## Amplitude method studies of effective field theories

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## Outline:

- Role model: gluon amplitudes
- Effective Field Theories
- From NLSM to Periodic Table of Scalar Theories
- Further avenues
- New soft theorem
- Summary


## Introduction: amplitudes

Objective of amplitude community:

## Study a priori known objects from different perspective

Example in mind: gluon amplitudes

- 1986: Parke and Taylor calculated 6-point gluon-scattering
- simplification: tree-level, no-fermions
- final result: extremely simple
- other way of calculation?


## Example: gluon amplitudes

At tree level:

- colour ordering $\rightarrow$ stripped amplitude


$$
M^{a_{1} \ldots a_{n}}\left(p_{1}, \ldots p_{n}\right)=\sum_{\sigma / Z_{n}} \operatorname{Tr}\left(t^{a_{\sigma(1)}} \ldots t^{a_{\sigma(n)}}\right) M_{\sigma}\left(p_{1}, \ldots, p_{n}\right)
$$

- $M_{\sigma}\left(p_{\sigma(1)}, \ldots, p_{\sigma(n)}\right)=M\left(p_{1}, \ldots, p_{n}\right) \equiv M(1,2, \ldots n)$
- propagators $\Rightarrow$ the only poles of $M_{\sigma}$
- thanks to ordering the only possible poles are:

$$
P_{i j}^{2}=\left(p_{i}+p_{i+1}+\ldots+p_{j-1}+p_{j}\right)^{2}
$$

## Pole structure

Weinberg's theorem (one particle unitarity): on the factorization channel

$$
\lim _{P_{1 j}^{2} \rightarrow 0} M(1,2, \ldots n)=\sum_{h_{l}} M_{L}(1,2 \ldots j, I) \times \frac{1}{P_{1 j}^{2}} \times M_{R}(I, j+1, \ldots n)
$$



## BCFW relations, preliminaries

## [Britto, Cachazo, Feng, Witten '05]

Reconstruct the amplitude from its poles (in complex plane)

- shift in two external momenta

$$
p_{i} \rightarrow p_{i}+z q, \quad p_{j} \rightarrow p_{j}-z q
$$

- keep $p_{i}$ and $p_{j}$ on-shell, i.e.

$$
q^{2}=q \cdot p_{i}=q \cdot p_{j}=0
$$

- amplitude becomes a meromorphic function $A(z)$
- only simple poles coming from propagators $P_{a b}(z)$
- original function is $A(0)$


Cauchy's theorem

$$
\frac{1}{2 \pi \mathrm{i}} \int \frac{d z}{z} A(z)=A(0)+\sum_{k} \frac{\operatorname{Res}\left(A, z_{k}\right)}{z_{k}}
$$

BCFW relations: factorization channels

Cauchy's theorem


$$
0=\frac{1}{2 \pi \mathrm{i}} \int \frac{d z}{z} A(z)=A(0)+\sum_{k} \frac{\operatorname{Res}\left(A, z_{k}\right)}{z_{k}}
$$

If $A(z)$ vanishes for $z \rightarrow \infty$

$$
A=A(0)=-\sum_{k} \frac{\operatorname{Res}\left(A, z_{k}\right)}{z_{k}}
$$

## BCFW relations

$$
P_{a b}^{2}(z)=0 \quad \text { if one and only one } i(\text { or } j) \text { in }(a, a+1, \ldots, b)
$$

Suppose $i \in(a, \ldots, b) \not \ni j$

$$
\begin{aligned}
P_{a b}^{2}(z)=\left(p_{a}+\ldots+p_{i-1}+p_{i}+z q+p_{i+1}\right. & \left.+\ldots+p_{b}\right)^{2}= \\
& =P_{a b}^{2}+2 q \cdot P_{a b} z=0
\end{aligned}
$$

solution

$$
z_{a b}=-\frac{P_{a b}^{2}}{2\left(q \cdot P_{a b}\right)} \quad \Rightarrow \quad P_{a b}^{2}(z)=-\frac{P_{a b}^{2}}{z_{a b}}\left(z-z_{a b}\right)
$$

