
Dynamical Axions and Gravitational Waves



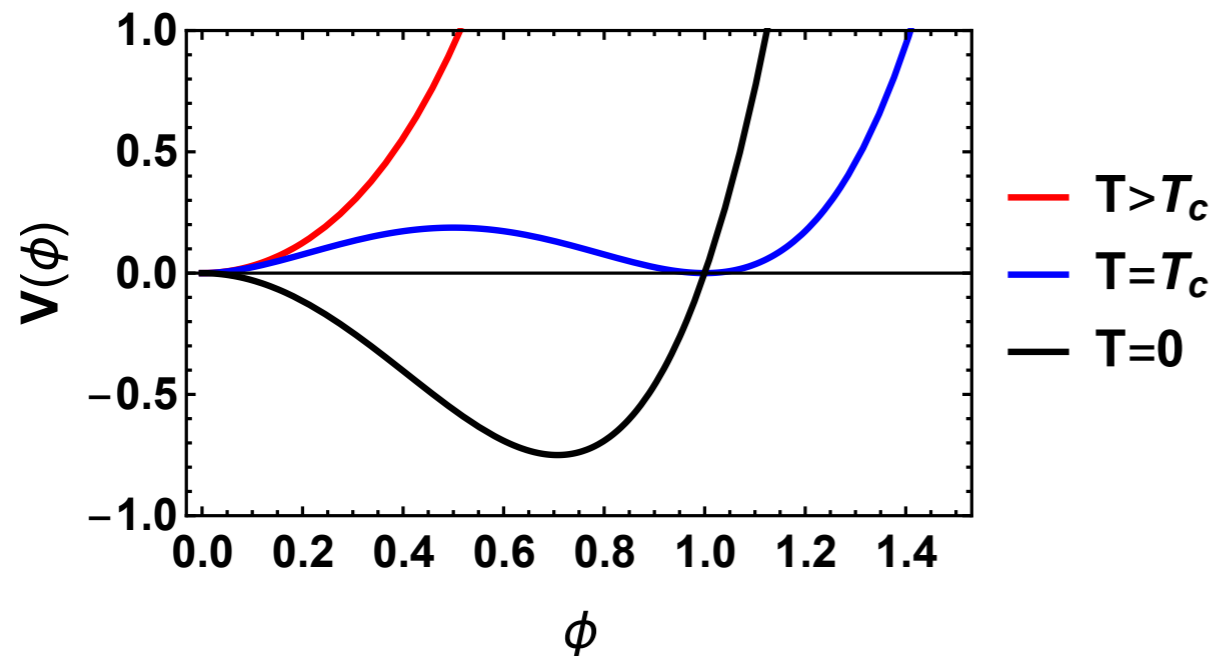
Instituto de
Física
Teórica
UAM-CSIC



Rachel Houtz
UC Davis
Seminar
October, 2019

In Collaboration with D. Croon (TRIUMF) and
V. Sanz (U. Sussex), arXiv:1904.10967

Gravitational wave phenomenology



- ❖ First order phase transitions can produce gravitational waves Witten (1984) C. Hogan (1986)

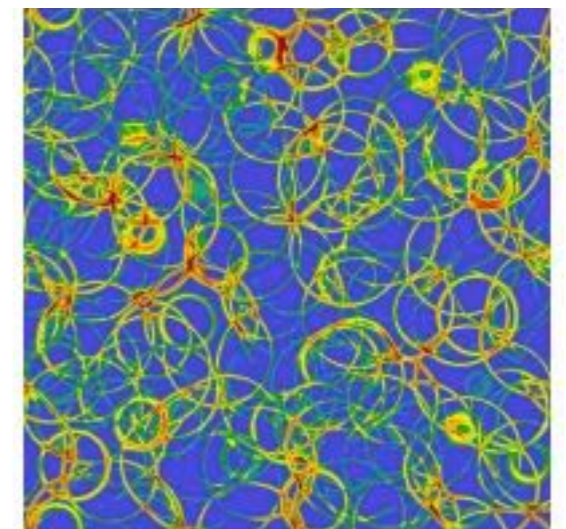
- ❖ Complementary probe of hidden sectors:

- Spontaneous symmetry breaking

J. Jaeckel, V. V. Khoze, P. Schwaller, arXiv:1504.07263
M. Spannowsky, arXiv:1602.03901 D. Croon, V. Sanz, G. White, arXiv:1806.02332

- Confining exotic color sectors

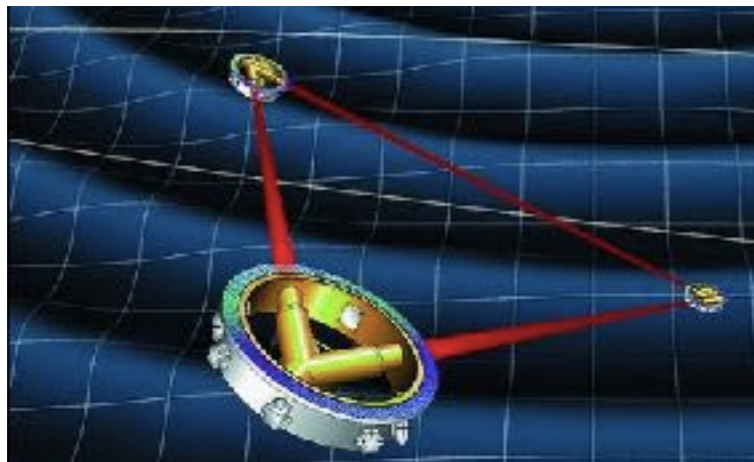
A. J. Helmboldt, J. Kubo, S. van der Woude, arXiv:1904.07891
Y. Bai, A. J. Long, S. Lu, arXiv:1810.04360



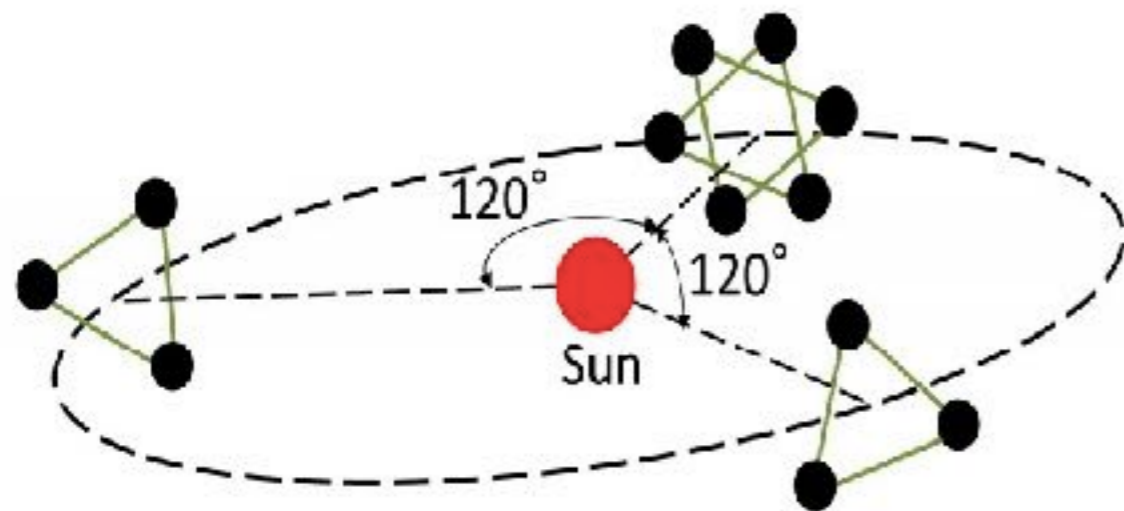
M. Hindmarsh, S. Huber, K. Rummukainen, D. Weir, arXiv:1504.03291

Future gravitational wave detectors

- ❖ Laser interferometers LISA, B-DECIGO, DECIGO and BBO



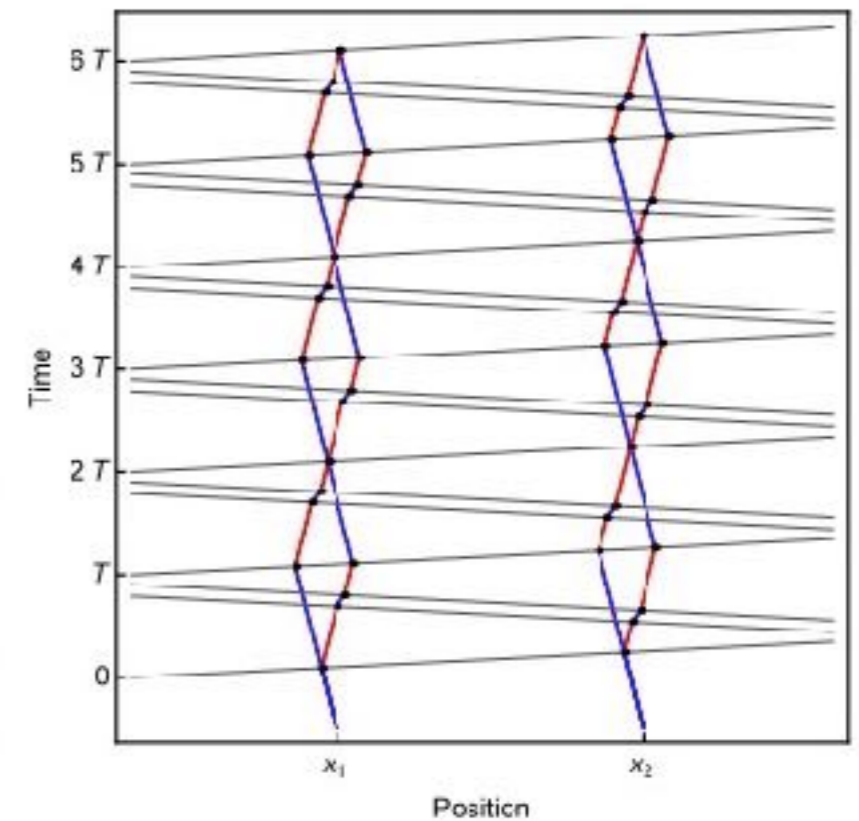
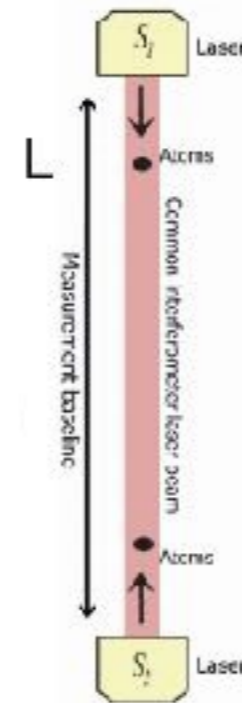
NASA illustration of LISA (from Wikipedia)



Yagi, 1302.2388

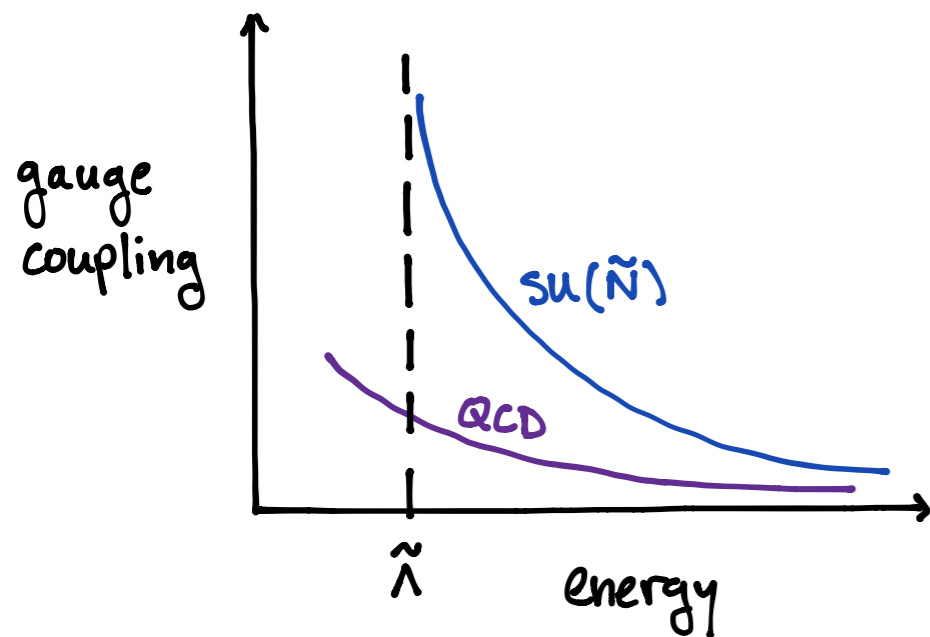
- ❖ Atom interferometers AION and MAGIS

Graham, Hogan, Kasevich, Rajendran, arXiv:1606.01860



Buchmueller, "A UK AION for the exploration of ultra-light dark matter and mid-frequency gravitational waves" (2018)

Gravitational Waves and Confining Sectors



- ❖ First order phase transition at confinement if $N_F \geq 3$

Pisarski, Wilczek (1984)

- ❖ Gravitational waves can probe confining exotic color sectors

- ❖ Use a low energy effective theory to try and parameterize the behavior of the potential at T_c

- Linear Sigma Model $N_F = 3$

Y. Bai, A. J. Long, S. Lu, arXiv:1810.04360

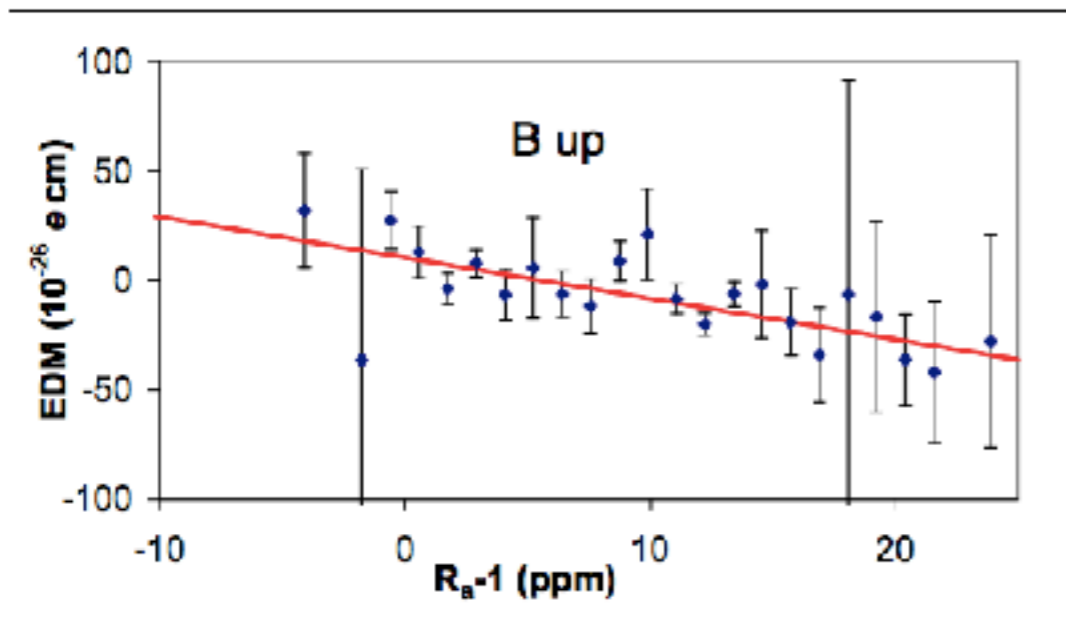
- Nambu-Jona-Lasinio Model $N_F = 3$

A. J. Helmboldt, J. Kubo, S. van der Woude, arXiv: 1904.07891

- ❖ Model building can motivate parameters in the low energy EFT

The strong CP problem and axions

$$\mathcal{L} = -\frac{1}{4}G_{\mu\nu}^a G^{a\mu\nu} + \frac{g^2\theta}{32\pi^2}G_{\mu\nu}^a \tilde{G}^{a\mu\nu} + \bar{q}Mq$$



