c - 2)\right)$$

containing $2N_f$ magnetic quarks q_i together with a meson singlet $M_{ij} = Q_iQ_j$ in the antisymmetric of the flavor group. For IR free magnetic dual, $N_c+3 \leq N_f < 3(N_c+1)/2$.

Can also build dimensional hierarchy models by adding a field U in the "traceless" antisymmetric $(N_c(2N_c-1)-1)$ of the gauge group, with a superpotential $W \propto \mathrm{Tr}(J_{2N_c}U)^3$ which restricts the mesons to

$$M_{ij} \equiv Q_iQ_j\ ,\ (M_U)_{ij} \equiv Q_iUQ_j\,. \tag{4.1}$$

Both are in the antisymmetric of the flavor group $SU(2N_f)$. Color indices are contracted with $J_{2N_c} = \mathbf{1}_{N_c} \otimes (i\sigma_2)$.

4.1 SUSY breaking

SUSY breaking goes through as a natural generalization of the above $SU(N)$ case. We turn on masses for the N_f flavors,

$$W_{el} = \sum_{k=1}^{N_f} m_k\,(Q^\alpha_{2k-1}J_{\alpha\beta}Q^\beta_{2k})\,. \tag{4.2}$$

Color indices are contracted with $J_{2N_c} = \mathbf{1}_{N_c} \otimes (i\sigma_2)$

The theory has a superpotential

$$W_{mag} = -h\,\mathrm{tr}(\mu^2\Phi) + h\,\mathrm{tr}(\Phi q^T q) \tag{4.3}$$

where the matrix μ^2 in the linear term is given by

$$\mu^2 = \mathrm{diag}(\mu_1^2, \ldots, \mu_{N_f}^2) \otimes (i\sigma_2)\ ,\ h\mu_i^2 \sim \Lambda m_i\,. \tag{4.4}$$

The metastable vacuum is obtained by turning on the maximum number of expectation values to cancel the largest F-terms,

$$\langle q^T q\rangle = \mathrm{diag}(\mu_1^2, \ldots, \mu_{\tilde{N}_c}^2) \otimes (i\sigma_2)\,. \tag{4.5}$$

Fluctuations around the vacuum are parametrized by

$$\Phi = \begin{pmatrix} Y_{2\tilde{N}_c\times 2\tilde{N}_c} & Z^T_{2\tilde{N}_c\times 2(N_f-\tilde{N}_c)} \\ -Z_{2(N_f-\tilde{N}_c)\times \tilde{2}N_c} & X_{2(N_f-\tilde{N}_c)\times 2(N_f-\tilde{N}_c)} \end{pmatrix} \ , \ q^T = \begin{pmatrix} \chi_{2\tilde{N}_c\times 2\tilde{N}_c} \\ \rho_{2(N_f-\tilde{N}_c)\times 2\tilde{N}_c} \end{pmatrix} . \quad (4.6)$$

The tree-level nonzero F-terms and vacuum energy are

$$W_X = -h \,\mathrm{diag}(\mu^2_{\tilde{N}_c+1}, \ldots, \mu^2_{N_f}) \otimes (i\sigma_2) \ , \ V_0 = 2 \sum_{j=\tilde{N}_c+1}^{N_f} (h\mu_j^2)^2 \,. \quad (4.7)$$

The expectation values $\langle \chi^T \chi \rangle$ are set by the largest μ_i^2 and the F-terms are controlled by the smaller ones. The nonzero expectation value $\langle \chi^T \chi \rangle$ higgses completely the magnetic gauge group $Sp(2\tilde{N}_c) \to 1$, and the SM gauge group is a weakly gauged subgroup from

$$SU(5)_{SM} \subset SU\left(2(N_f - \tilde{N}_c)\right) \quad (4.8)$$

Notice that (Z, ρ) give $2\tilde{N}_c(\Box + \overline{\Box})$ of $SU(2(N_f - \tilde{N}_c))$ and X is an antisymmetric.

The field X is a pseudo-modulus; it is flat at tree-level but generically receives quantum corrections and will be lifted. (Y, χ) are supersymmetric at tree level. ρ couples directly to the pseudo-modulus X which has a nonzero F-term. Also, ρ and Z have a supersymmetric mass $W \supset h\langle\chi\rangle Z\rho$ and have nontrivial SM quantum numbers. Therefore, in the macroscopic theory (ρ, Z) are composite messengers with supersymmetric mass $M = \langle\chi\rangle$ and splittings given by $|W_X|^{1/2}$.