Thus

$$
\operatorname{Res}\left(A, z_{a b}\right)=\sum_{s} A_{L}^{-s}\left(z_{a b}\right) \times \frac{-z_{a b}}{P_{a b}^{2}} \times A_{R}^{s}\left(z_{a b}\right)
$$

and for allowed helicities it factorizes into two subamplitudes

## BCFW relations

Using Cauchy's formula, we have finally as a result

$$
A=\sum_{k, s} A_{L}^{-s_{k}}\left(z_{k}\right) \frac{1}{P_{k}^{2}} A_{R}^{s_{k}}\left(z_{k}\right)
$$

- based on two-line shift (convenient choice: adjacent $i, j$ )
- recursive formula (down to 3-pt amplitudes)
- number of terms small $=$ number of factorization channels
- all ingredients are on shell


## BCFW Example: gluon amplitudes

\# od diagrams for $n$-body gluon scatterings at tree level

| $n$ | 3 | 4 | 5 | 6 | 7 | 8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| \# diagrams (inc.crossing) | 1 | 4 | 25 | 220 | 2485 | 34300 |
| \# diagrams (col.ordered) | 1 | 3 | 10 | 38 | 154 | 654 |
| \# BCFW terms | - | 1 | 2 | 3 | 6 | 20 |

[C.Cheung: TASI Lectures '17]
[KK, Novotny, Trnka '13]

## BCFW recursion relations: problems

We have assumed that

$$
A(z) \rightarrow 0, \quad \text { for } \quad z \rightarrow \infty
$$

More generally we have to include a boundary term in Cauchy's theorem.
This is intuitively clear: we can formally use the derived BCFW recursion relations to obtain any higher $n$ amplitude starting with the leading interaction. But this does not have to be the correct answer.

## BCFW recursion relations: problems

 example: scalar-QED$$
\mathcal{L}=-\frac{1}{4} F_{\mu \nu} F^{\mu \nu}-|D \phi|^{2}-\frac{1}{4} \lambda|\phi|^{4}
$$



Due to the power-counting the boundary term is proportional to

$$
B \sim 2 e^{2}-\lambda
$$

In order to eliminate the boundary term we have to set $\lambda=2 e^{2}$, then the original BCFW works.
I.e. we needed some further information (e.g. supersymmetry) to determine the $\lambda$ piece.

## Effective field theories

## Effective field theories: general picture

Now we have infinitely many unfixed " $\lambda$ " terms. Schematically

$$
\mathcal{L}=\frac{1}{2}(\partial \phi)^{2}+\lambda_{4}\left(\partial^{m_{4}} \phi\right)^{4}+\lambda_{6}\left(\partial^{m_{6}} \phi\right)^{6}+\ldots
$$

Example: 6pt scattering, Feynman diagrams


Corresponding amplitude:

$$
\mathcal{M}_{6}=\sum_{I=\text { poles }} \lambda_{4}^{2} \frac{\cdots}{P_{I}}+\lambda_{6}(\ldots)
$$

$\lambda_{6}$ part: not fixed by the pole behaviour.
Task: to find a condition in order to link these two terms

## Effective field theories: introduction

Usual steps:
Symmetry $\rightarrow$ Lagrangian $\rightarrow$ Amplitudes $\rightarrow$ physical quantities (cross-section, masses, decay constants, ...)
In our work - opposite direction:
Amplitudes $\rightarrow$ Lagrangian $\rightarrow$ Symmetry
Our aim: classification of interesting EFTs
works done in collaborations with Clifford Cheung, Jiri Novotny, Chia-Hsien Shen, Jaroslav Trnka and Congkao Wen

## Effective field theories: scalar theories

As simple as possible: a spin-0 massless degree of freedom with a three-point interaction.