C. A. Baker *et al*, hep-ex/0602020

- ❖ Dynamical solution employing $U(1)_{PQ}$ results in the axion

R. Peccei, H. Quinn (1977)

$$\mathcal{L} \ni \frac{g^2}{32\pi^2} \frac{a}{f_a} G_{\mu\nu}^a \tilde{G}^{a\mu\nu}$$

F. Wilczek (1978) S. Weinberg (1978)

$$\bar{\theta} = \theta + \arg \det M \quad \bar{\theta} < 10^{-10}$$

Dynamical axions

- ❖ Massless quark solution to the strong CP problem

G. 't Hooft (1976)

If $m_\psi = 0$, then a chiral rotation of ψ results in a shift

$$\theta \frac{g^2}{32\pi^2} G\tilde{G} \rightarrow (\theta - 2\alpha) \frac{g^2}{32\pi^2} G\tilde{G} \quad \rightarrow \theta \text{ can be removed by field redefinition}$$

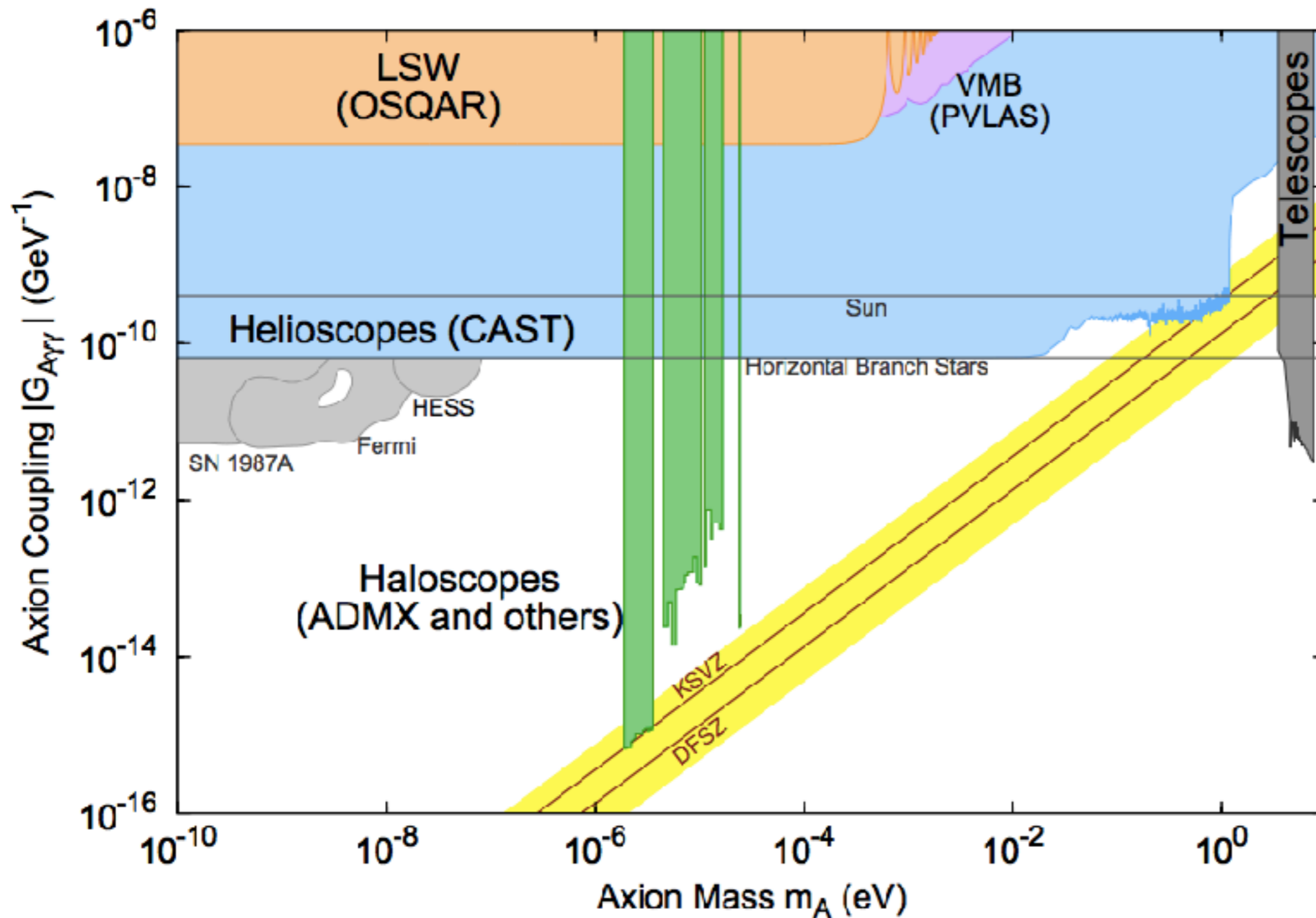
- ❖ Below confinement, massless quarks will form bound states, one of which is the dynamical axion

MK Gaillard, B. Gavela, R. Houtz, P. Quilez, R. del Rey, arXiv:1805.06465



The **axion** is a pNGB of a global chiral $U(1)$ symmetry with anomalous couplings to $G\tilde{G}$ of a confining group

Invisible axion parameter space

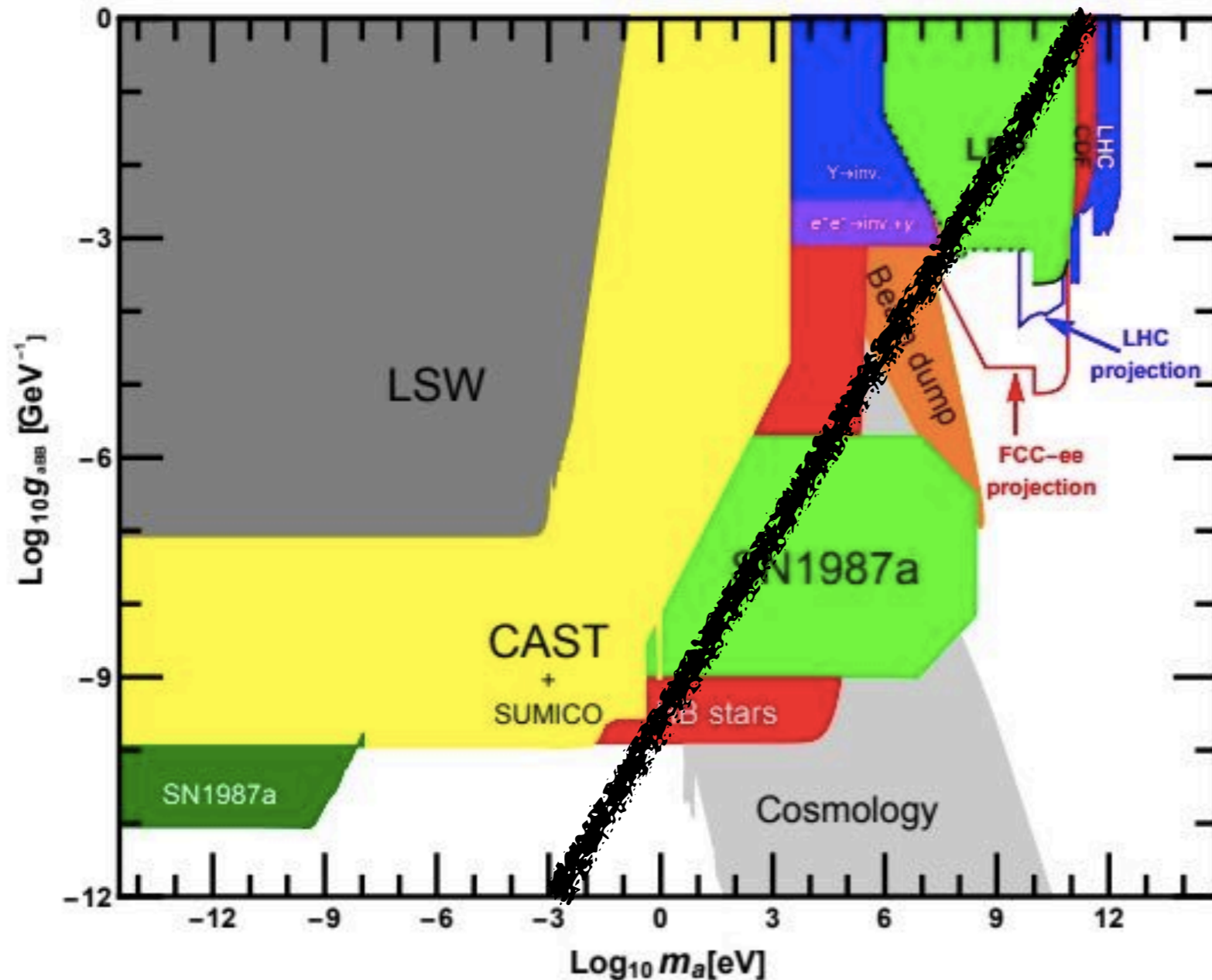


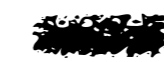
J. Kim (1979), M. Shifman, A. Vainshtein, V. Zakharov (1980)

M. Dine, W. Fischler, M. Srednicki (1981), Zhitnitsky (1980)

A. Ringwald, L. J. Rosenberg, G. Rybka, Particle Data Group (2018)

Invisible axion parameter space



 Extrapolation of the invisible axion expected region

J. Jaeckel, M. Spannowsky arXiv:1509.00476

I. Brivio, HEFT 2017

Motivation for exotic confining groups

(1) Additional color interactions can alter the m_a, f_a relationship

$$m_a^2 f_a^2 \approx m_\pi^2 f_\pi^2 \longrightarrow + \sim \Lambda_{\text{new}}^4$$

(2) Hide massless quark in bound states


$$\sim \Lambda_{\text{new}}$$

$$m_\psi = 0$$

Dimopoulos, Susskind (1979)

Tye (1981) Rubakov (1997)

Berezhiani, Gianfagna, Giannotti, hep-ph/0009290

Fukuda, Harigaya, Ibe, Yanagida, arXiv:1504.06084

Gherghetta, Nagata, Shifman, arXiv:1604.01127

Dimopoulos, Hook, Huang, Marques-Tavares, arXiv:1606.03097

Agrawal, Howe, arXiv:1706:04195

K. Choi, JE Kim (1985)

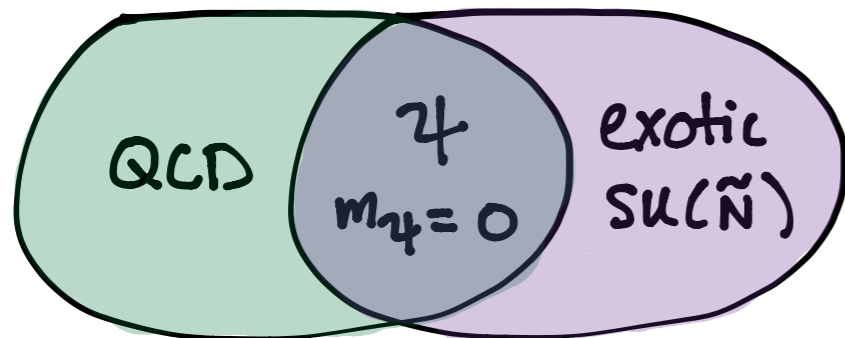
A. Hook, arXiv:1411.3325

Agrawal, Howe, 1712.05803

MK Gaillard, B. Gavela, R. Houtz, P. Quilez, R. del Rey, arXiv:1805.06465

Generic properties of dynamical axion models

❖ Massless messenger fields



❖ $N_F \geq 3$ at $SU(\tilde{N})$ confinement

❖ First order PT at $\tilde{\Lambda} \sim 3 \text{ TeV}$

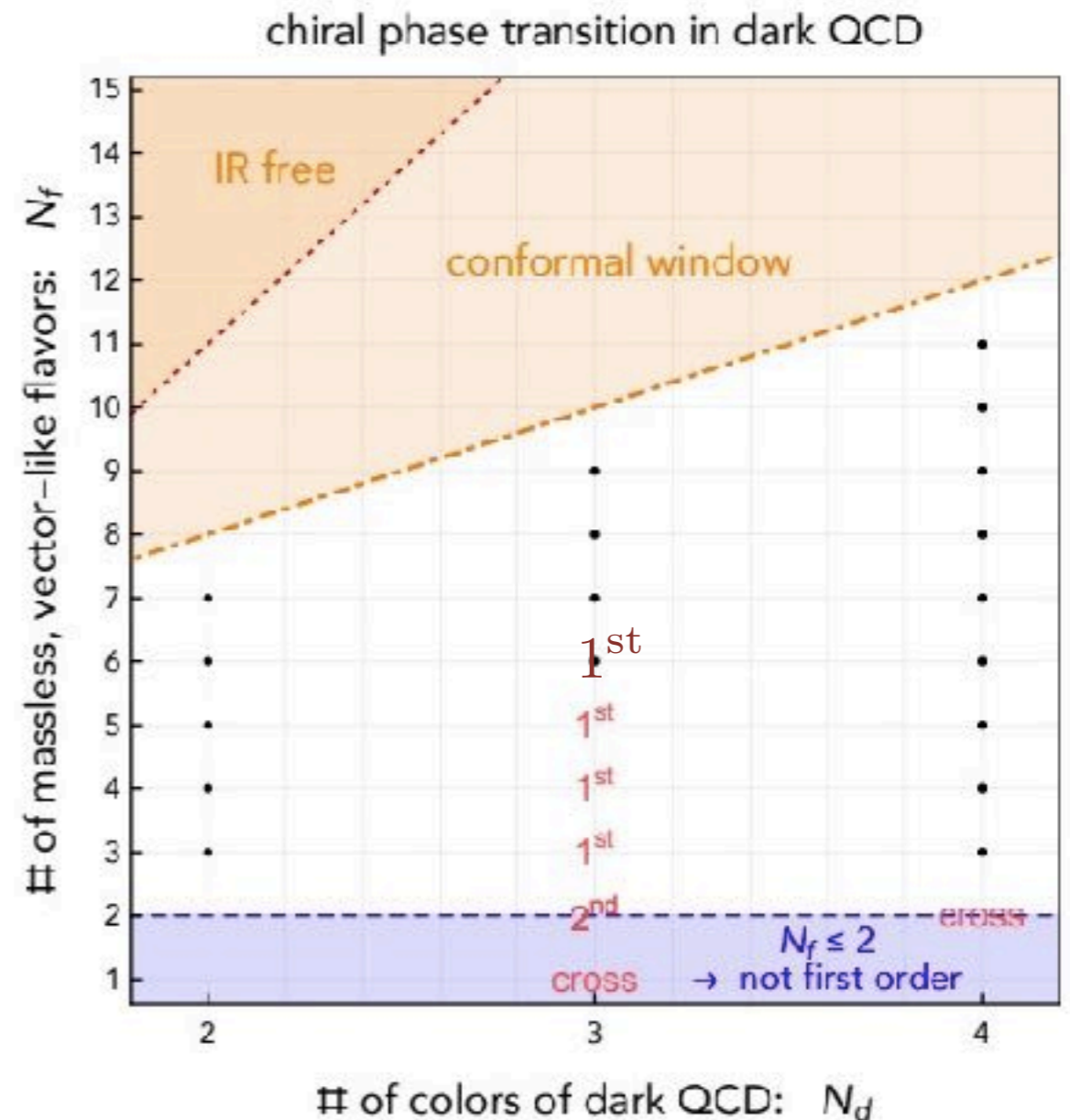
Pisarski, Wilczek (1984)

