(T8)

QM 115B Lec 18

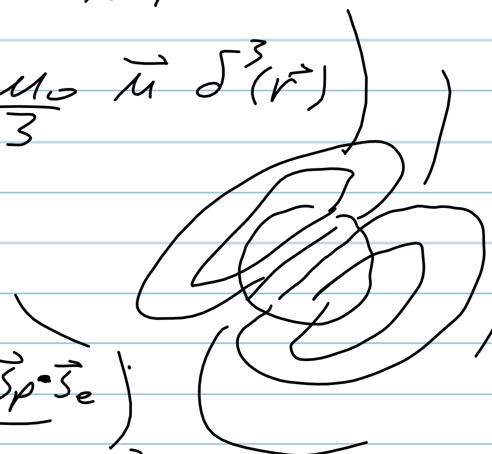
Hyperfine Splitting

$$\vec{\mu}_e = -\frac{g_e e}{2m_e}\vec{S}_e \qquad \vec{\mu}_p = \frac{g_p e}{2m_p}\vec{S}_p$$

$$g_e \approx 2 \qquad g_p = 5.59$$

$$\vec{B}_p = \frac{\mu_0}{4\pi r^3}\left(3(\vec{\mu}\cdot\hat{r})\hat{r} - \vec{\mu}\right) + \frac{2\mu_0}{3}\vec{\mu}\,\delta^3(\vec{r})$$

$$H'_{hf} = -\vec{\mu}_e \cdot \vec{B}_p$$



$$H'_{hf} = \frac{\mu_0 g_p e^2}{8\pi m_p m_e}\left(\frac{3(\vec{S}_p\cdot\hat{r})(\vec{S}_e\cdot\hat{r}) - \vec{S}_p\cdot\vec{S}_e}{r^3}\right) + \frac{\mu_0 g_p e^2}{3m}\vec{S}_p\cdot\vec{S}_e\,\delta^3(\vec{r})$$

$$E'_{hf} = \frac{\mu_0 g_p e^2}{8\pi m_p m_e}\left\langle \frac{3(\vec{S}_p\cdot\hat{r})(\vec{S}_e\cdot\hat{r}) - \vec{S}_p\cdot\vec{S}_e}{r^3}\right\rangle$$

$= 0$ for $l=0$ Prob 6.27

$$+ \frac{\mu_0 g_p e^2}{3 m_p m_e}\langle \vec{S}_p\cdot\vec{S}_e\rangle |\psi_{100}(0)|^2, \qquad |\psi_{100}(0)|^2 = \frac{1}{\pi a^3}$$

$$\vec{S} = \vec{S}_p + \vec{S}_e \qquad S^2 = S_p^2 + S_e^2 + 2\vec{S}_p\cdot\vec{S}_e$$

$$\vec{S}_p\cdot\vec{S}_e = \frac{1}{2}(S^2 - S_p^2 - S_e^2) = \frac{1}{2}\left(S^2 - \frac{3}{4}\hbar^2 - \frac{3}{4}\hbar^2\right)$$

$$E'_{hf} = \frac{4}{3} g_p \alpha^4 \frac{m_e^2 c^2}{m_p \hbar^2}\begin{cases}\hbar^2\frac{1}{2}\left(1(1+1) - \frac{3}{2}\right) & \text{triplet}\\ \hbar^2\frac{1}{2}\left(0 - \frac{3}{2}\right) & \text{singlet}\end{cases}$$

$$= \alpha^4 m_e c^2\left(\frac{4}{3} g_p \frac{m_e}{m_p}\right)\begin{cases}\frac{1}{4} & \text{triplet}\\ -\frac{3}{4} & \text{singlet}\end{cases}$$

$$\Delta E = 5.88 \times 10^{-6}\ \text{eV}$$

Stark Effect

$\vec{\mathcal{E}} = \mathcal{E}\hat{z}$ $\qquad H' = e\mathcal{E}z$

find first order shifts in $n=1$ $n=2$ levels

$n=1$ groundstate $\quad l=0 \quad m=0$
non-degenerate

$$E'_{100} = \langle 100 | H' | 100 \rangle$$

$$= e\mathcal{E} \langle 100 | z | 100 \rangle$$

$$= e\mathcal{E} \int \psi^*_{100} \, z \, \psi_{100} \, d^3\vec{r} = 0$$

Parity $\quad z \to -z$

$$|\psi_{100}|^2 \to |\psi_{100}|^2$$

Proof: $\quad Y_0^0 = \frac{1}{\sqrt{4\pi}} \qquad z = r\cos\theta$

$$\int_0^{2\pi}\int_0^{\pi} |Y_0^0|^2 \cos\theta \sin\theta \, d\theta \, d\phi = \frac{2\pi}{4\pi}\int_0^{\pi} \cos\theta \sin\theta \, d\theta$$