General formula for three-particle amplitude

$$
A\left(1^{h_{1}} 2^{h_{2}} 3^{h_{3}}\right)= \begin{cases}\langle 12\rangle^{h_{3}-h_{1}-h_{2}}\langle 23\rangle^{h_{1}-h_{2}-h_{3}}\langle 31\rangle^{h_{2}-h_{3}-h_{1}}, & \Sigma h_{i} \leq 0 \\ {[12]^{h_{1}+h_{2}-h_{3}}[23]^{h_{2}+h_{3}-h_{1}}[31]^{h_{3}+h_{1}-h_{2}},} & \Sigma h_{i} \geq 0\end{cases}
$$

Used a spinor-helicity notation, e.g. $p_{i} \cdot p_{j} \sim\langle i j\rangle[i j]$
For scalars $\left(h_{i}=0\right)$ this is a constant - corresponding to $\mathcal{L}_{\text {int }}=\lambda \phi^{3}$.
All derivatives can be removed by equations of motions (boxes)

$$
\mathcal{L}_{i n t}=\left(\partial_{\alpha} \ldots \partial_{\omega} \phi\right)\left(\partial^{\alpha} \ldots \partial^{\omega} \phi\right) \phi \quad \rightarrow \quad \mathcal{L}_{i n t}=(\square \phi)(\ldots)
$$

## Effective field theories: scalar theories

We start with ( $m$ counts number of derivatives)

$$
\mathcal{L}_{\text {int }}=\lambda_{4} \partial^{m} \phi^{4}
$$

n.b. we want to connect this four-point vertex with the 6 -point contact terms


This rules out again no-derivative terms, as the powercounting dictates:

$$
\partial^{m} \times \frac{1}{\partial^{2}} \times \partial^{m} \quad \rightarrow \quad \partial^{2 m-2} \phi^{6}
$$

## Simplest example: two derivatives, single scalar

$$
\mathcal{L}=\frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi+\lambda_{4} \partial^{2} \phi^{4}+\lambda_{6} \partial^{2} \phi^{6}+\ldots
$$

How to connect $\lambda_{4}$ and $\lambda_{6}$ ?
Well Lagrangian, an infinite series, looks complicated, but it is not the case. It represents a free theory:

$$
\mathcal{L}=\frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi \underbrace{\left(1+\lambda_{4} \phi^{2}+\ldots\right)}_{F(\phi)}
$$

$F(\phi)$ can be removed by a field redefinition
Non-trivial simplest example:

- more derivatives
- more flavours $\left(\phi \rightarrow \phi_{1}, \phi_{2}, \ldots\right)$


## More flavours

$$
\mathcal{L}=\frac{1}{2} \partial_{\mu} \phi^{i} \partial^{\mu} \phi^{i}+\lambda_{i j k l} \partial_{\mu} \phi^{i} \partial^{\mu} \phi^{j} \phi^{k} \phi^{\prime}+\lambda_{i_{1} \ldots 1_{6}} \partial_{\mu} \phi^{i} \partial^{\mu} \phi^{i_{2}} \phi^{3_{3}} \ldots \phi^{i_{6}}+\ldots
$$

- Can be used for systematic studies of two species, three species, etc.
- Very complicated generally
- Assume some simplification, organize using a group structure

$$
\phi=\phi^{a} T^{a}
$$

- motivated by the 'gluon case': flavour ordering [kא,Novotny, Trma ' ${ }^{13]}$

$$
A^{a_{1} \ldots a_{n}}=\sum_{p e r m} \operatorname{Tr}\left(T^{a_{1}} \ldots T^{a_{n}}\right) A\left(p_{1}, \ldots p_{n}\right)
$$