❖ Quadratically divergent mass terms for pions

$$m^2(\pi) \sim \tilde{\Lambda}^2$$

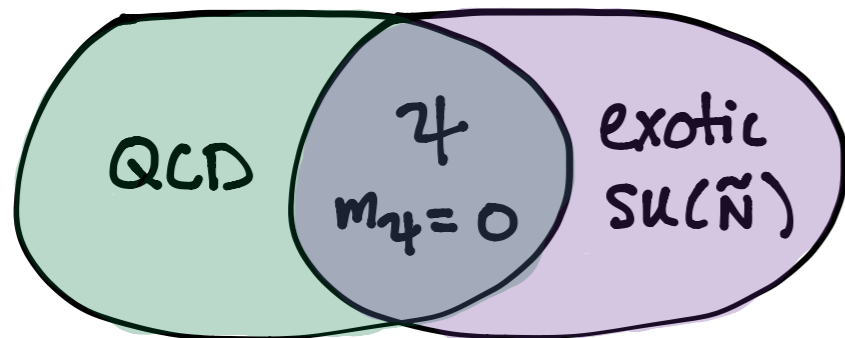
Plot lifted from: Bai, Long, Lu, arXiv:1810.04360

$N_F = 6$: Iwasaki, Kanaya, Sakai, Yoshié, hep-lat/9504019



Generic properties of dynamical axion models

❖ Massless messenger fields



❖ $N_F \geq 3$ at $SU(\tilde{N})$ confinement

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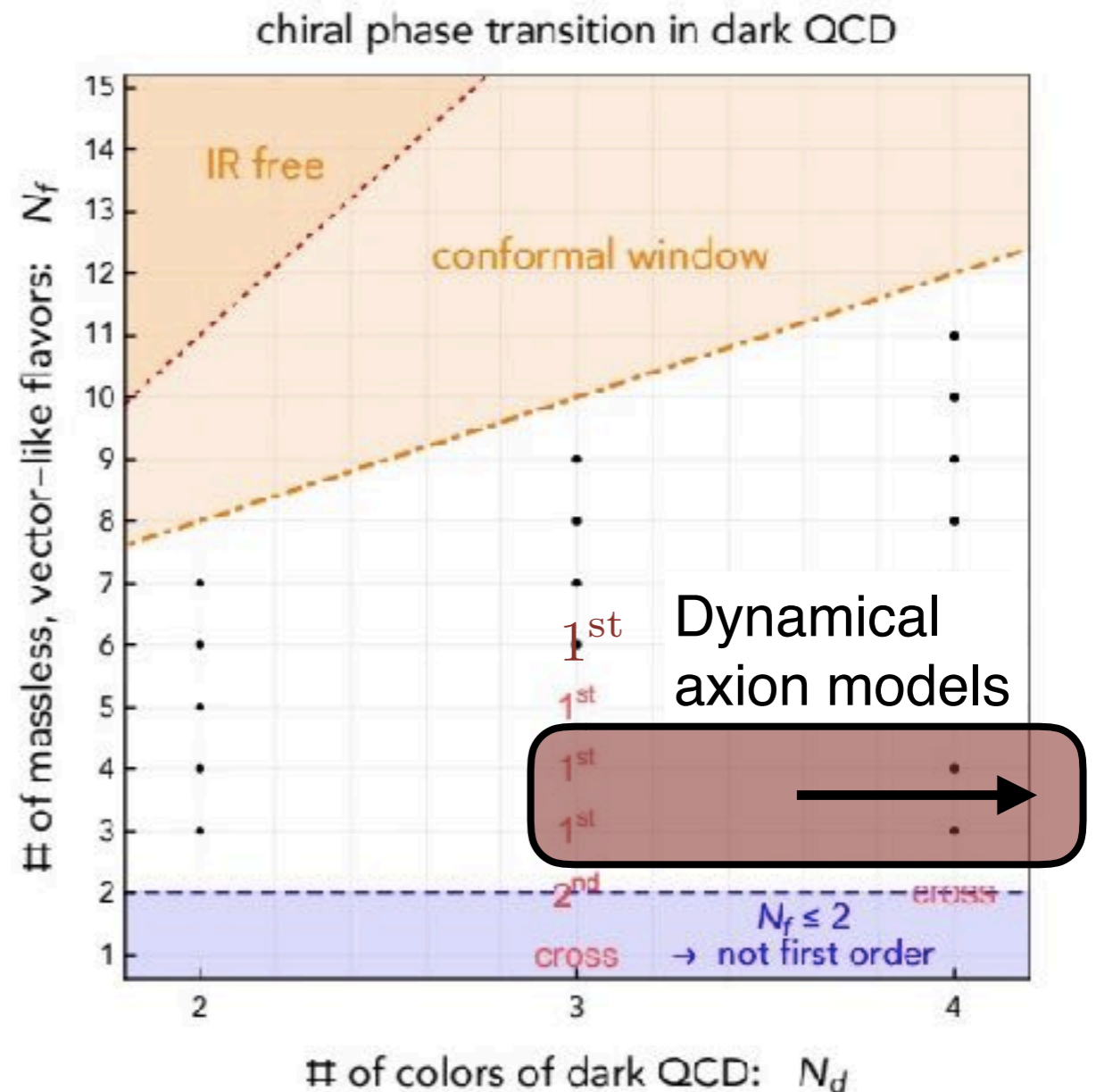
Pisarski, Wilczek (1984)

❖ Quadratically divergent mass terms for pions

A Feynman diagram showing a loop of a messenger field (represented by a wavy line) between two quark lines (represented by dashed lines). To the right of the diagram is the equation $m^2(\pi) \sim \tilde{\Lambda}^2$.

Plot lifted from: Bai, Long, Lu, arXiv:1810.04360

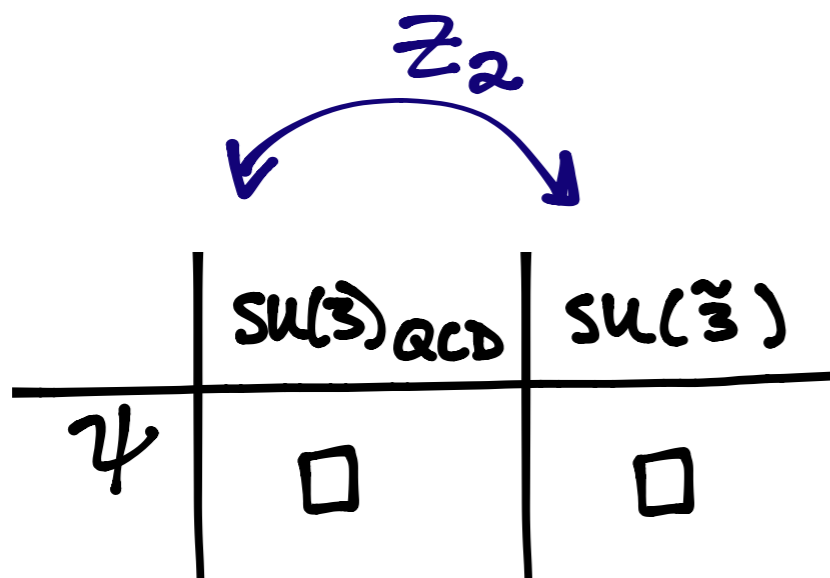
$N_F = 6$: Iwasaki, Kanaya, Sakai, Yoshié, hep-lat/9504019



Dynamical axions $N_F = 3$

Massless quark content

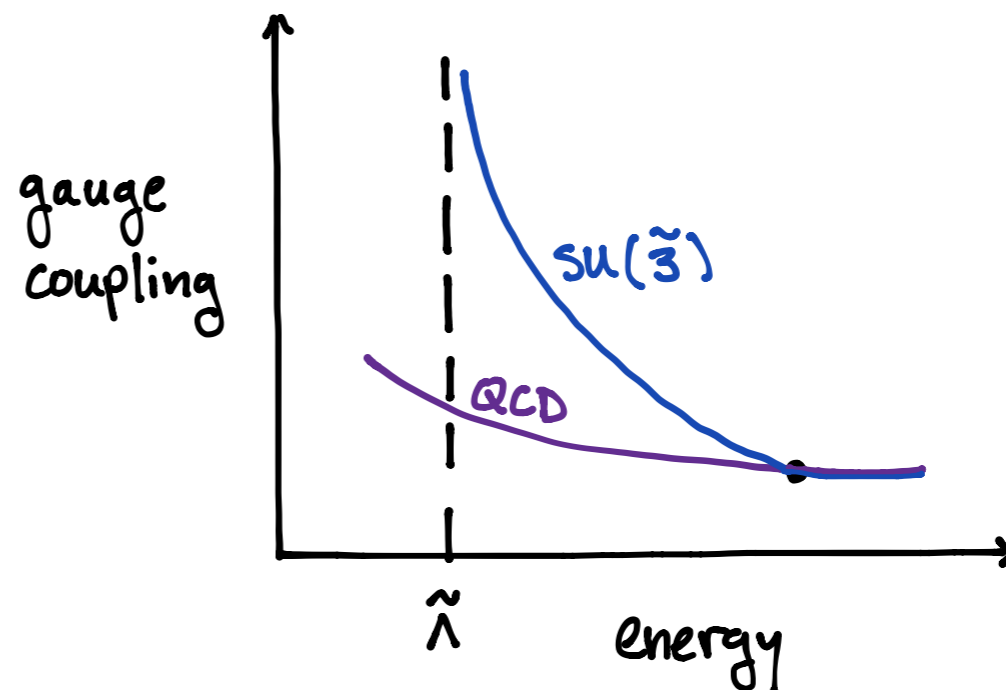
A. Hook, arXiv:1411.3325



- ❖ Set up one Higgs VEV to be very large:

$$\tilde{v} \gg v \quad \longrightarrow \quad \tilde{m}_q \gg m_q$$

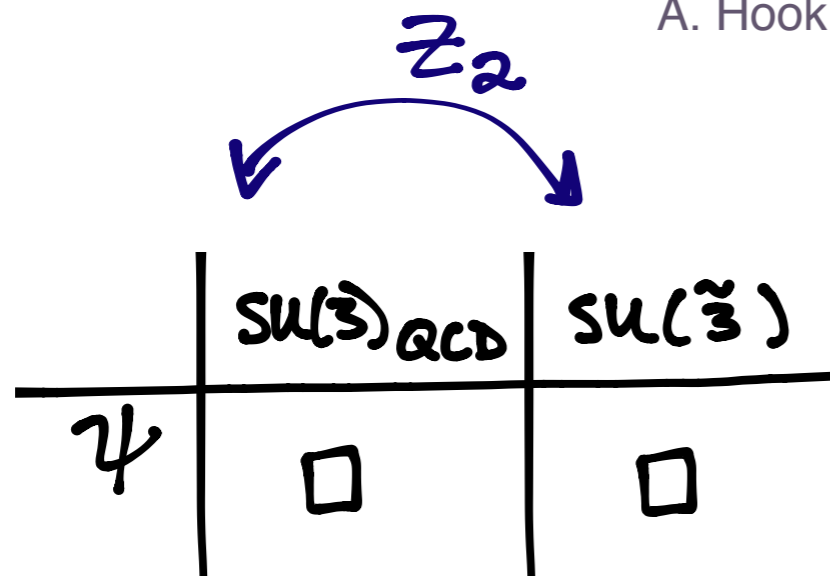
- ❖ Complete Z_2 copy of the SM
- ❖ Massless quark $m_\psi = 0$
- ❖ $\tilde{\Lambda} \gg \Lambda_{QCD}$



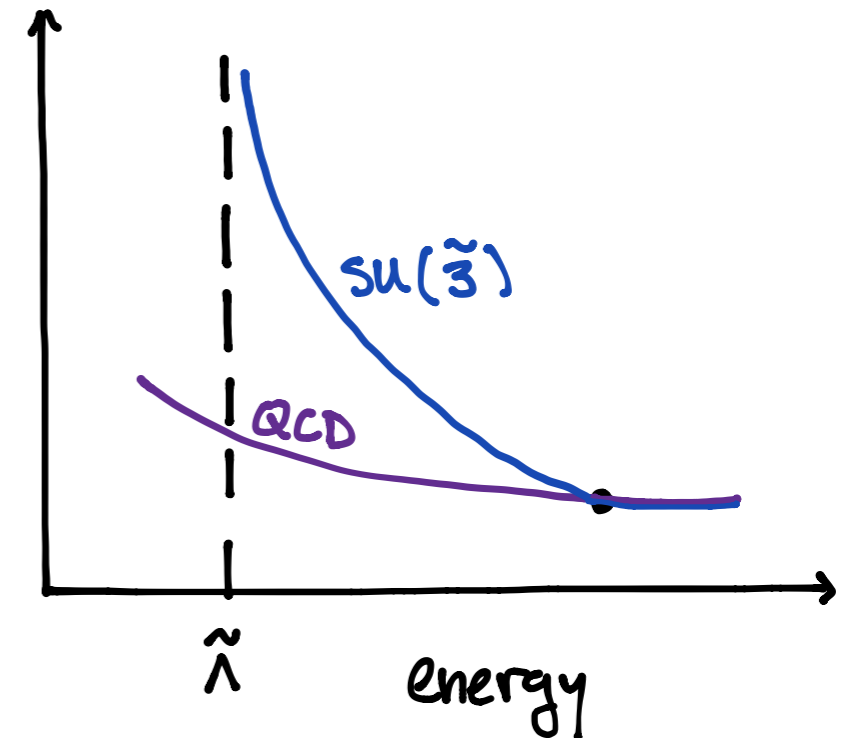
Dynamical axions $N_F = 3$

Massless quark content

A. Hook, arXiv:1411.3325



gauge coupling



- ❖ Chiral symmetry breaking pattern at confinement:

$$SU(3)_L \times SU(3)_R \times U(1)_L \times U(1)_R \rightarrow SU(3)_V \times U(1)_V$$

- ➔ Bound states composed of massless quarks live at $\tilde{\Lambda}$
- ➔ Possible light pNGB states