Up to order one numerical factors, the CW mass is

$$m^2_{CW} \approx \frac{h^2}{16\pi^2} \frac{(h\mu_2^2)^2}{\mu_1^2} \,. \quad (4.9)$$

4.2 SM Embedding

Now we can try inputting the SM fields. Because we can make a chiral embedding, we can attempt to be ambitious and get the chiral content of the

SM dynamically. It turns out there are only a few ways to do this without generating Landau poles.

I will briefly mention two less-ambitious ways, and one ambitious way. The unambitious ways are to focus on ten-centered models

These are three of the four simplest models with SM index – the fourth is less attractive and I will omit.

First Model

$Sp(2N_c = 8)$ gauge theory with $N_f = 7$ flavors of fundamentals. Flavor symmetry is $SU(14)$. Dual is $Sp(2\tilde{N}_c)$ gauge theory with $\tilde{N}_c = N_f - N_c - 2 = 1$ and $2N_f$ magnetic quarks transforming as conjugates of the electric quarks, plus the gauge singlet meson.

The embedding of $SU(5)$ in the flavor symmetry of the UV theory is

	$Sp(8)$	$SU(5)_{SM}$
Q_i^α	$\square$	$(\mathbf{5}+\mathbf{5}+\mathbf{1}+\mathbf{1})+\mathbf{1}+\mathbf{1}$
S_a	$\mathbf{1}$	$\overline{\mathbf{15}}+\overline{\mathbf{10}}+4\times\overline{\mathbf{5}}$

where the parenthesis denotes the $Sp(12)$ subgroup of the flavor symmetry that remains unbroken in the nonsupersymmetric vacuum.

The structure of the magnetic dual is

	$Sp(2)$	$SU(5)_{SM}$
M_{ij}	$\mathbf{1}$	$8\times\mathbf{5}+3\times\mathbf{10}+\mathbf{15}+6\times\mathbf{1}$
q_i^α	$\square$	$(\bar{\mathbf{5}}+\bar{\mathbf{5}}+\mathbf{1}+\mathbf{1})+\mathbf{1}+\mathbf{1}$
S_a	$\mathbf{1}$	$\overline{\mathbf{15}}+\overline{\mathbf{10}}+4\times\overline{\mathbf{5}}$

The resulting composite messengers comprise

$$(\rho\oplus Z)\sim 2\times(2\times\mathbf{5}+2\times\bar{\mathbf{5}}) \tag{4.10}$$

and the lower block of the meson transforms as a 12×12 antisymmetric tensor, decomposing under $SU(5)$ as

$$X\sim 2\times\mathbf{10}+[\mathbf{10}+\mathbf{15}+4\times\mathbf{5}+\mathbf{1}]\ . \tag{4.11}$$

The only massless composites in the IR are then the first two **10**'s in X, giving the required SM matter fields. Given this matter content, the messenger index for this theory is $N_{mess} = 4$, more than compatible with perturbative gauge coupling unification

Second Model

Consider now an $Sp(2N_c = 8)$ gauge theory with $N_f = 7$ flavors of fundamentals. The embedding in the UV theory is

	$Sp(8)$	$SU(5)_{SM}$
Q_i^α	$\square$	$(\mathbf{10}+\mathbf{1}+\mathbf{1})+\mathbf{1}+\mathbf{1}$
S_a	$\mathbf{1}$	$\mathbf{45}$

where again the SM elementary fields are not shown.

The structure of the magnetic dual is

	$Sp(2)$	$SU(5)_{SM}$
M_{ij}	$\mathbf{1}$	$4\times\mathbf{10}+\overline{\mathbf{45}}+6\times\mathbf{1}$
q_i^α	$\square$	$(\overline{\mathbf{10}}+\mathbf{1}+\mathbf{1})+\mathbf{1}+\mathbf{1}$
S_a	$\mathbf{1}$	$\mathbf{45}$

For the model considered here, we have composite messengers

$$(\rho \oplus Z) \sim 2\times(\mathbf{10}+\overline{\mathbf{10}}+4\times\mathbf{1}) \tag{4.12}$$

and the lower block of the meson transforms as a 12×12 antisymmetric tensor, decomposing under $SU(5)$ as

$$X \sim 2\times\mathbf{10}+\left[\overline{\mathbf{45}}+\mathbf{1}\right]\ . \tag{4.13}$$

Given this matter content, the messenger index for this theory is $N_{mess} = 6$, just compatible with perturbative gauge coupling unification.