## More flavours: stripped amplitude

first non-trivial case 6 pt scattering:

power-counting is ok:

$$
\lambda_{4}^{2} p^{2} \frac{1}{p^{2}} p^{2}+\lambda_{6} p^{2}
$$

in order to combine the pole and contact term we need to consider some limit. The most natural candidate: We will demand soft limit, i.e.

$$
A \rightarrow 0, \quad \text { for } \quad p \rightarrow 0
$$

$\Rightarrow \lambda_{4}^{2} \sim \lambda_{6}$

## Standard direction(s)

Assuming the shift symmetry

$$
\phi^{a} \rightarrow \phi^{a}+\epsilon^{a}
$$

$\Rightarrow$ Noether current

$$
A_{\mu}^{a}=\frac{\delta \mathcal{L}}{\delta \partial^{\mu} \phi^{a}}
$$

$\Rightarrow$ Ward identity $\Rightarrow$ LSZ

$$
\langle 0| A_{\mu}^{a}(x)\left|\phi^{b}(p)\right\rangle=i F \delta^{a b} p_{\mu} \mathrm{e}^{-i p x}
$$

$\Rightarrow$ Adler zero

$$
\lim _{p \rightarrow 0}\left\langle f \mid i+\phi^{a}(p)\right\rangle=0
$$

$\Rightarrow$ CCWZ: non-linear sigma model

$$
\mathcal{L}=\frac{F^{2}}{2} \operatorname{Tr}\left(\partial_{\mu} U^{\dagger} \partial^{\mu} U\right), \quad U=\mathrm{e}^{\frac{i}{\digamma} \phi^{a} T^{a}}
$$

[Weinber'66], [lan Low '14-'15]

## Natural classification: $\sigma$ and $\rho$

Soft limit of one external leg of the tree-level amplitude

$$
A\left(t p_{1}, p_{2}, \ldots, p_{n}\right)=\mathcal{O}\left(t^{\sigma}\right), \quad \text { as } \quad t p_{1} \rightarrow 0
$$

Interaction term

$$
\mathcal{L}=\partial^{m} \phi^{n}
$$

Then another natural parameter is (counts the homogeneity)

$$
\rho=\frac{m-2}{n-2} \quad \text { "averaging number of derivatives" }
$$

e.g. $\mathcal{L}=\partial^{m} \phi^{4}+\partial^{\widetilde{m}} \phi^{6}$

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$2 m-2-2=\tilde{m}-2 \Rightarrow \frac{2 m-4}{4}=\frac{\tilde{m}-2}{4} \Rightarrow$

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$2 m-2-2=\tilde{m}-2 \Rightarrow \frac{2 m-4}{4}=\frac{\tilde{m}-2}{4} \Rightarrow \rho=\widetilde{\rho}$
rho is same if they talk to each other

Non-trivial cases

$$
\text { for: } \mathcal{L}=\partial^{m} \phi^{n}: \quad m<\sigma n
$$

or

$$
\sigma>\frac{(n-2) \rho+2}{n}
$$

i.e.

| $\rho$ | $\sigma$ at least |
| :---: | :---: |
| 0 | 1 |
| 1 | 2 |
| 2 | 2 |
| 3 | 3 |

i.e. non-trivial regime for $\rho \leq \sigma$

## First case: $\rho=0$ (i.e. two derivatives)

Schematically for single scalar case

$$
\mathcal{L}=\frac{1}{2}(\partial \phi)^{2}+\sum_{i} \lambda_{4}^{i}\left(\partial^{2} \phi^{4}\right)+\sum_{i} \lambda_{6}^{i}\left(\partial^{2} \phi^{6}\right)+\ldots
$$

similarly for multi-flavour $\left(\phi_{i}: \phi_{1}, \phi_{2}, \ldots\right)$.
non-trivial case

$$
\sigma=1
$$

Outcome:

- single scalar: free theory
- multiple scalars (with flavour-ordering): non-linear sigma model n.b. it represents a generalization of [Susskind, Frye '70], [Ellis, Renner '70]