Berezhiani, Gianfagna,
Giannotti, hep-ph/0009290

SU($\tilde{3}$) confinement

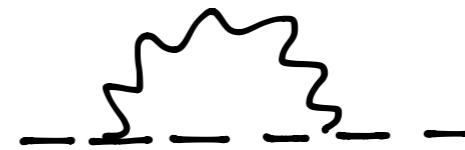
- ❖ Spontaneous chiral symmetry breaking:

$$SU(3)_L \times SU(3)_R \times U(1)_L \times U(1)_R \rightarrow SU(3)_V \times U(1)_V$$

- ❖ Resulting Goldstone Bosons: $9 \rightarrow 8_c + 1_c = \pi_8 + \eta'$

Explicit symmetry breaking effects:

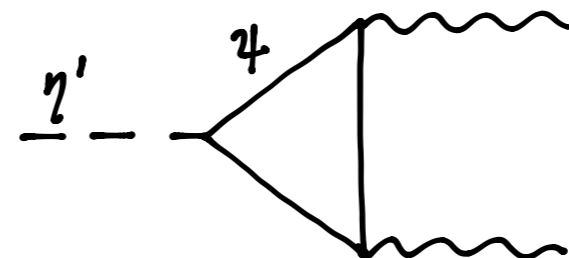
- (1) QCD explicitly breaks $SU(3)_V$



$$m^2(\pi_8) \sim \tilde{\Lambda}^2$$

- (2) $G\tilde{G}$ explicitly breaks $U(1)_A$

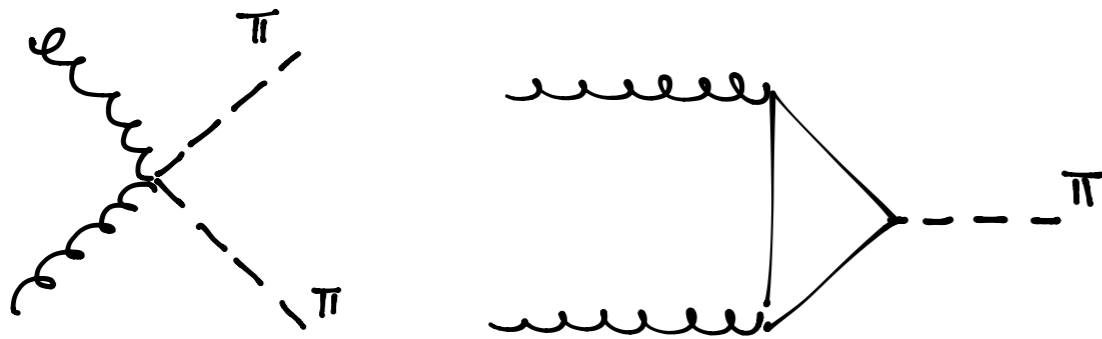
➔ The η' is a **visible dynamical axion**



$$m^2(\eta') \sim \tilde{\Lambda}^2$$

Collider phenomenology

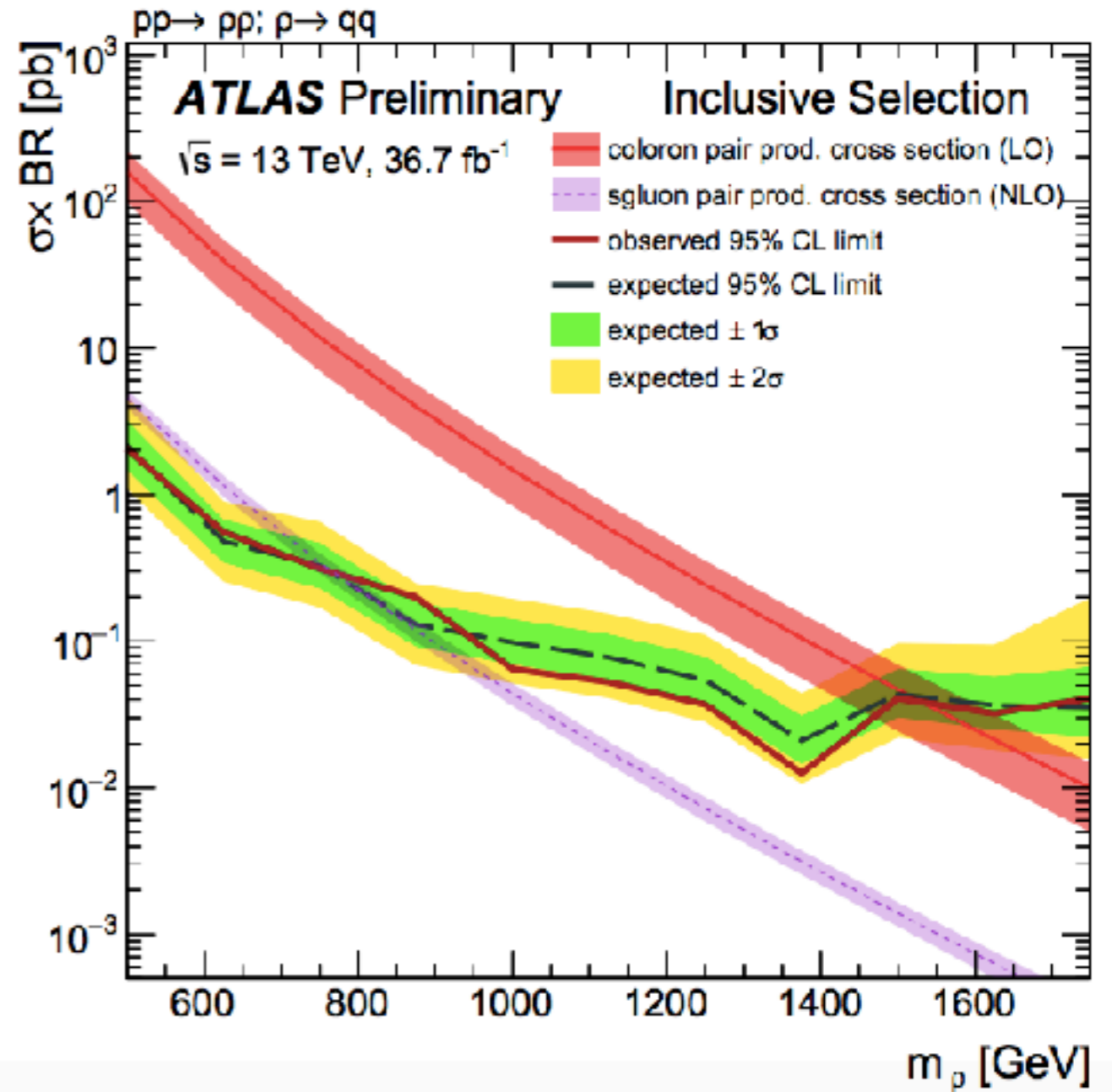
- ❖ Collider accessible states: QCD colored “pions”



- ❖ We have a bound on color octet scalars

$$m^2(\delta_c) \approx \frac{9}{4\pi} \alpha_{\text{QCD}} \Lambda_{\text{diag}}^2$$

$$\tilde{\Lambda} \lesssim 2.9 \text{ TeV}$$



Dynamical axions $N_F = 4$

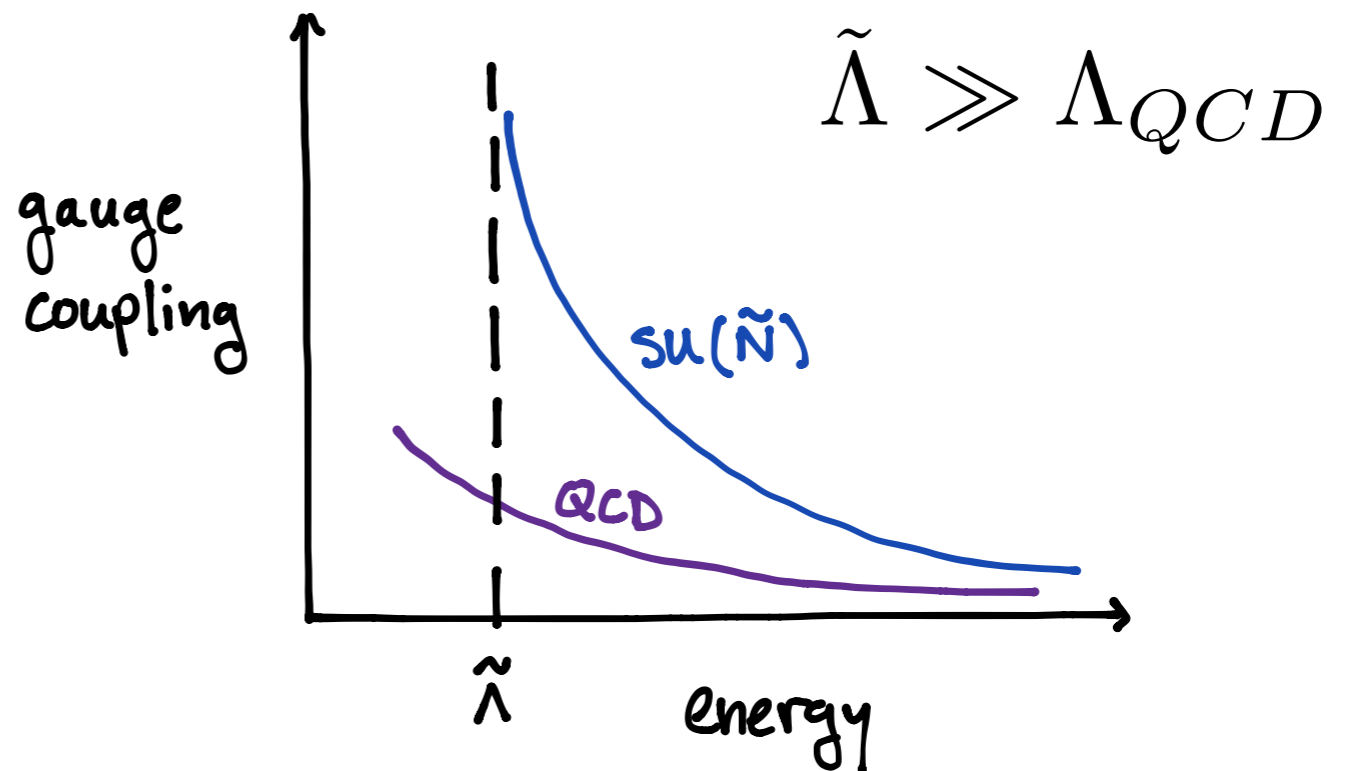
Massless quark content K. Choi, J. E. Kim (1985)

	$SU(3)_{QCD}$	$SU(\tilde{N})$
ψ	\square	\square
χ	$\underline{1}$	\square

- Require two massless quarks to independently rotate away both:

$$\theta_{QCD} \text{ \& \ } \tilde{\theta}$$

- Add an exotic confining gauge group, not related to QCD by a symmetry



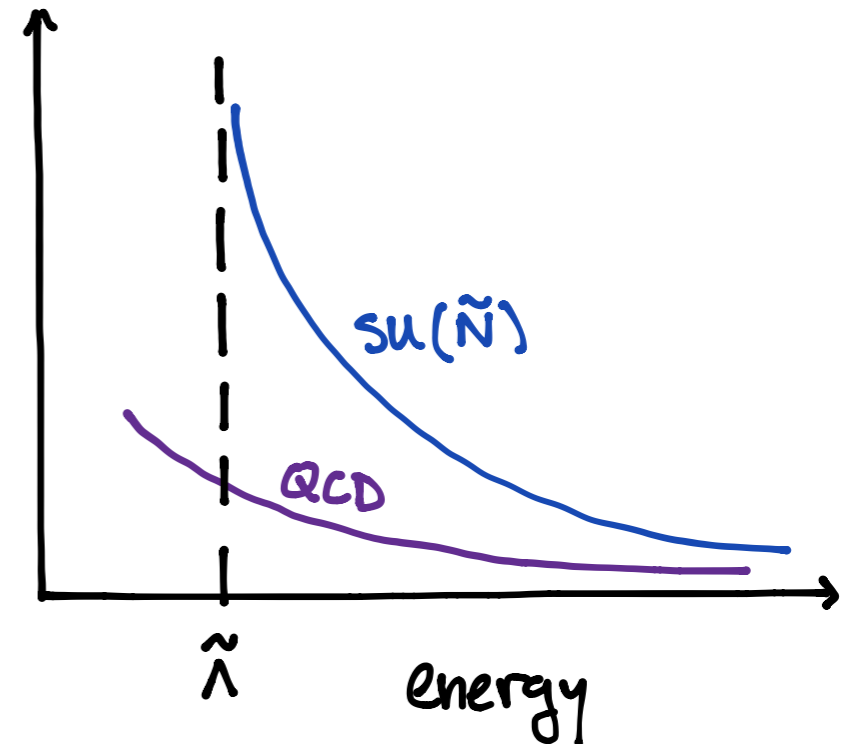
Dynamical axions $N_F = 4$

Massless quark content

K. Choi, J. E. Kim (1985)

	$SU(3)_{\text{QCD}}$	$SU(\tilde{N})$
ψ	\square	\square
χ	$\underline{1}$	\square

gauge
coupling



- ❖ Chiral symmetry breaking pattern at confinement:

$$SU(4)_L \times SU(4)_R \times U(1)_L \times U(1)_R \rightarrow SU(4)_V \times U(1)_V$$

- ➔ Bound states composed of massless quarks live at $\tilde{\Lambda}$
- ➔ Possible light pNGB states

Berezhiani, Gianfagna,
Giannotti, hep-ph/0009290

SU(\tilde{N}) confinement

Spontaneous symmetry breaking: $U(4)_L \times U(4)_R \rightarrow U(4)_V$

❖ Resulting Goldstone Bosons: $15 + 1 \rightarrow 8_c + 3_c + \bar{3}_c + 1_c + 1_c$
 $= \pi_8 + \pi_3 + \bar{\pi}_3 + \eta'_\psi + \eta'_\chi$

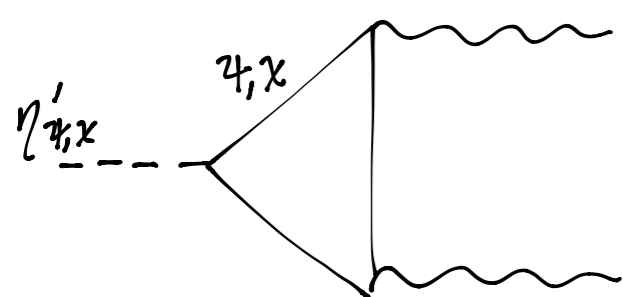
Explicit symmetry breaking effects:

(1) QCD explicitly breaks $SU(4)_V$



$$m^2(\pi_8, \pi_3) \sim \tilde{\Lambda}^2$$

(2) $G\tilde{G}$ explicitly breaks $U(1)_A$

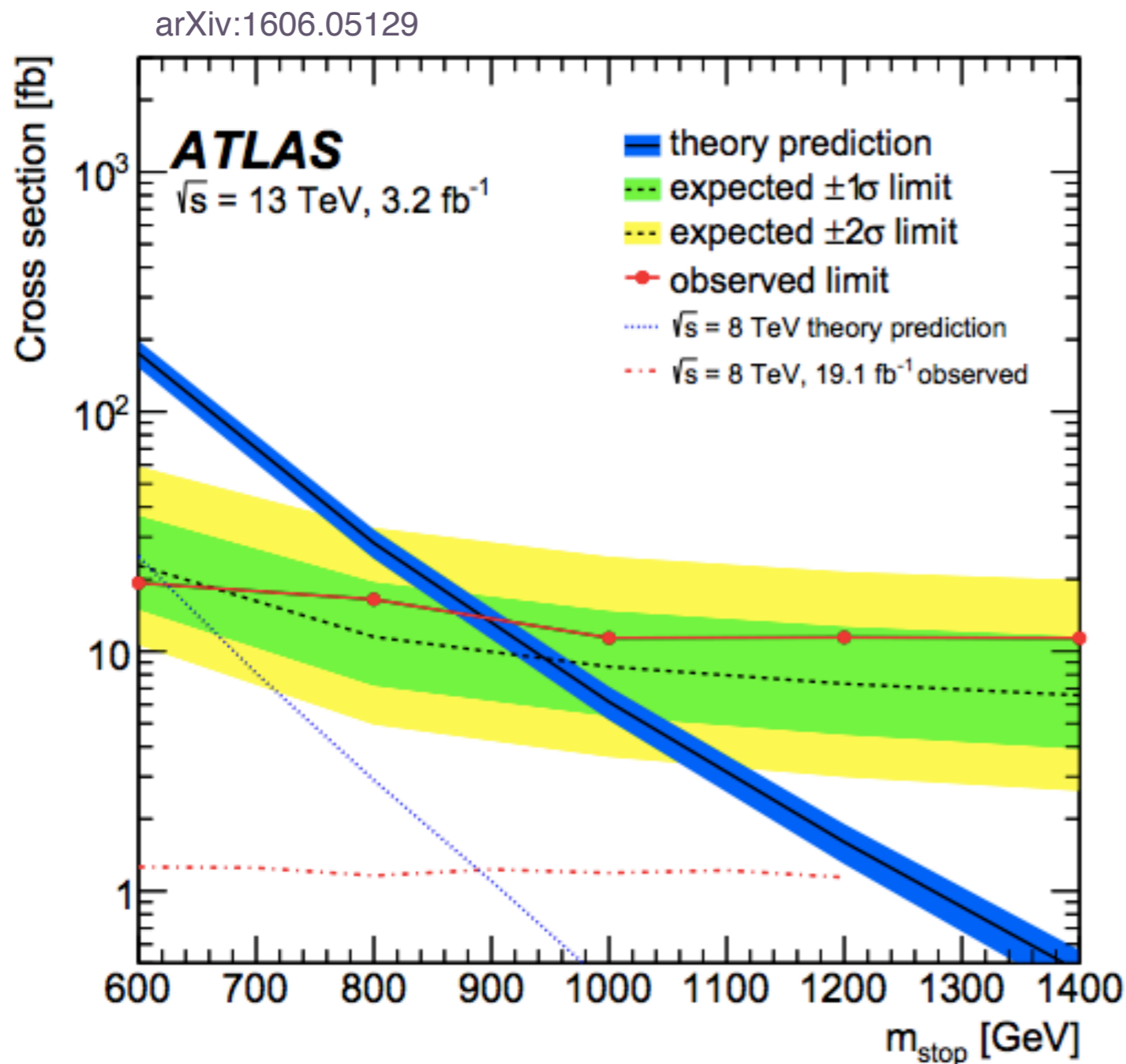


$$m^2(\eta'_1) \sim \tilde{\Lambda}^2$$

$$m(\eta'_2) \tilde{f} \sim m_\pi f_\pi$$

- ❖ The anomaly only gives mass to one η'
- ❖ The light eta is an **invisible dynamical axion**

Collider phenomenology: R-hadron searches



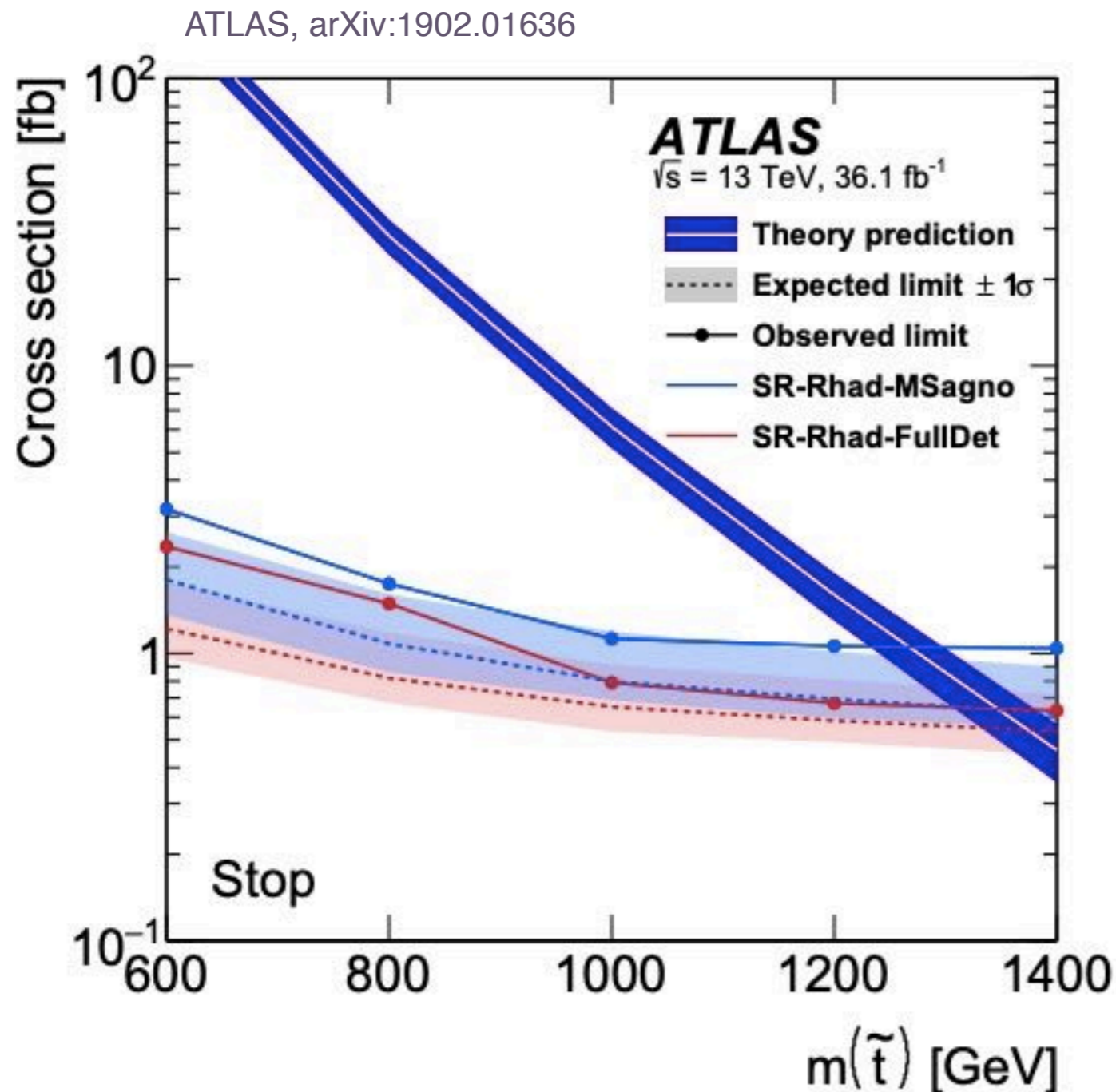
❖ We have a bound on color triplet scalars

$$m(\pi_d) \gtrsim 890 \text{ GeV}$$

$$m^2(3_c) \approx \frac{\alpha_c}{\pi} \Lambda_{\text{diag}}$$

$$\Lambda_{\text{diag}} \gtrsim 3 \text{ TeV}$$

Collider Phenomenology: R-Hadron Searches



- ❖ We have an updated bound on color triplet scalars

$$m(\pi_d) \gtrsim 1345 \text{ GeV}$$

$$m^2(3_c) \approx \frac{\alpha_c}{\pi} \Lambda_{\text{diag}}^2$$

$$\Lambda_{\text{diag}} \gtrsim 7 \text{ TeV}$$

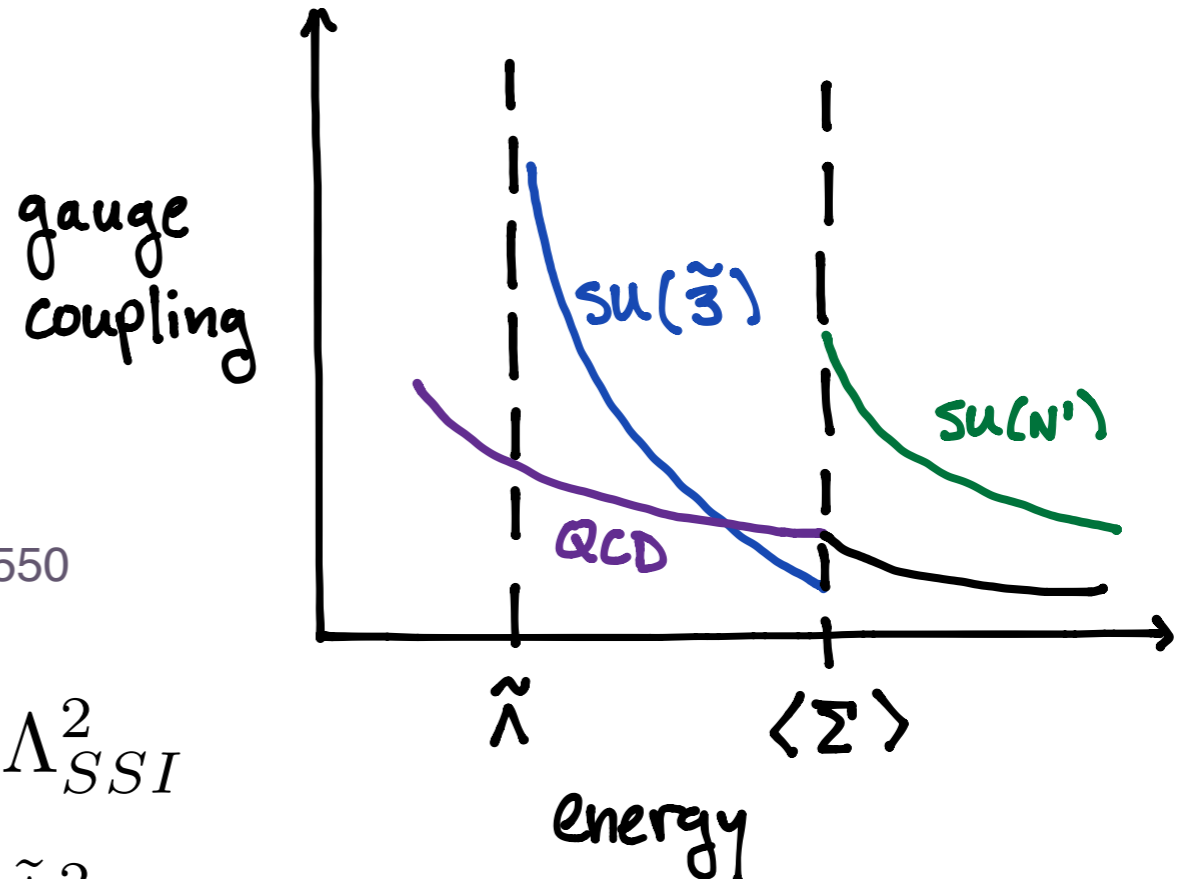
Visible axion models $N_F = 4$

- ❖ New physics at high energies can induce sizable instanton corrections to the axion mass

P. Agrawal, K. Howe arXiv:1710.04213, 1712.05803

J. Fuentes-Martin, M. Reig, A. Vicente, arXiv:1907.02550

$$\begin{array}{ccc}
 m^2(\eta'_1) \sim \tilde{\Lambda}^2 & & m^2(\eta'_1) \sim \Lambda_{SSI}^2 \\
 m(\eta'_2) \sim m_\pi \frac{f_\pi}{\tilde{f}} & \rightarrow & m^2(\eta'_2) \sim \tilde{\Lambda}^2
 \end{array}$$



MK Gaillard, B. Gavela, RH, P. Quilez, R. del Rey, arXiv:1805.06465


- ❖ Possible to have a combination of anomalous effects raise the mass of the lightest η'

Color Unified Dynamical Axion



MK Gaillard, B. Gavela, RH,
P. Quilez, R. del Rey, arXiv:1805.06465

$$SU(6) \times SU(3') \xrightarrow{\Lambda_{\text{CUT}}} SU(3)_c \times SU(3)_{\text{diag}}$$

	SU(6)	SU(3')	SU(2) _L			SU(3)	SU(3) _{diag}	SU(2) _L
Ψ	20	1	1	Λ _{CUT} →	4	□	□̄	1
χ	1	□	1		χ	1	□	1
					24 _ν	1	1	1

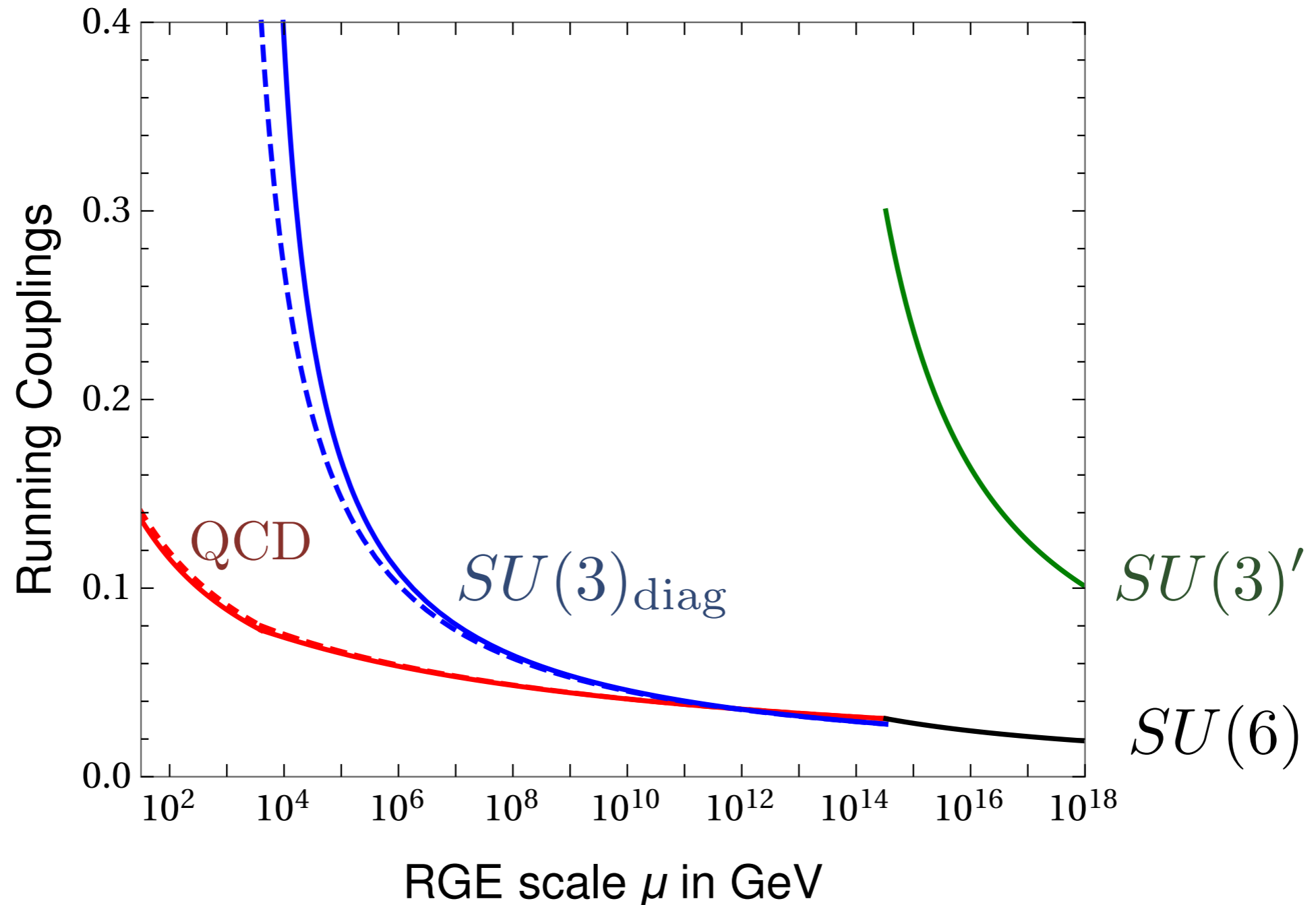


Small-sized instantons
charged under $SU(3)'$

Confining gauge groups

Color Unified Dynamical Axion



The CUDA Dynamical Axions

- ❖ The dynamical axions above and below Λ_{CUT}

$$\mathcal{L} \ni -\frac{\alpha_6}{8\pi} \frac{\sqrt{6}\eta'_\psi}{f_d} G_6 \tilde{G}_6 - \frac{\alpha'}{8\pi} \frac{2\eta'_\chi}{f_d} G' \tilde{G}' \quad \xrightarrow{\Lambda_{CUT}}$$