To this set of fields we must add the usual complement of elementary Standard Model fields: two $\bar{\mathbf{5}}$ for the first two generations, as well as one $\bar{\mathbf{5}}+\mathbf{10}$ pair for the elementary third generation. Given this field content, we must also add one **45** in order for the theory to be anomaly-free. Conveniently, this pairs with the $\overline{\mathbf{45}}$ contained in X to obtain a mass at the duality scale, leaving no superfluous fields charged under $SU(5)$ at low energies. In this case it is amusing to note that the massive $\mathbf{45}$ may be used to generate a Georgi-Jarlskog texture.

Third Model

Finally, there is a model with all chiral SM fields and no need for spectators. In contrast to the ten-centered models considered above, these spectator-free models automatically contain both $\mathbf{10}$ and $\bar{\mathbf{5}}$ representations.

The most minimal such model has a magnetic gauge group with $\tilde{N}_c = 1$ and two composite SM generations; this corresponds to

$$N_f = 10\ ,\ N_c = 7\ ,\ n_1 = 5\,. \tag{4.14}$$

The UV theory is

	$Sp(14)$	$SU(5)_{SM}$
Q_i^α	$\square$	$(\mathbf{10}+\bar{\mathbf{5}}+\mathbf{1}+\mathbf{1}+\mathbf{1})+\mathbf{1}+\mathbf{1}$
S_a	$\mathbf{1}$	$-$

with magnetic dual

	$Sp(2)$	$SU(5)_{SM}$
M_{ij}	$\mathbf{1}$	$5\times\mathbf{10}+5\times\bar{\mathbf{5}}+\mathbf{45}+\overline{\mathbf{45}}+\overline{\mathbf{10}}+\mathbf{5}+10\times\mathbf{1}$
q_i^α	$\square$	$(\overline{\mathbf{10}}+\mathbf{5}+\mathbf{1}+\mathbf{1}+\mathbf{1})+\mathbf{1}+\mathbf{1}$
S_a	$\mathbf{1}$	$-$

The composite messengers in this theory consist of

$$(\rho\oplus Z)\sim 2\times(\mathbf{10}+\overline{\mathbf{10}}+\mathbf{5}+\bar{\mathbf{5}}+3\times\mathbf{1}) \tag{4.15}$$

and the lower block of the meson transforms as

$$X\sim 2\times(\mathbf{10}+\bar{\mathbf{5}})+\left[\mathbf{45}+\overline{\mathbf{45}}+\mathbf{10}+\bar{\mathbf{5}}+\overline{\mathbf{10}}+\mathbf{5}+3\times\mathbf{1}\right]\,. \tag{4.16}$$

Excluding the singlet necessary for supersymmetry breaking, the extra representations inside the brackets are vector-like and are made massive by deforming

$$\Delta W \propto \mathrm{tr}\, X^2$$

Unfortunately, the messenger index for this theory is $N_{mess} = 8$, rendering it incompatible with perturbative gauge coupling unification; SM gauge couplings hit a Landau pole around 10^{12} GeV in this theory.

5 Summary & Conclusions

Let me summarize. We have seen that it is possible to generate the fermion hierarchy through compositeness of dynamics that breaks SUSY. Some SM fields are pseudomoduli of SUSY breaking. The scalars of composite generations get masses from one loop CW potential and gauge mediation. Elementary fields get masses from GM. The spectrum works nicely if

$$\mu_\phi \sim 1 \text{ TeV}, \quad \sqrt{F} \sim \mu \sim \mathcal{O}(100 - 200 \text{ TeV}), \tag{5.1}$$

so that the direct SUSY breaking contribution from the CW potential to the first and second generation sfermions is

$$m_{CW} \sim 10 \text{ TeV}. \tag{5.2}$$

while the GM contribution to elementary fields is

$$m_{GM} \sim 1 \text{ TeV}$$

The gravitino mass in this theory is simply given by

$$m_{3/2} \sim \sqrt{\frac{N_f - \tilde{N}_c}{3} \frac{h\mu^2}{M_P}} \tag{5.3}$$

For the low SUSY breaking scale considered here, the gravitino is light and has a mass of

$$m_{3/2} \sim 10 \text{ eV}, \tag{5.4}$$

which makes it cosmologically quite safe.

Very unusual, heavy flavor signals at LHC. Prompt NLSP decay to gravitino

Have written down Ordinary Gauge Mediation models, but can imagine writing GGM models using weakly coupled realization.

A natural question is, does this work with FCNCs? The answer is yes. The soft masses are diagonal in a basis in which the Yukawa textures are generated. Diagonalizing Yukawas then generates off-diagonal soft masses that mix generations, but we can compute FCNCs. Everything can be rendered safe (modulo CP violation).

In general these theories are ugly. But there is a small set of models that work nicely, without too many extra fields or spectators.