## Second case: $\rho=1, \sigma=2$ (double soft limit)

1. focus on the lowest combination and fix the form:

$$
\mathcal{L}_{\text {int }}=c_{2}(\partial \phi \cdot \partial \phi)^{2}+c_{3}(\partial \phi \cdot \partial \phi)^{3} \quad \text { condition: } c_{3}=4 c_{2}^{4}
$$



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2. find the symmetry

$$
\phi \rightarrow \phi-b_{\rho} x^{\rho}+b_{\rho} \partial^{\rho} \phi \phi \quad \text { (again up to } 6 \mathrm{pt} \text { so far) }
$$

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\phi \rightarrow \phi-b_{\rho} x^{\rho}+b_{\rho} \partial^{\rho} \phi \phi \quad \text { (again up to 6pt so far) }
$$

3. ansatz of the form

$$
c_{n}(\partial \phi \cdot \partial \phi)^{n}+c_{n+1}(\partial \phi \cdot \partial \phi)^{n} \partial \phi \cdot \partial \phi
$$

4. in order to cancel: $2(n+1) c_{n+1}=(2 n-1) c_{n}$
i.e. $c_{1}=\frac{1}{2} \Rightarrow c_{2}=\frac{1}{8}, c_{3}=\frac{1}{16}, c_{4}=\frac{5}{128}, \ldots$

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$$
\text { i.e. } c_{1}=\frac{1}{2} \Rightarrow c_{2}=\frac{1}{8}, c_{3}=\frac{1}{16}, c_{4}=\frac{5}{128}, \ldots
$$

solution:

$$
\mathcal{L}=-\sqrt{1-(\partial \phi \cdot \partial \phi)}
$$

This theory known as a scalar part of the Dirac-Born-Infeld [1934] - DBI action
Scalar field can be seen as a fluctuation of a 4-dim brane in five-dim Minkowski space


Remark: soft limit and symmetry are "equivalent"

## Third case: $\rho=2, \sigma=2$ (double soft limit)

Similarly to previous case we will arrive to a unique solution: the Galileon Lagrangian

$$
\begin{gathered}
\mathcal{L}=\sum_{n=1}^{d+1} d_{n} \phi \mathcal{L}_{n-1}^{\text {der }} \\
\mathcal{L}_{n}^{\operatorname{der}}=\varepsilon^{\mu_{1} \ldots \mu_{d}} \varepsilon^{\nu_{1} \ldots \nu_{d}} \prod_{i=1}^{n} \partial_{\mu_{i}} \partial_{\nu_{i}} \phi \prod_{j=n+1}^{d} \eta_{\mu_{j} \nu_{j}}=-(d-n)!\operatorname{det}\left\{\partial^{\nu_{i}} \partial_{\nu_{j}} \phi\right\} .
\end{gathered}
$$

It possesses the Galilean shift symmetry

$$
\phi \rightarrow \phi+a+b_{\mu} x^{\mu}
$$

(leads to EoM of second-order in field derivatives)

## Surprise: $\rho=2, \sigma=3$ (enhanced soft limit)

- general galileon: three parameters (in 4D)
- only two relevant (due to dualities [de Rham, Keltner, Tolley '14] [KK, Novotny '14])


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- we demanded $\mathcal{O}\left(p^{3}\right)$ behaviour
- we have verified: possible up to very high-pt order
- suggested new theory: special galileon [Cheung,KK,Novotny,Trnka 1412.4095]


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- suggested new theory: special galileon [Cheung,KK,Novotny,Trnka 1412.4095]
- symmetry explanation: hidden symmetry [K. Hinterbichler and A. Joyce 1501.07600]

$$
\phi \rightarrow \phi+s_{\mu \nu} x^{\mu} x^{\nu}-12 \lambda_{4} s^{\mu \nu} \partial_{\mu} \phi \partial_{\nu} \phi
$$

# New recursion for effective theories 

[Cheung, KK, Novotny, Shen, Trnka 2015]

The high energy behaviour forbids a naive Cauchy formula

$$
A(z) \neq 0 \quad \text { for } \quad z \rightarrow \infty
$$

Can we instead use the soft limit directly?