$$\mathcal{L} \ni -\frac{\alpha_c}{8\pi} \frac{\sqrt{6}\eta'_\psi}{f_d} G_{QCD} \tilde{G}_{QCD} - \frac{\alpha_{diag}}{8\pi} \left(\frac{2\eta'_\chi}{f_d} + \frac{\sqrt{6}\eta'_\psi}{f_d} \right) G_{diag} \tilde{G}_{diag}$$

- ❖ Mass contributions to the dynamical axions

$$\mathcal{L}_{eff} = \Lambda_{SSI}^4 \cos \left(\frac{2\eta'_\chi}{f_d} \right) + \Lambda_{diag}^4 \left(\frac{2\eta'_\chi}{f_d} + \frac{\sqrt{6}\eta'_\psi}{f_d} \right)$$

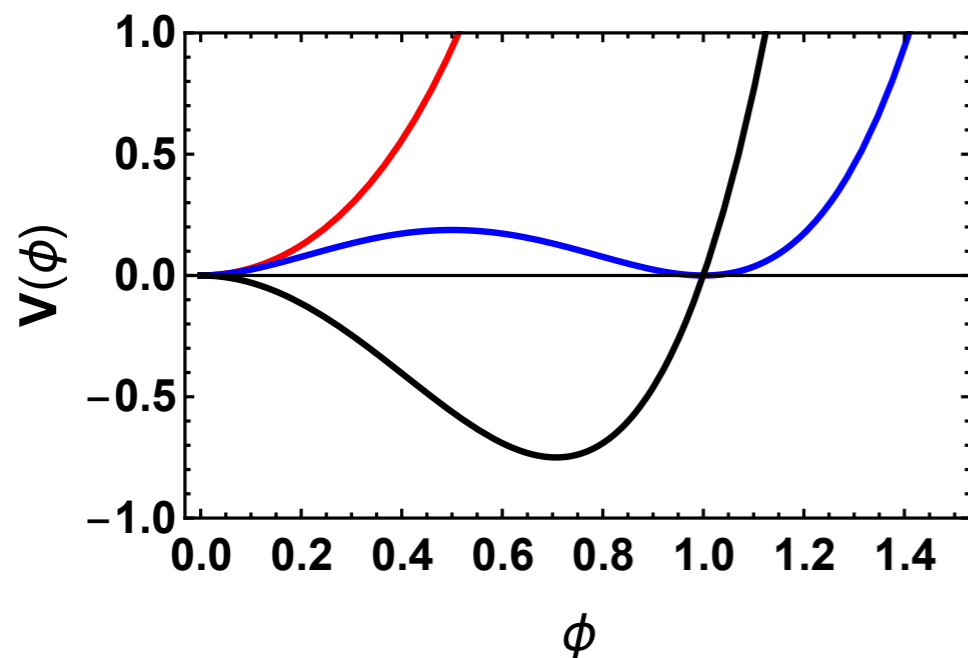
The Phase Transition

Phase transition at confinement

- ❖ Model the phase transition using the linear sigma model

$$V(\Sigma) = -m_\Sigma^2 \text{Tr}(\Sigma\Sigma^\dagger) + \frac{\lambda}{2} [\text{Tr}(\Sigma\Sigma^\dagger)]^2 + \frac{\kappa}{2} (\Sigma\Sigma^\dagger\Sigma\Sigma^\dagger),$$

- ❖ Spontaneous chiral symmetry breaking $\Sigma_{ij} \sim \langle \bar{\psi}_{Rj} \psi_{Li} \rangle$



$$\Sigma_{ij} = \frac{\varphi + i\eta'}{\sqrt{2N_F}} \delta_{ij} + X^a T_{ij}^a + i\pi^a T_{ij}^a$$

$$\langle \varphi \rangle = 0, \quad T \gg 0 \quad \text{Chiral symmetry restored}$$


$$\langle \varphi \rangle = f_\Sigma, \quad T \leq T_c \quad \text{Chiral symmetry}$$

Phase transition at confinement

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$$\Sigma_{ij} = \frac{\varphi + i\eta'}{\sqrt{2N_F}} \delta_{ij} + X^a T_{ij}^a + i\pi^a T_{ij}^a$$


Captures dynamics of the phase transition:

$$\langle \varphi \rangle = 0, \quad T \gg 0$$

Chiral symmetry restored

$$\langle \varphi \rangle = f_\Sigma, \quad T \leq T_c$$

~~Chiral symmetry~~

Phase transition at confinement

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Heavy fields corresponding to unbroken generators

$$\textcircled{\psi} \text{---} \textcircled{\psi} \sim \tilde{\Lambda}^2$$

Phase transition at confinement

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$$\Sigma_{ij} = \frac{\varphi + i\eta'}{\sqrt{2N_F}} \delta_{ij} + X^a T_{ij}^a + i\pi^a T_{ij}^a$$

(pseudo) Goldstone Boson fields

- ➔ Masses due explicit symmetry breaking effects:
QCD and $U(1)_A$

Phase transition at confinement

- ❖ Model the phase transition using the linear sigma model

$$V(\Sigma) = -m_\Sigma^2 \text{Tr}(\Sigma\Sigma^\dagger) + \frac{\lambda}{2} [\text{Tr}(\Sigma\Sigma^\dagger)]^2 + \frac{\kappa}{2} (\Sigma\Sigma^\dagger\Sigma\Sigma^\dagger),$$

- ❖ Goldstone boson masses

$$N_F = 3$$

$$m_{\pi^a}^2 = 0$$

$$m_{\eta'}^2 = 0$$

$$N_F = 4$$

$$m_{\pi_3}^2 = 0$$

$$m_{\pi_8}^2 = 0$$

$$m_{\eta'_\chi} = 0$$

$$m_{\eta'_\psi} = 0$$

Phase transition at confinement

- ❖ Symmetry breaking parameters μ_Σ , ξ , μ_{SSI} determine the masses of the pNGB's π and η'

$$V(\Sigma) = -m_\Sigma^2 \text{Tr}(\Sigma\Sigma^\dagger) + \frac{\lambda}{2} [\text{Tr}(\Sigma\Sigma^\dagger)]^2 + \frac{\kappa}{2} (\Sigma\Sigma^\dagger\Sigma\Sigma^\dagger),$$

- $-(\mu_\Sigma \det\Sigma + h.c.)$ ← Include the explicit $U(1)_A$ symmetry breaking from instanton effects
- $-\xi \text{Tr} Q^a \Sigma\Sigma^\dagger Q^{a\dagger}$ ← Include the explicit symmetry breaking from QCD charges
- $-\mu_{SSI} \text{Tr}(P_\chi \Sigma P_\chi \Sigma^\dagger P_\chi)$ ← Include the new mass contributions from small-sized instantons

Phase transition at confinement

- ❖ Model the phase transition using the linear sigma model

$$V(\Sigma) = -m_\Sigma^2 \text{Tr}(\Sigma\Sigma^\dagger) + \frac{\lambda}{2} [\text{Tr}(\Sigma\Sigma^\dagger)]^2 + \frac{\kappa}{2} (\Sigma\Sigma^\dagger\Sigma\Sigma^\dagger), \\ - (\mu_\Sigma \det\Sigma + h.c.) - \xi \text{Tr} Q^a \Sigma\Sigma^\dagger Q^{a\dagger}$$

- ❖ Goldstone boson masses

$$N_F = 3$$

$$N_F = 4$$

$$m_{\pi^a}^2 = 3\xi$$

$$m_{\pi_3} = \frac{4}{3}\xi$$

$$m_{\eta'_\psi}^2 = \frac{4m_\Sigma^2\mu_\Sigma}{\kappa + 4\lambda - \mu_\Sigma}$$

$$m_{\eta'}^2 = \frac{3\mu_\Sigma}{2\kappa + 6\lambda} \begin{pmatrix} \mu_\Sigma & \dots \end{pmatrix}$$

$$m_{\pi_8}^2 = 3\xi$$

$$m_{\eta'_\chi} = 0$$

Phase transition at confinement

- ❖ Model the phase transition using the linear sigma model

$$\begin{aligned}
 V(\Sigma) = & -m_\Sigma^2 \text{Tr} (\Sigma \Sigma^\dagger) + \frac{\lambda}{2} [\text{Tr} (\Sigma \Sigma^\dagger)]^2 + \frac{\kappa}{2} (\Sigma \Sigma^\dagger \Sigma \Sigma^\dagger), \\
 & - (\mu_\Sigma \det \Sigma + h.c.) - \xi \text{Tr} Q^a \Sigma \Sigma^\dagger Q^{a\dagger} \\
 & - \mu_{SSI} \text{Tr} (P_\chi \Sigma P_\chi \Sigma^\dagger P_\chi)
 \end{aligned}$$

$$N_F = 3$$

$$N_F = 4$$

$$m_{\pi^a}^2 = 3\xi$$

$$m_{\pi_3}^2 = \frac{4}{3}\xi + \frac{1}{4}\mu_{SSI}$$

$$m_{\eta'_\psi}^2 = \frac{(4m_\Sigma^2 - \mu_{SSI})\mu_\Sigma}{\kappa + 4\lambda - \mu_\Sigma}$$

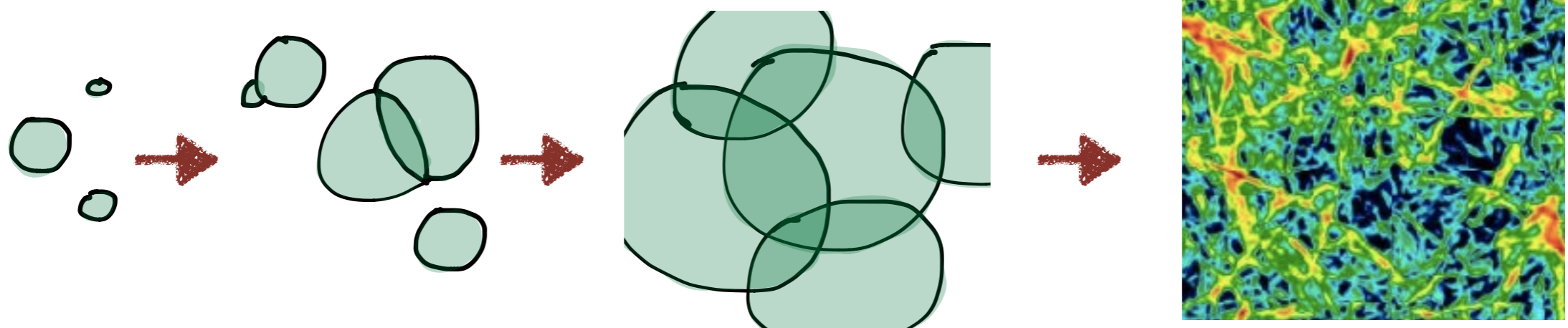
$$m_{\eta'}^2 = \frac{3\mu_\Sigma}{2\kappa + 6\lambda} \begin{pmatrix} \mu_\Sigma \dots \end{pmatrix}$$

$$m_{\pi_8}^2 = 3\xi + \frac{1}{4}\mu_{SSI}$$

$$m_{\eta'_\chi}^2 = \frac{1}{2}\mu_{SSI}$$

Phase transition in the early universe

- ❖ Dynamics from tunneling to the $T = 0$ vacuum



D. Weir, “The sound of gravitational waves from a [confinement] phase transition,” saoghal.net/slides/ectstar/

$$\Omega_{GW}|_{\text{peak}} = \Omega_{GW} \left(\alpha, \frac{\beta}{H} \right)$$