$x = \cos\theta$
$dx = -\sin\theta$

$$= \frac{1}{2}\int_{-1}^{1} (-x)\, dx$$

$$= \frac{1}{2}\left[\frac{x^2}{2}\right]_{-1}^{1}$$

$$= 0$$

$n=2 \qquad \psi_{200} \quad \psi_{211} \quad \psi_{210} \quad \psi_{21-1}$

4-fold degeneracy

$\langle 2\, l' m' | z | 2\, l m \rangle \qquad z = r\cos\theta$

z independent of ϕ

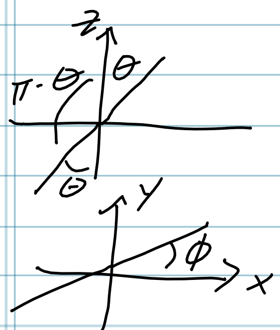
16 3D integrals

$$\int Y_l^{m'} Y_l^{m} f(\theta) \sin\theta \, d\theta \, d\phi \propto \delta_{mm'}$$

$P \quad \vec{r} \to -\vec{r}$

$\quad z \to -z$

$$P\, \psi_{nl'm}\, \psi_{nlm} = -\psi_{nl'm}\, \psi_{nlm}$$



$P \quad r \to r$

$\quad \theta \to \pi - \theta$

$\quad \phi \to \phi + \pi$

Y_0^0 even

$$Y_1^0 = c\cos\theta \xrightarrow{P} c\cos(\pi-\theta) = -c\cos\theta$$

$$Y_1^{\pm 1} = c\sin\theta\, e^{\pm i\phi} \to c\sin(\pi-\theta)\, e^{\pm i(\pi+\phi)}$$

$$= c\sin\theta\, e^{\pm i\pi} e^{\pm i\phi}$$

$$= -Y_1^{\pm 1}$$

$$Y_l^m \to (-1)^l Y_l^m$$

only $\langle 200 | z | 210 \rangle = \langle 210 | z | 200 \rangle^* \neq 0$

$$\langle 200 | z | 210 \rangle = \int_0^\infty r^2 R_{20}^*(r) R_{21}(r)\, r\, dr \int \cos\theta\, Y_0^{0*} Y_1^0 \, d\Omega$$

$$= \int_0^\infty r^3 \frac{1}{\sqrt{2a^3}} \left(1 - \frac{1}{2a} r\right) e^{-r/2a} \frac{1}{\sqrt{24a^3}} \frac{r}{a} e^{-r/2a} dr \int_0^{2\pi}\int_0^{\pi} \frac{\cos\theta}{\sqrt{4\pi}} \sqrt{\frac{3}{4\pi}} \cos\theta \sin\theta \, d\theta$$

$$= \frac{1}{\sqrt{48}} \int_0^\infty \left(\frac{r^4}{a^4} - \frac{r^5}{2a^5}\right) e^{-r/a} dr \; \frac{2\pi\sqrt{3}}{4\pi} \int_0^\pi \cos^2\theta \sin\theta \, d\theta$$

(81) $\int_0^\infty dy\, y^m e^{-by} = \frac{m!}{b^{m+1}}$

$y = r/a$ $\qquad x = \cos\theta$

$$\langle 200|z|210\rangle = \frac{a}{\sqrt{48}} \int_0^\infty \left(y^4 - \frac{y^5}{2}\right) e^{-y} dy \frac{\sqrt{3}}{2} \int_{-1}^{1} dx\, x \cdot x$$

$$= \frac{a}{\sqrt{48}} \left(4! - \frac{5!}{2}\right) \frac{\sqrt{3}}{2} \left[\frac{x^3}{3}\right]_{-1}^{1}$$

$$= \frac{a}{\sqrt{48}} (24 - 60) \frac{1}{\sqrt{3}} = \frac{-36a}{4 \cdot 3} = -3a$$

$$W = -3ae\mathcal{E} \begin{array}{c} \begin{array}{cccc} 200 & 211 & 210 & 21-1 \end{array} \\ \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

$$\begin{vmatrix} -\lambda & 1 \\ 1 & -\lambda \end{vmatrix} = \lambda^2 - 1 \qquad \lambda \pm 1 \qquad \lambda = 0$$

eigenvec. $\lambda = 0$ $\qquad \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$

$\lambda = 1 \quad \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix} \qquad \lambda = -1 \quad \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ -\frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}$

$$|\psi_2\rangle_A = \frac{1}{\sqrt{2}}\left(|200\rangle + |210\rangle\right) \qquad E_{2A} = -\frac{mc^2\alpha^2}{8} - 3e\varepsilon$$

$$|\psi_2\rangle_B = |211\rangle$$

$$|\psi_2\rangle_C = |21\,{-1}\rangle \qquad E_{2C} = E_{2B} = -\frac{mc^2\alpha^2}{8}$$

$$|\psi_2\rangle_D = \frac{1}{\sqrt{2}}\left(|200\rangle - |210\rangle\right) \qquad E_{2D} = -\frac{mc^2\alpha^2}{8} + 3e$$