The high energy behaviour forbids a naive Cauchy formula

$$
A(z) \neq 0 \quad \text { for } \quad z \rightarrow \infty
$$

Can we instead use the soft limit directly? yes!
The standard BCFW not applicable, we propose a special shift:

$$
p_{i} \rightarrow p_{i}\left(1-z a_{i}\right) \quad \text { on all external legs }
$$

This leads to a modified Cauchy formula

$$
\oint \frac{d z}{z} \frac{A(z)}{\Pi_{i}\left(1-a_{i} z\right)^{\sigma}}=0
$$

note there are no poles at $z=1 / a_{i}$ (by construction).
Now we can continue in analogy with BCFW

## Further avenues

- similarly for vector EFT:

$$
\mathcal{L}_{\mathrm{BI}}=1-\sqrt{(-1)^{D-1} \operatorname{det}\left(\eta_{\mu \nu}+F_{\mu \nu}\right)}
$$

(see [Cheung, KK, Novotny, Shen, Trnka, Wen '18])

- so far avoided the fermionic degrees of freedom (see e.g. Elvang et al.'18)
- higher orders in NLSM (see [Bijnens,KK, Sjö 2019])
- multiple flavours - especially without flavour ordering
- only two-flavour case fully classified
- preliminary study of the mixed scalar-vector case (Galileon-BI): more promising than the pure Galileon-like BI
- spin-2: similar to Galileon-like studies - no exceptional candidate
- non-abelian Born-Infeld
- non-zero masses (technically possible)
- loop corrections - focused on the exceptional theories


## BUT...

[KK, Novotny, Shifman, Trnka 2019 and in prep.]

- it seems we have a powerful method to classify effective field theories
- more efficient than the standard group oriented methods: spontaneous symmetry breaking, (non-)compact groups, (semi-)simple, CCWZ construction $\ldots \longrightarrow$ complicated monomial structure, where equivalence is not transparent
- e.g. for two flavours - two-derivative counting: only one non-trivial theory: $O(3) / O(2)$
- more problematic for three flavours - finished (work in progress with J.Bijnens): again only $O(4) / O(3)$ and combinations of $O(3) / O(2)$ plus one free scalar
- but what about completely broken $O(3)$ :

$$
S U(2) / 1
$$

## $\operatorname{SU}(2) / 1$

- it describes three GBs
- CCWZ construction
- $\Rightarrow$ Lagrangian
- it is neither equivalent to $O(4) / O(3)$ nor to $O(3) / O(2)$, nor any their flavour combinations
- on top of it: the amplitudes don't have adler zero!
- What have we missed?


## General discussion

- answer is then easy - we missed "non-zero" Adler zero
- beyond the scope of our classification
- so our method is not that general
- can we extend it?


## Adler zero: textbook derivation

GB couples to the associated Noether current

$$
\langle 0| J^{\mu}(x)|\phi(p)\rangle=-i p^{\mu} F \mathrm{e}^{-i p \cdot x}
$$

For the process $i \rightarrow f+\phi(p)$ we have:

$$
\langle f| J^{\mu}(0)|i\rangle=F \frac{p^{\mu}}{p^{2}} \mathcal{A}(f+\phi(p), i)+R^{\mu}(p)
$$

The current conservation $p_{\mu}\langle f| J^{\mu}(0)|i\rangle=0$ yields

$$
\mathcal{A}(f+\phi(p), i)=-\frac{1}{F} p_{\mu} R^{\mu}(p)
$$

and thus finally

$$
\lim _{p \rightarrow 0} \mathcal{A}(f+\phi(p), i)=0
$$

if $R(p)$ regular in the limit.