$\alpha =$ Latent heat, $\frac{\Delta\mathcal{L}}{\rho_{\text{rad}}}$

$\frac{\beta}{H} =$ Parameterizes speed of the phase transition

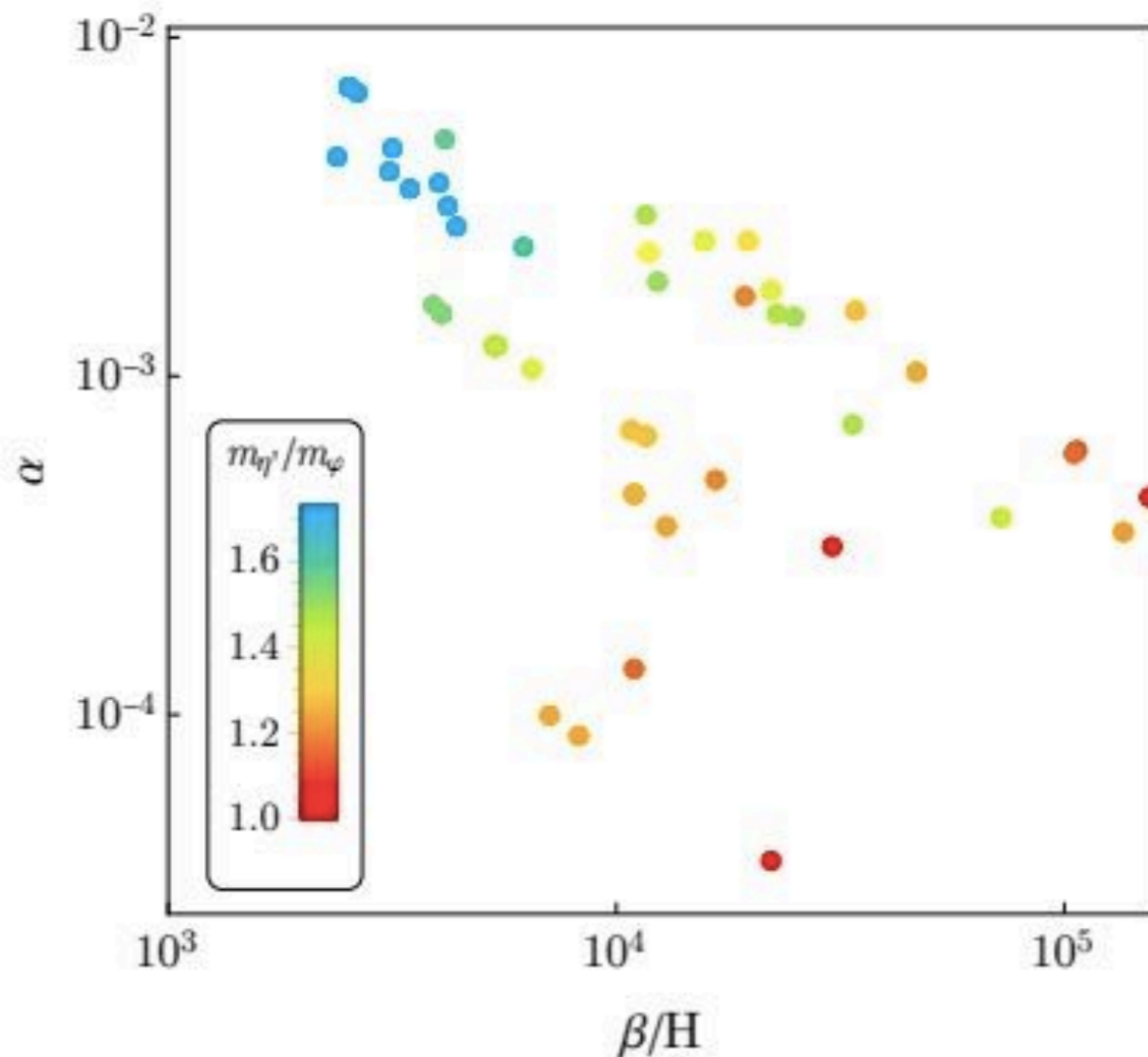
$$f_{GW}|_{\text{peak}} = f_{GW} \left(T_N, \frac{\beta}{H} \right)$$

$T_N =$ Nucleation temperature

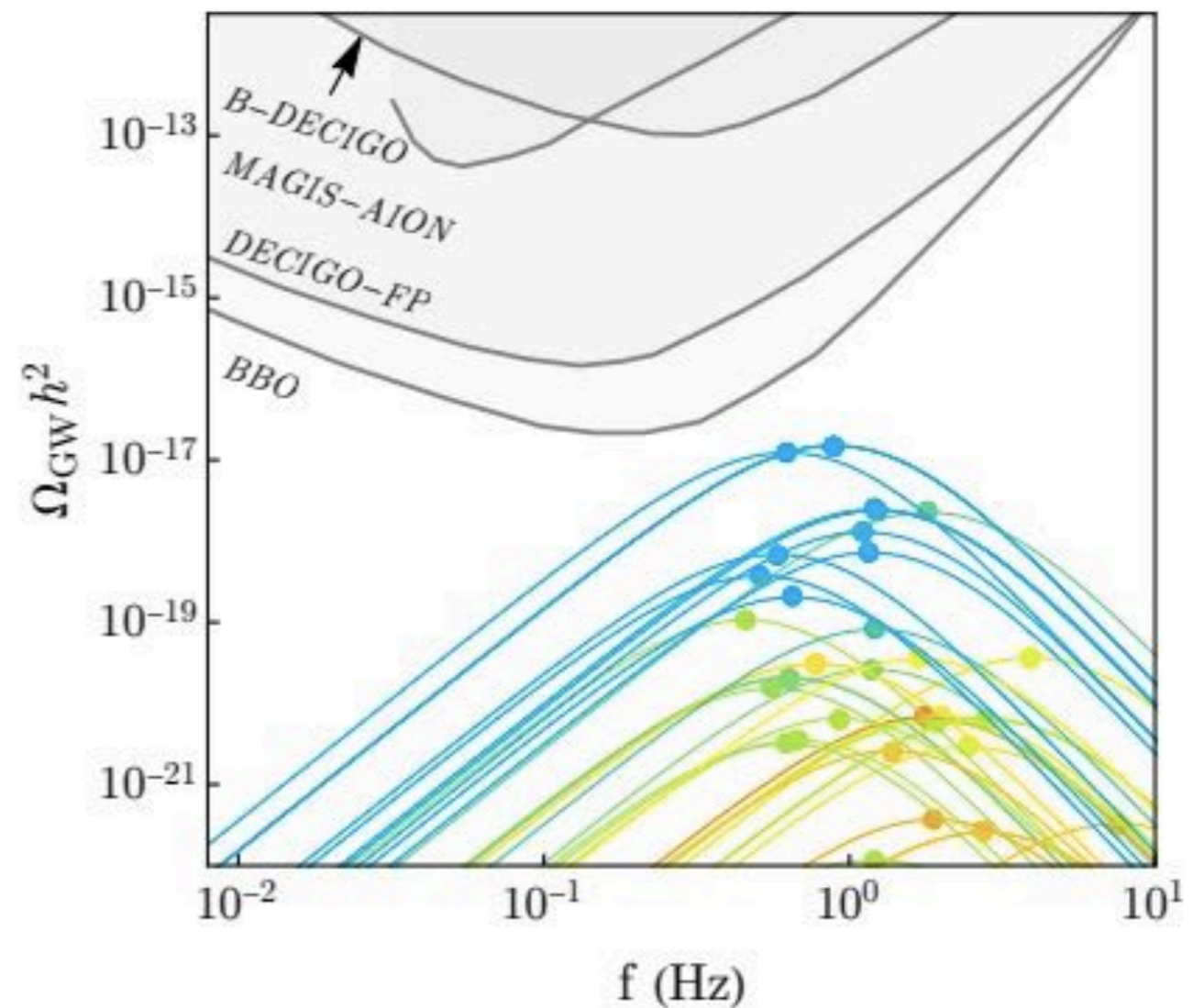
Gravitational wave signal $N_F = 3$

D. Croon, R. Houtz, V. Sanz, arXiv:1904.10967

Thermal parameters



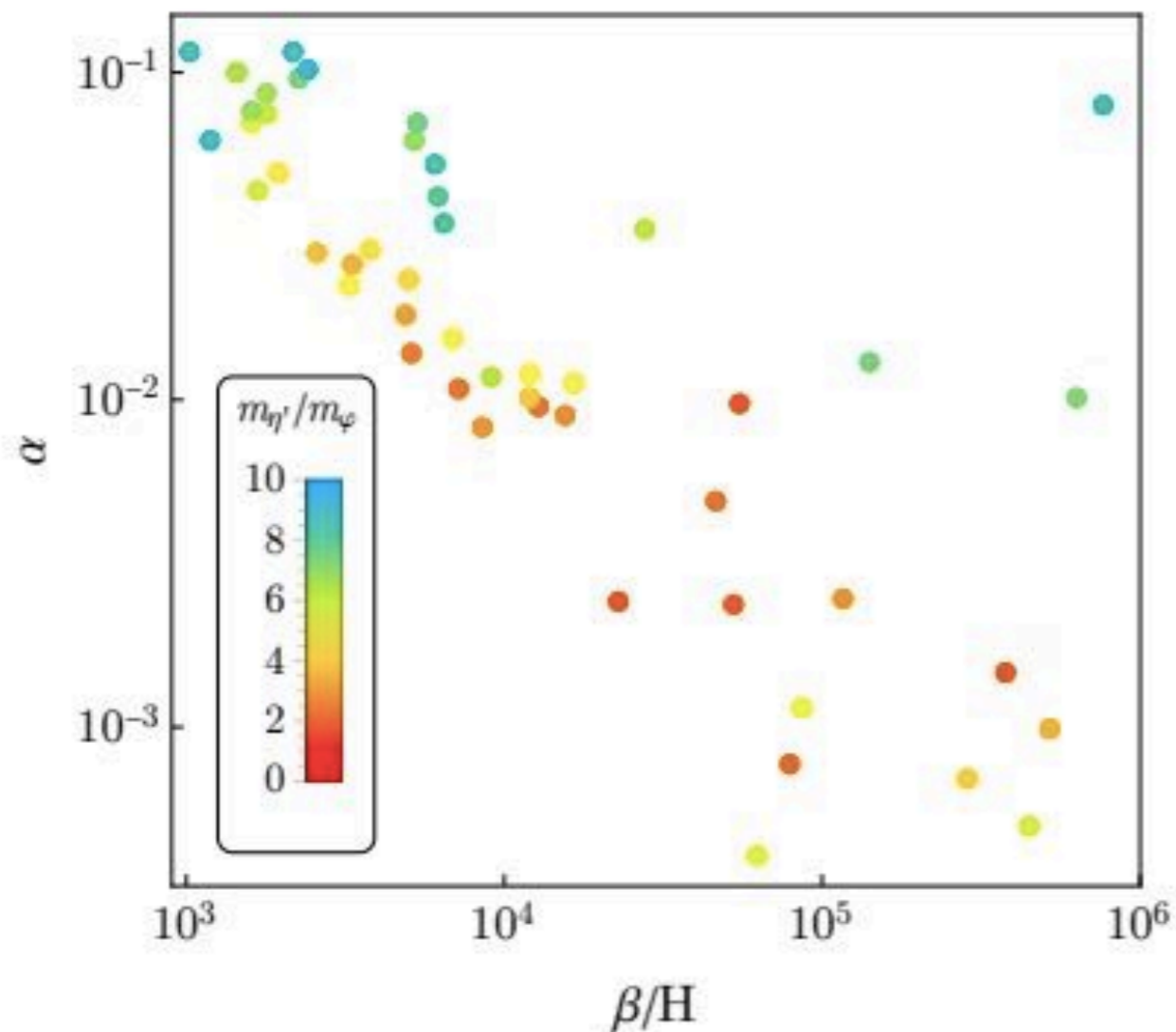
GW signal $\tilde{\Lambda} \sim 3$ TeV



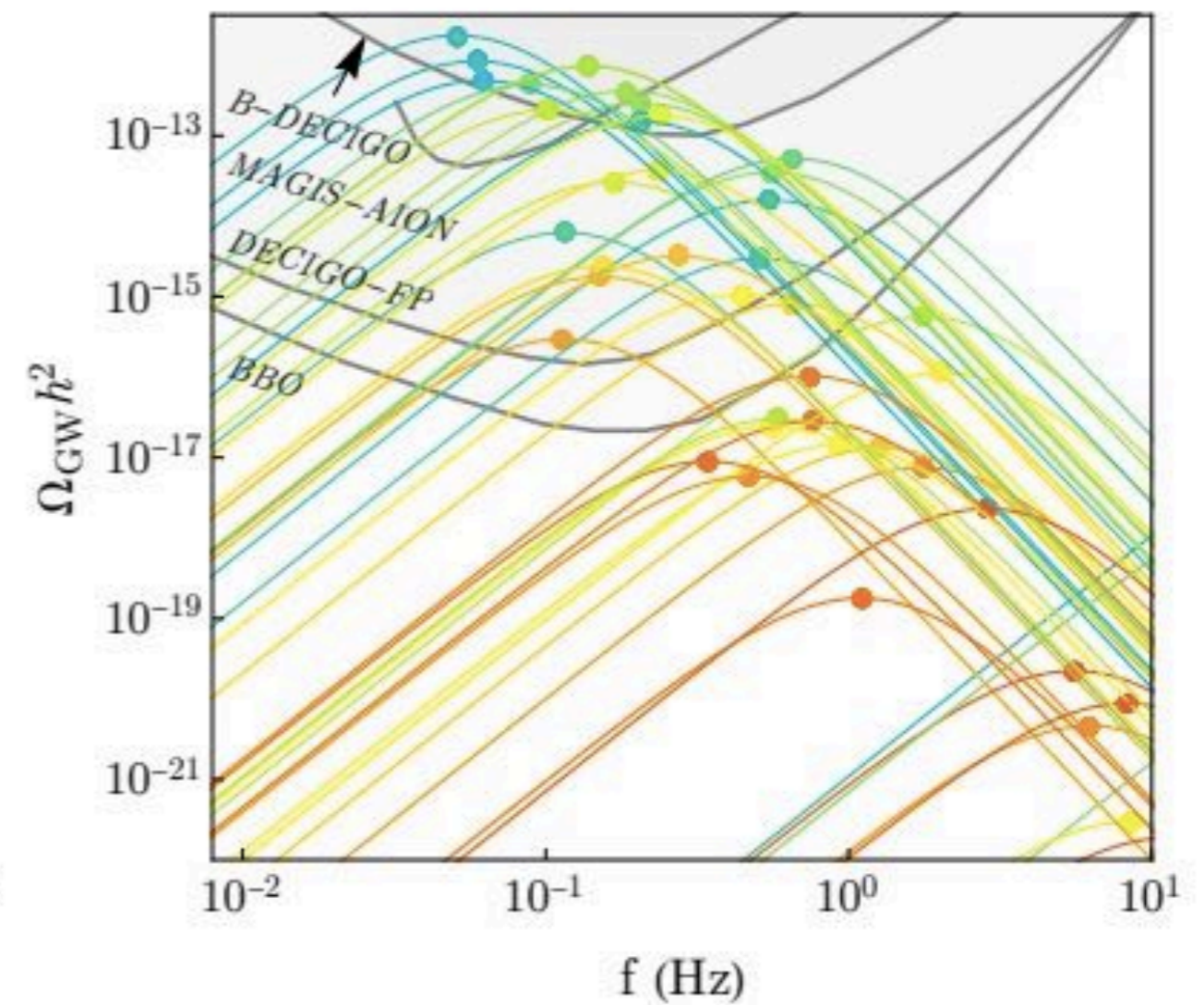
Gravitational wave signal $N_F = 4$

D. Croon, R. Houtz, V. Sanz, arXiv:1904.10967


Thermal parameters



GW signal $\tilde{\Lambda} \sim 3$ TeV



Summary

- ❖ Prospects for gravitational wave signals for dynamical axion models with $\tilde{\Lambda} \sim 3 \text{ TeV}$
- ❖ Features of dynamical axion models: 
 - Massless quark messenger between QCD and $SU(\tilde{N})$
 - At least three light flavors and a first order phase transition
 - Generic parameters in the theory below $\tilde{\Lambda}$, for example the exotic pion masses due to gluon interactions
- ❖ The gravitational wave signature is sensitive to the explicit $U(1)_A$ breaking parameter μ_Σ in the linear sigma model

Temperature dependence of μ_Σ

- ❖ The size of the $\mu_\Sigma \det \Sigma$ term is important for the GW signal

$$m_{\eta'} \sim \mu_\Sigma$$

- ❖ Explicit $U(1)_A$ breaking comes from instantons G. 't Hooft (1976)