## Adler zero: loophole

When

$$
\lim _{p \rightarrow 0} p_{\mu} R^{\mu} \neq 0 \quad ?
$$

Two possibilities:

- there are cubic vertices

- Noether current is quadratic in fields



## $S U(2) / 1$ : new soft theorem

- n.b.: three GBs, can be rotated to: two charged $\phi^{ \pm}$and one neutral $\phi$.
- Simplification: charge conservation + shift symmetry in the neutral mode.
- standard Adler zero for the neutral mode
- we can focus only on the $\phi^{+}$shift. Ansatz:

$$
\begin{aligned}
& \lim _{p_{1} \rightarrow 0} \phi^{+}\left(p_{1}\right) \ldots \phi^{+}\left(p_{n}\right) \phi^{-}\left(q_{1}\right) \ldots \phi^{-}\left(q_{n}\right) \phi\left(k_{1}\right) \ldots \phi\left(k_{m}\right)= \\
& x \sum_{i=1}^{m} \phi^{+}\left(\mathbf{k}_{\mathbf{i}}\right) \ldots \phi^{+}\left(p_{n}\right) \phi^{-}\left(q_{1}\right) \ldots \phi^{-}\left(q_{n}\right) \phi\left(k_{1}\right) \notin\left(k_{i}\right) \phi\left(k_{m}\right) \\
+ & y \sum_{i=1}^{n} \phi^{+}\left(p_{1}\right) \ldots \phi^{+}\left(p_{n}\right) \phi^{-}\left(q_{1}\right) \text { 中(qi) } \phi^{-}\left(q_{n}\right) \phi\left(\mathbf{q}_{\mathbf{i}}\right) \phi\left(k_{1}\right) \ldots \phi\left(k_{m}\right)
\end{aligned}
$$

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$$
\begin{aligned}
& \lim _{p_{1} \rightarrow 0} \phi^{+}\left(p_{1}\right) \ldots \phi^{+}\left(p_{n}\right) \phi^{-}\left(q_{1}\right) \ldots \phi^{-}\left(q_{n}\right) \phi\left(k_{1}\right) \ldots \phi\left(k_{m}\right)= \\
& x \sum_{i=1}^{m} \phi^{+}\left(\mathbf{k}_{\mathbf{i}}\right) \ldots \phi^{+}\left(p_{n}\right) \phi^{-}\left(q_{1}\right) \ldots \phi^{-}\left(q_{n}\right) \phi\left(k_{1}\right) \phi\left(k_{i}\right) \phi\left(k_{m}\right) \\
+ & y \sum_{i=1}^{n} \phi^{+}\left(p_{1}\right) \ldots \phi^{+}\left(p_{n}\right) \phi^{-}\left(q_{1}\right) 中\left(q_{i}\right) \phi^{-}\left(q_{n}\right) \phi\left(\mathbf{q}_{\mathbf{i}}\right) \phi\left(k_{1}\right) \ldots \phi\left(k_{m}\right)
\end{aligned}
$$

- IT WORKS!
- $y=-x$
- verified on amplitudes up to 7-pt
- can be proved from Lagrangian
- new generalization for the formal Lagrangian (work in progress)


## Summary

- We have offered a new tool for effective field theories
- motivated by the amplitude methods employed for renormalizable theories
- analogy between gravity and soft scalar theories (Bonifacio et al. 'today)
- used for classification of scalar theories
- one new theory discovered: special galileon
- one exceptional theory for spin-1 particles: BI
- generalization of Adler zero
- work in progress: classification can be extended for generalized Adler zero


## Summary

- We have offered a new tool for effective field theories
- motivated by the amplitude methods employed for renormalizable theories
- analogy between gravity and soft scalar theories (Bonifacio et al. 'today)
- used for classification of scalar theories
- one new theory discovered: special galileon
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Thank you!