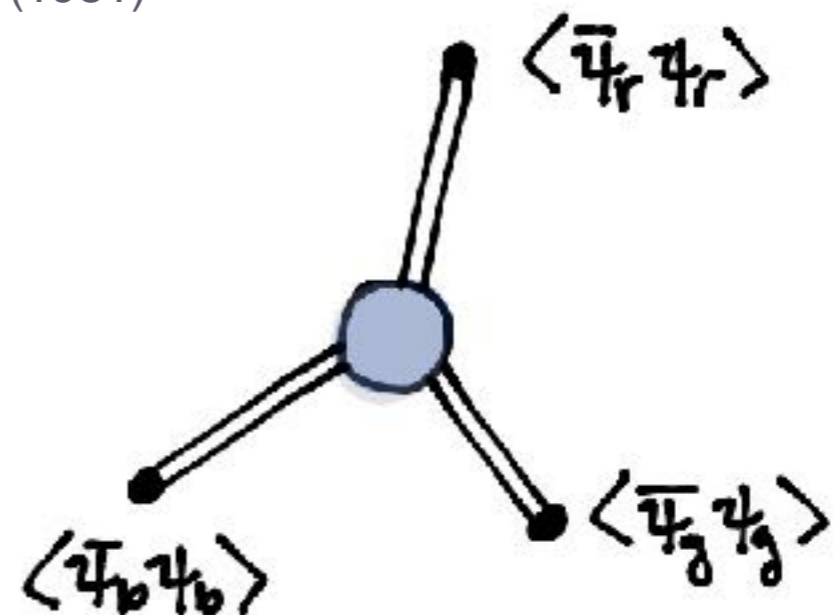
- ❖ When $T \gg T_c$ and g_{QCD} is perturbative, the dilute instanton gas approximation holds Gross, Pisarski, Yaffe, (1981)

- ❖ For example, when the axion mass is lifted by small size instanton effects:

$$m_{\eta'} \sim \frac{\Lambda_{SSI}^2}{\tilde{f}} \sim \tilde{f}^6 \int d\rho \rho^4 d(\rho, T)$$

M. A. Shifman, A. I. Vainshtein, V. I. Zakharov (1980)

Callan, Dashen, Gross, (1978)



Temperature dependence of μ_Σ

- ❖ A related parameter is the topological susceptibility

$$\chi(T) = \int \frac{d\rho}{\rho^5} d(\rho, T) \quad \rightarrow \quad m_{\eta'} \sim \tilde{f}^6 \int d\rho \rho^4 d(\rho, T)$$

D. Croon, R. Houtz, V. Sanz, arXiv:1904.10967

- ❖ Studied by the lattice community

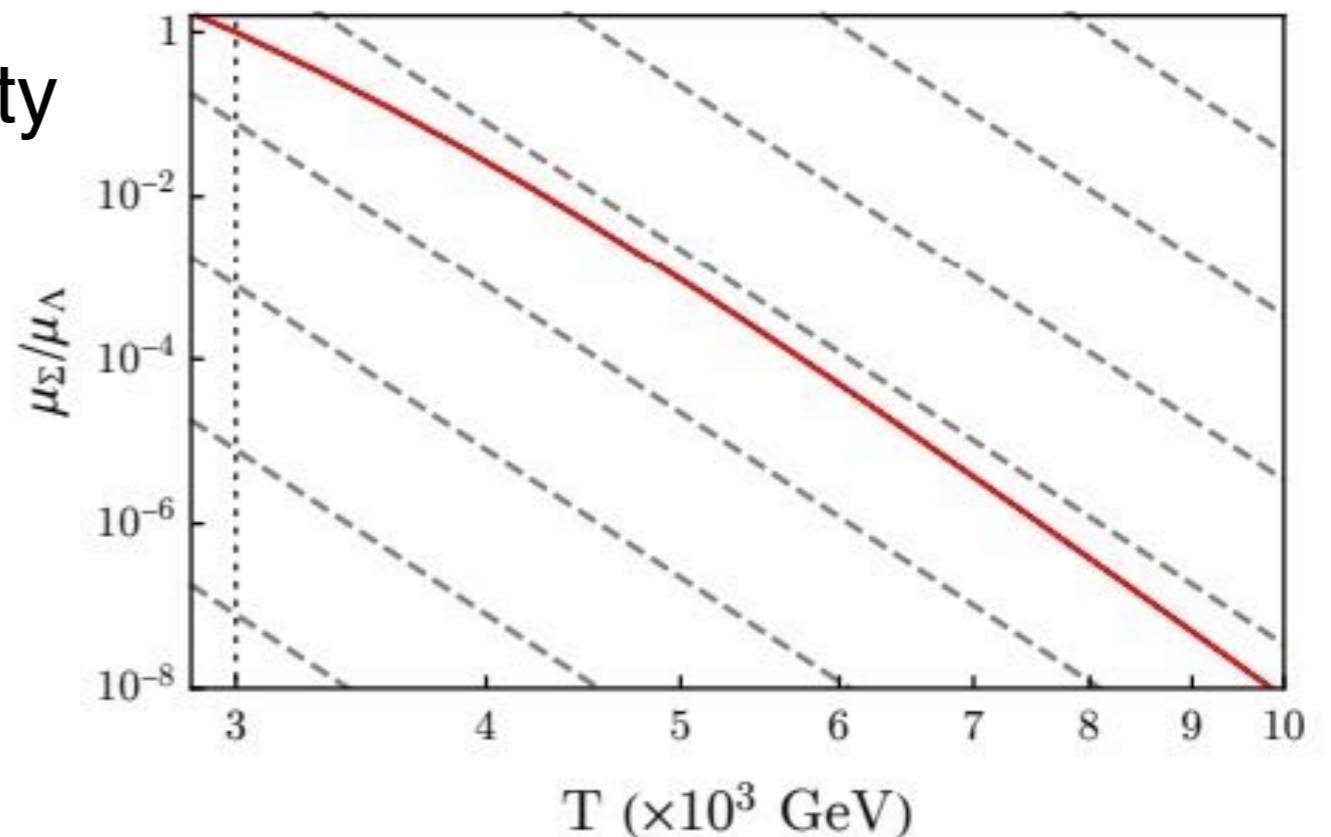
$$\chi(T) \sim T^{-8} \quad \rightarrow$$

J. Frison, R. Kitano, H. Matsufuru, S. Mori, N. Yamada, arXiv:1606.07175

M. Dine, P. Draper, L. Stephenson-Haskins, D. Xu, arXiv:1705.00676

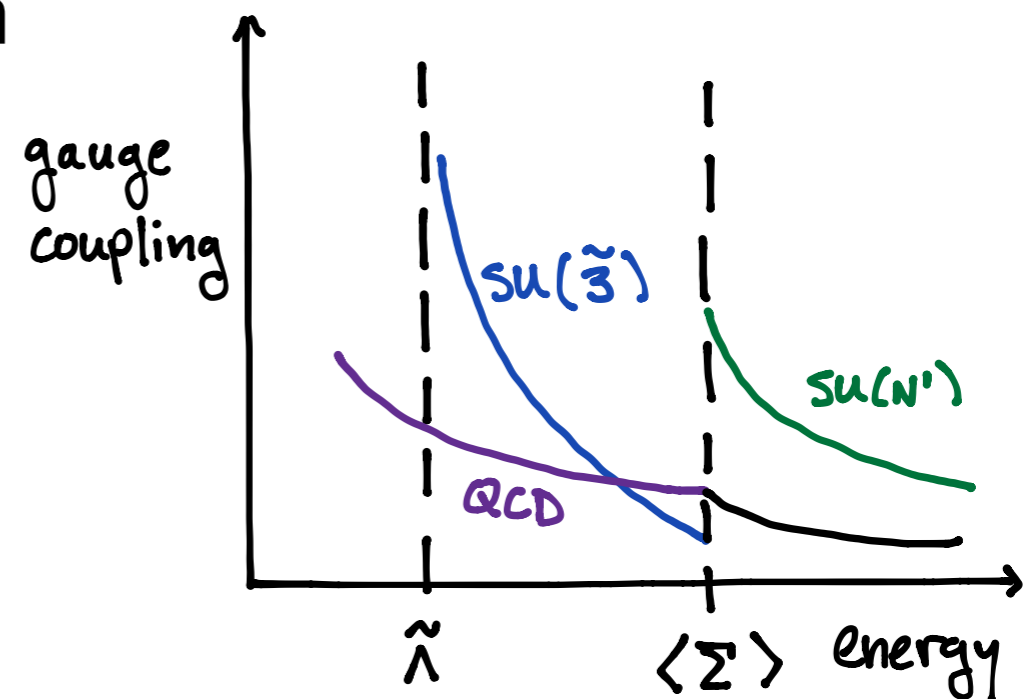
- ❖ DGA breaks down near confinement

- ❖ Does μ_Σ contribute significantly during the phase transition?



Conclusions

- ❖ Extra color groups provide a window to richer phenomenology
- ❖ Gravitational waves can probe exotic confining color groups
- ❖ Visible axions $m_a^2 f_a^2 \approx m_\pi^2 f_\pi^2 \longrightarrow + \sim \Lambda_{\text{new}}^4$
- ❖ A dynamical axion made of massless quarks is a viable solution to the strong CP problem
- ❖ GW signal favors models where high energy effects of extra color groups provide another source of axion mass

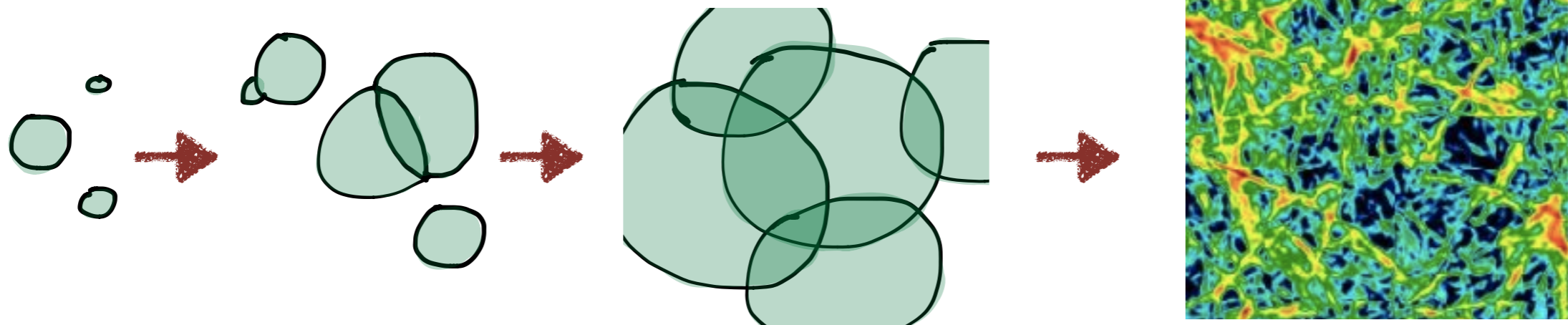


Thank you!

Back-up slides

Phase transition in the early universe

- ❖ Dynamics from tunneling to the $T = 0$ vacuum



- ❖ Speed of the phase transition:

$$\frac{\beta}{H} \sim T \left. \frac{d(S_E/T)}{dT} \right|_{T=T_N}$$

- ❖ Latent heat of the phase transition:

$$\alpha \sim \frac{1}{\rho_N} \left(\Delta V - \frac{T}{4} \Delta \frac{dV}{dT} \right) \Big|_{T=T_N}$$

D. Weir, "The sound of gravitational waves from a [confinement] phase transition," saoghal.net/slides/ectstar/

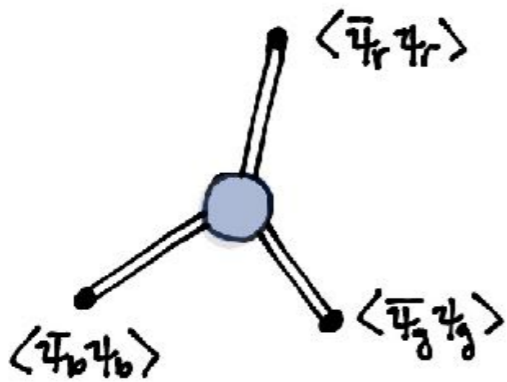
- ❖ The GW spectrum:

$$\Omega_{GW}(\alpha, \beta, f)$$

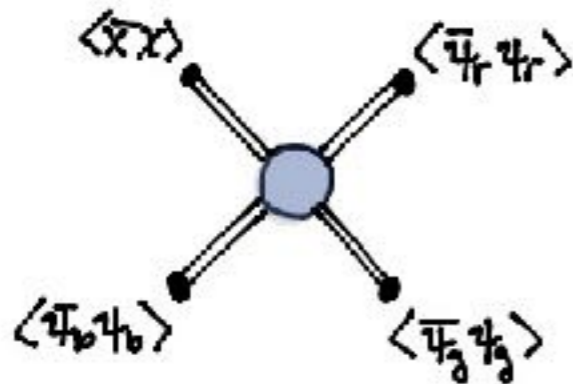
Small size instantons with fermions

- ❖ Adding fermion effects gives an instanton suppression

$$d(\rho, T) = C_{inst} \left(\frac{2\pi}{\alpha(1/\rho)} \right)^6 e^{-\frac{2\pi}{\alpha(1/\rho)}}$$



$$\Lambda_{SSI}^4 = \int \frac{d\rho}{\rho^5} d(\rho, T) \left(\frac{2}{3} \pi^2 \rho^3 \langle \bar{\psi} \psi \rangle \right)^3$$



$$\Lambda_{SSI}^4 = \int \frac{d\rho}{\rho^5} d(\rho, T) \left(\frac{2}{3} \pi^2 \rho^3 \langle \bar{\psi} \psi \rangle \right)^4$$

$$\mathcal{L}_{eff} = \Lambda_{SSI}^4 \cos \left(2 \frac{\eta'_\chi}{f_d} \right) + \Lambda_{diag}^4 \cos \left(2 \frac{\eta'_\chi}{f_d} + \sqrt{6} \frac{\eta'_\psi}{f_d} \right) + \Lambda_{QCD}^4 \cos \left(\sqrt{6} \frac{\eta'_\psi}{f_d} \right)$$

The pseudoscalar mass matrix

The $SU(3)'$
instanton
contribution

The $SU(3)_{\text{diag}}$
instanton
contribution

$$M_{\eta'_\chi, \eta'_\psi, \eta'_{\text{QCD}}}^2 = \begin{pmatrix} 4 \frac{(\Lambda_{SSI}^4 + \Lambda_{\text{diag}}^4)}{f_d^2} & 2\sqrt{6} \frac{\Lambda_{\text{diag}}^4}{f_d^2} & 0 \\ 2\sqrt{6} \frac{\Lambda_{\text{diag}}^4}{f_d^2} & 6 \frac{(\Lambda_{\text{diag}}^4 + \Lambda_{\text{QCD}}^4)}{f_d^2} & 2\sqrt{6} \frac{\Lambda_{\text{QCD}}^4}{f_\pi f_d} \\ 0 & 2\sqrt{6} \frac{\Lambda_{\text{QCD}}^4}{f_\pi f_d} & 4 \frac{\Lambda_{\text{QCD}}^4}{f_\pi^2} \end{pmatrix}$$

$$\Lambda_{SSI} \lesssim 10^4 \text{ TeV}$$

$$\Lambda_{\text{diag}} \sim \text{few TeV}$$

$O(1)$ prime Yukawa couplings

